How does Titan retain a finite orbital eccentricity?

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Introduction: There is appreciable evidence for a significant hydrocarbon ocean on the surface of Titan [1, 2, 3, 4]. However, it has long been appreciated that tidal dissipation within a putative hydrocarbon ocean on Titan easily yields an orbital eccentricity damping time $_e$ which is short compared to the age of the solar system [5]. Unless Titan's present eccentricity (e = 0.0288) were acquired recently, it requires that either: the ocean has a configuration which limits dissipation [6, 7, 8], or some mechanism exists which effectively maintains the eccentricity against dissipative damping. We argue for the latter. Specifically, the proximity of Jupiter and Saturn to a 5:2 mean motion resonance may provide a sufficient excitation source, and thereby effectively remove dynamical constraints on the dissipation and configuration of the Titan ocean.

Isolated Orbit: We first review the argument for circularization of an isolated orbit by tidal dissipation. For a Keplerian orbit, the energy **E** and scalar angular momentum **L** are

$$E = -G\, m_1 m_2 / 2a$$

$$L = \frac{m_1 m_2}{m_1 + m_2} a^2 n \sqrt{1-e^2}$$

where **G** is the gravitational constant, $\mathbf{m_j}$ are the masses, **a** is the semimajor axis, **n** is the mean motion and **e** is the eccentricity. Tidal dissipation reduces the energy of the orbit, and thereby shrinks the semimajor axis, but does not change the angular momentum. Expressing eccentricity in terms of energy and angular momentum, we obtain

$$e^2 = 1 + 2E\,L^2\,F \quad with$$

$$F = \frac{m_1 + m_2}{G^2 (m_1\, m_2)^3}$$

and the timescale for eccentricity damping is substantially shorter than that for energy (semimajor axis) damping

$$\frac{\tau_e}{\tau_a} = \frac{2e^2}{1-e^2} \approx 2e^2$$

Critical to this derivation is the assumption that angular momentum of the orbit is conserved. We now consider cases in which that is no longer true, due to interaction and angular momentum exchange with other bodies.

Influence of Sun and oblate Saturn: While an isolated pair of gravitating point masses will move along a fixed Keplerian elliptical orbit, the Saturn-Titan pair does not quite fit that description. The oblate figure of Saturn causes the apsidal line of the orbit to advance at a rate whose leading order term is

$$\frac{d\varpi}{dt} = \frac{3}{2} n\, J_2 \left(\frac{R}{a}\right)^2 = 0.4916\,\deg/\,yr$$

where J_2 and R are the degree two zonal gravity coefficient, and equatorial radius of Saturn. Averaged over the orbital period of Saturn, the solar torque acts as an additional effective oblateness of Saturn, with contribution

$$\frac{d\varpi}{dt} = \frac{3}{4} n \left(\frac{n_{Saturn}}{n}\right)^2 = 0.0136\,\deg/\,yr$$

The net effect of these torques is a steady rotation or advance of the apsidal line, with a period of 712 years.

Influence of other satellites: Titan is only rather weakly coupled to the other large satellites in the Saturn system, but does have an influence on their orbits, and they, in turn, have a minor influence on Titan's orbit. We can model the coupled oscillations of the orbital eccentricities and apsidal lines by averaging the satellite positions over their respective orbital periods, and examining the interactions of rotating eccentric mass rings. The resulting secular perturbation equations are most conveniently written in terms of the complex variables

$$h_j = e_j (\cos\varpi_j + i \sin\varpi_j)$$

and the oscillations are governed by a system of linear differential equations which can be written as [9,10]

$$\frac{dh}{dt} = i\, A \cdot h$$

where **h** is the vector of satellite orbital components and the coupling matrix **A** is real.

Each satellite orbit exerts a torque on each other orbit. These interaction torques are in the direction of attempting to align the apsidal lines, but due to the angular momentum of the system, they result in periodic oscillations in the orbital elements.

The behavior of the system is governed by the eigenvalues and eigenvectors of the matrix **A**. In the absence of dissipative effects, the eigenvalues are all real, and the free oscillations of the system are undamped. For the system including Tethys, Dione, Rhea, Titan, and Iapetus, the eigenvalues are {72.31, 30.77, 10.05, 0.5149, 0.1233} deg/yr, with corresponding periods of {4.98, 11.7, 35.8, 699, 2920} years. These are the free oscillation periods of the system, and each satellite has a response at each frequency. However, it is also true that each of these normal modes has a relatively localized response, and the satellite apsidal rates are nearly equal to the respective eigenvalues.

If there is dissipation in the system, then all of the eigenvalues will be complex, and the oscillations will eventually damp out. However, adding dissipation to one of the bodies will mainly influence only the single eigenvalue mostly closely associated with that body. That one mode of oscillation will be strongly damped, and the others will be much less influenced. Thus, interactions with its satellite peers cannot rescue Titan from a declining eccentricity if it experiences significant dissipation within its ocean.

Influence of other planets: Other planets in the solar system perturb the orbit of Saturn, and if the magnitude and period of the perturbations are right, they may influence Titan's orbital eccentricity. Jupiter and Saturn are close to a 5:2 mean motion resonance, and the resulting long period perturbations on the solar system have many interesting results. Historically, the first effect to be recognized was the "great inequality" in the motions of Jupiter and Saturn, which initially appeared to be a flaw in Newton's theory of gravitation, until it was recognized by Laplace as a long period perturbation. More recently, it has been appreciated that this near resonance plays a significant role in destabilizing the asteroid population at the Hecuba gap, or 2:1 mean motion resonance with Jupiter [11, 12, 13]. It is also implicated in contributing to the chaotic dynamics of the solar system as a whole [14, 15, 16].

The current rate of circulation of the near resonant argument is

$$\sigma^{5:2}{}_0 = 2n_J - 5n_S = -0.4309 \deg/ yr$$

with a corresponding period of 835 years. This spectral line is actually a multiplet, with splitting due to motions of the apsidal lines of Jupiter and Saturn

$$\sigma^{5:2}{}_k = \sigma^{5:2}{}_0 + k\dot{\varpi}_S - (k-3)\dot{\varpi}_J,\ k = 0,1,2,3$$

In addition, the central line has probably drifted over the age of the solar system [17, 18], especially during the late phases of planetary accretion.

In the current context, the important point is that the apparent orbit of the Sun about Saturn will have an eccentricity which varies on a time scale which is quite close to the mean apsidal motion rate of Titan. That allows the eccentricity of Titan to be excited, in much the same way that asteroidal eccentricities in the Hecuba gap are amplified.

For the excitation to be effective, the forcing period must be quite close to the period of the apsidal motion of Titan. The nominal central period of the 5:2 Jupiter-Saturn interaction is too far from the Titan apsidal period to be effective. However, the apsidal rates of Jupiter and Saturn, which control the side-band frequencies, vary on a variety of time scales. The mean apsidal periods for Jupiter and Saturn are {305.0, 46.35} 10^3 years, and the largest variations are due to secular interactions with other planets.

Another important perturbation arises from interaction with a 7:1 mean motion near-resonance between Jupiter and Uranus. As these two near resonances interact, the instantaneous period of each of them oscillates. This family of resonances contributes to chaotic motion in the outer solar system [14, 15] and some of the side bands appear to have actually librated in the past.

Implications: The finite eccentricity of Titan remains something of a mystery. However, if adequate excitation mechanisms can be identified, explored, and properly quantified, it may emerge that the persistence of a finite eccentricity will be seen to place less stringent constraints on oceanic tidal dissipation than has been previously supposed.

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