HYPERVELOCITY IMPACT ENERGY LOSS AND TRACK SHAPE IN AEROGELS: THEORY AND EXPERIMENT. G. Domínguez, A. J. Westphal, *Space Sciences Laboratory, University of California at Berkeley, CA USA*, S. M. Jones, *Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA USA*, M. L. F. Phillips, *Pleasanton Ridge Research Corporation, Hayward, CA USA*.

We have developed and tested a theoretical model of impact cratering in aerogels that derives the impact crater dimensions directly from the energy and momentum deposition of the projectile [1]. We outline our work below.

1. Energy loss of projectiles in Aerogel

The energy and momentum loss of the projectile can be used to derive the shock wave strength at the interface between the aerogel and the projectile. Energy loss is given by the following:

$$\frac{dE}{dz} = -(F_h + F_c) = -\frac{C_d}{2}\rho_0 v_g^2 A - P_c A \qquad (1)$$

where ρ_0 is the uncompressed density of the aerogel, v_g is the instantaneous velocity, A is the cross sectional area of the projectile, and P_c is defined as the mechanical crushing strength of the aerogel.

This equation of energy loss can be solved exactly for velocity and kinetic energy. To a very good approximation, near the surface of the aerogel, the projectile kinetic energy can be expressed:

$$E(z) = E_0 e^{-\frac{z}{\lambda}} \qquad (2)$$

where $E_0 = \frac{1}{2} m_g v_0^2$ is the initial kinetic energy of the projectile and λ is given by:

$$\lambda = \frac{1}{C_d}\frac{4}{3}\frac{\rho_g}{\rho_0} r_g \qquad (3)$$

In reality, the energy loss of projectiles is complicated by projectile ablation and aerogel accumulation onto the projectile [1].

2. Energy Loss and Shock Wave Formation in Aerogels

The energy and momentum loss of the projectile is transferred to the aerogel material that the projectile intercepts. Energy and momentum conservation can be used to show that, at the surface of the projectile, the shock compressed aerogel must have axial and radial components given by:

$$v_{s,z}(r = rg, z) \quad = \quad C_d \frac{v_g(z)}{2} \qquad (4)$$

$$v_{s,r}(r = rg, z) \quad = \quad v_g(z)\left(C_d - \frac{C_d^2}{4}\right)^{\frac{1}{2}} \qquad (5)$$

These two equations imply that the shock wave travels at a fixed angle, independent of velocity (and hence z), given by:

$$\tan\theta_s = \frac{C_d}{2(C_d - \frac{C_d{}^2}{4})^{\frac{1}{2}}} \qquad (6)$$

This result is illustrated by Figure 1.

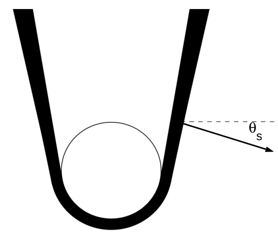

Figure 1: Shock wave propagation into aerogel at points away from the surface. $\theta_s = 30^o$ when $C_d = 1$.

3. Cylindrical Shock Wave Attenuation in Aerogels

If the aerogel is approximated as a perfect compressible material, consisting of a low density ($\sim$ 20 kg m^{-3}) and a high density phase ($\sim$ 2500 kg m^{-3}), the aerogel shock strength can be shown to attenuate as:

$$P \propto M^{-2} \qquad (7)$$

M is the mass of the shock wave and n is an index that is a function of the aerogel's porosity [1,2].

For cylindrical shock waves, the equations above are equivalent to:

$$P \propto R^{-4} \qquad (8)$$

The expansion of this shock wave stops this ram pressure equals the crushing strength of the aerogel. This, together with the geometry shown in Figure 1, leads us to conclude that far away from the surface, the track radius away from the surface and transition region is given by:

$$r_T(z) = r_g \cos\theta_s \left(\frac{C_d \rho_0 v(z)^2}{2P_c}\right)^{\frac{1}{4}} \qquad (9)$$

4. Crushing Strength vs. Density

The maximum size of the impact cavity is ultimately determined by the mechanical strength of the aerogel. To determine how this crushing strength scales with density, we simultaneously exposed 14, 20, 30, 60 and 100 mg cm^{-3} to 20 micron (diameter) borosilicate glass beads. We then measured the maximum size of the track radius by imaging the sample and calibration with an optically encoded computer controlled microscope.

