Resonant Rotation of the Two-layer Mercury Model

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Present significance of the study of rotation of Mercury considered as a core-mantle system arises from planned Mercury missions. New high accurate data on Mercury's structure and its physical fields are expected from BepiColombo mission (Anselmi et al., 2001). Investigation of resonant rotation of Mercury, begun by Colombo G. (1966), will play here main part.

New approach to the study of Mercury dynamics and the construction of analytical theory of its resonant rotation is suggested. Within these approach Mercury is considered as a system of two non-spherical interacting bodies: a core and a mantle.The mantle of Mercury is considered as non-spherical, rigid (or elastic) layer. Inner shell is a liquid core, which occupies a large ellipsoidal cavity of Mercury. This Mercury system moves in the gravitational field of the Sun in resonant traslatory-rotary regime of the resonance 3:2. We take into account only the second harmonic of the force function of the Sun and Mercury. For the study of Mercury rotation we have been used specially designed canonical equations of motion in Andoyer and Poincare variables, more convenient for the application of mentioned methods. Approximate observational and some theoretical evaluations of the two main coefficients of Mercury gravitational field J_2 and C_{22} are known. From observational data of Mariner-10 mission were obtained some first evaluations of these coefficients: $J_2 = (8 \pm 6) \cdot 10^{-5}$ (Esposito et al., 1977); $J_2 = (6 \pm 2) \cdot 10^{-5}$ and $C_{22} = (1.0 \pm 0.5) \cdot 10^{-5}$ (Anderson et al., 1987). Some theoretical evaluation of ratio of these coefficients has been obtained on the base of study of periodic motions of the system of two non-spherical gravitating bodies (Barkin, 1976). Corresponding values of coefficients consist: $J_2 = 8 \cdot 10^{-5}$ and $C_{22} = 0.33 \cdot 10^{-5}$. We have no data about non-sphericity of inner core of Mercury.

There are also some evaluations of moments of inertia Mercury and its core: $C/(mR^2) = 0.35, C_m/C = 0.5 \pm 0.07$ (Peal, 1996). Here C and C_m are the moments of inertia of the full Mercury and of its core, m and R is a mass and a mean radius of Mercury.

Based on two methods, we consider the rotation of Mercury in the gravitational field of the Sun. First method of perturbation has been effectively applied to the construction of a rotational theory of the Earth for its models as two or three layer celestial body moving in gravitational fields of the Moon, Sun and planets in wide set of papers ranging in 1999-2001 years of Ferrandiz J.M. and Getino J.(2001). Another method is an analytical method of construction of the resonant rotational motion of synchronous satellites and Mercury, considered as non-spherical rigid bodies. This method has been applied earlier to construction of an analytical theory of rotation of the Moon considered as rigid non-spherical body (Barkin, 1989). Here we modified these methods to apply them to the study of the resonant rotation of a two-layer Mercury. By this we use very effective for the application of perturbation methods and dynamical geometrical illustration the canonical equations in Andoyer and Poincare variables. Main resonant properties of Mercury motion were been described first as generalized Cassini's laws (Colombo, 1966). But Colombo and some another scientists considered Mercury as rigid non-spherical body sometimes taking into account tidal deformation. Here we have been obtained and formulated these laws and their generalization for a two-layer model of Mercury.

1. Vectors of angular velocities and angular momentums of the core and Moon coincide with its polar axis of inertia. 2. The mantle-core system of Mercury rotates as one rigid body about polar axis of inertia in direction of its orbital motion with constant angular velocity equal 3/2 from the mean orbital motion of Mercury. 3. Mean ascending node of the Mercury orbit on invariable (ecliptic) plane coincides with the mean descending node of the general plane orthogonal to vectors of angular momentums of the core and Mercury. 4. Angular momentums of Mercury and its core make a constant small angle with the normal to ecliptic plane which depends from dynamical oblatenesses of Mercury, from a precession velocity of Mercury orbit plane and oth.

On the next step we have evaluated frequencies of free oscillations of core-mantle system of Mercury. Based on the mentioned data about Mercury (Barkin, 1976) we have been obtained the following model values of moments of inertia of Mercury and its core: A=0.3499534, B=0.3499667, C=0.35; A_c=0.1749767, C_c=0.175000 (1 unit=mR^2, m and R is a mass and a mean radius of Mercury). Here we used model values for moments of inertia of the core using also some analogy with axysimmetrical model of the core of the Moon from the paper Williams et al.(2003). Corresponding periods of free oscillations were determined on the base specially constructed equations of developed theory. They are equal: T_1=260543$\cdot T_{rot}$ days and $T_2 = 0.999468 \cdot T_{rot}$ (T_{rot}=58.6462 days is a period of Mercury rotation). Last period determines long period of relative oscillation of the core and mantle T_r=302 years. The mentioned period T_1=713 years.

Algorithm of calculations of perturbations of first and high order in Mercury resonant rotation in neighborhood of Cassini-Colombo motion have been developed.

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