We found that the crushing pressure of aerogels, similarly prepared by S. M. Jones, to scale as:

$$P_c(\rho) \simeq 23 \left(\frac{\rho}{14 \text{ mg cm}^{-3}}\right)^{2.04} \text{kPa} \qquad (10)$$

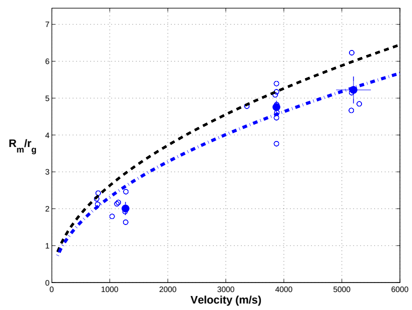

Figure 2: Velocity scaling of R_m/r_g for shots into 50 mg cm^{-3} aerogels. Blue circles indicate individual data points, while solid blue dots indicate statistical averages. Dashed line (black) indicates prediction of track shape model when P_c is determined using Eq. (10). A better fit is obtained if P_c is extrapolated from the experimentally derived value of the 60 mg cm^{-3} aerogel assuming that $P_c \propto \rho_0^2$. Note that the data appear to be consistent with $R_m \propto v^{1/2}$, as predicted by track shape model.

5. Testing the model's velocity scaling

We independently verified our model by measuring the maximum normalized radius of an impact crater, R_m/r_g, in samples of 50 mg cm^{-3}aerogel. Most of our data points were derived from impacts with 20-μm glass beads, although when available, we also included measurements of 2 and 5 μm in diameter glass beads. Figure 2 shows that $R_m \propto v^{1/2}$ as expected.

The figure above displays data points of impacts into 50 mg/cc aerogel at various velocities. The black line is the model prediction if the crushing strength of the aerogel is given by the power law fit in the section above. Blue line uses an extrapolation of the crushing pressure from the 60 mg cm^{-3}data point and appears to be a better fit.

6. Scaling of impacts in Aerogels

In summary, the track length scales as:

$$L_T \propto \left(\frac{\rho_g}{\rho_0}\right) \ln\left(\frac{v_0}{v_c(\rho_0)}\right) \tag{11}$$

where $v_c = \left(\frac{2P_c}{\rho_0}\right)^{1/2}$. The track radius away from the surface and transition region scales as:

$$r_T \propto r_g \left(\frac{v}{v_c}\right)^{\frac{1}{2}} \tag{12}$$

The track volume of an impact crater in aerogel is given by:

$$V_T = \int_0^{L_T} \pi \left(r_T(z)\right)^2 dz \tag{13}$$

Far away from the surface, the track radius is given by $r_T(z) \sim \left(\frac{\rho_0 v^2(z)}{2P_c}\right)^{\frac{1}{4}}$ and the volume of the track takes on the analytic form of:

$$V_T \sim \pi r_g^2 \left(\frac{\rho_0}{2P_c}\right)^{\frac{1}{4}} (4\lambda)(v_0)^{\frac{1}{2}} \left(1 - e^{\frac{-L_T}{4\lambda}}\right) \tag{14}$$

where $\lambda = \left(\frac{4}{3}\right)\left(\frac{\rho_g}{\rho_0}\right) r_g$ as before.

Using Eq. (11), the track volume can be expressed as:

$$V_T \sim V_p \left(\frac{\rho_g}{\rho_0}\right)\left(\frac{v_0}{v_c}\right)^{1/2} \tag{15}$$

$$\propto E_k v_0^{-3/2} v_c^{-1/2} \propto m v^{\frac{1}{2}} \tag{16}$$

Thus, the track volume is not a straightforward function of the projectile's kinetic energy E_k and has a weak velocity dependence.

7. Conclusion

We have developed and tested a theoretical model of impact cratering in aerogels. The correspondence between experiment and theory is good, indicating that the formation of an impact cavity, at least for the microscopic regime explored in this study, is well-described by a model in which an outgoing, momentum-conserving cylindrical shock attenuates, similar to the attenuation of supernova shock waves in the interstellar medium during the so-called "snowplow"phase, until it can no longer overcome the crushing strength of the aerogel.

The general theoretical method proposed here could be used to understand impact cratering in porous solids. Unlike sand, aerogel has the ability to conserve it's shape well after the impact. Thus, studies of impacts into aerogels, whose densities can be synthesized and varied readily, could be used to understand impact cratering in porous solids as a function of porosity.

These method presented here could be used to provide estimates or limits of the impact velocities of microscopic dust grains captured in aerogel collectors.

Finally, this work should help us understand the behavior of the calorimetric aerogel collectors/detectors which are capable of recording the kinetic energy of captured projectiles [3].

References

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3. Domínguez, G., Westphal, A. J., Phillips, M. L. F., Jones, S. M., Jul. 2003. A Fluorescent Aerogel for Capture and Identification of Interplanetary and Interstellar Dust. *The Astrophysical Journal* **592**, 631–635.