

EARLY CLEARING OF THE ASTEROID BELT. Wm. R. Ward, Space Studies Department, Southwest Research Institute, Boulder, CO 80302.

Most proposed mechanisms for the excitation and clearing of the asteroid belt implicate Jupiter as somehow responsible. However, mean motion resonances with Jupiter do not span more than $\sim 15\%$ of the belt. Objects could diffuse into the resonances by mutual scattering or non-gravitational forces (gas and/or Yarkovsky drag)[1], but this would not prevent accretion from proceeding for much of the belt. Secular resonances could have swept through the belt during the dissipation of the solar nebula [2-4]. Their passage causes a strong excitation of asteroid eccentricities, resulting in high-velocity erosive impacts among the belt population. However, this is primarily a “one shot” mechanism, and although it could abort accretion everywhere for a while, high velocities might not be sustained long enough to sufficiently deplete the belt. Jupiter zone planetesimals could penetrate the belt and excite eccentricities, but they tend to be ejected by Jupiter quickly [5]. Some, however, may be trapped and can stir up the belt, increasing its diffusion rate into mean motion resonances [6-7]. Alternatively, embryos may have accreted in the belt itself and provided this function, only to eventually become scattered into the resonances themselves and removed by Jupiter [8-9].

A point of commonality of all the above ideas is a starting condition consisting of a primordial belt perturbed by a fully formed Jupiter. However, if Jupiter formed via the core accretion model, then the system may have passed through a prolonged earlier state consisting of the primordial belt adjacent to a population of embryos forming the Jovian core. Assuming that oligarchic objects appeared in the Jovian zone ahead of the asteroid region, perhaps because of the increased solids due to ice condensation in the former, *e.g.*, [10], the effects of the mean motion resonances from an *ensemble* of embryos on the asteroid population could be significant.

The disturbing function for a single embryo can be written,

$$GM_e/|r - r_e| = (GM_e/a)\sum_m b_{1/2}^{(m)}(r/a)\cos m(\theta - \Omega_e t), \quad (1)$$

where M_e is the embryo mass, a is its semimajor axis, Ω is its mean motion, and $b_{1/2}^{(m)}(\alpha) = (2/\pi)\int_0^\pi \cos m\theta (1 + \alpha^2 - 2\alpha \cos \theta)^{-1/2}$ is a Laplace coefficient. The strongest m^{th} order inner Lindblad resonances are located at $r/a = 1 - 2/3m$,

and the resonance density is therefore $dm/dr = 3m^2/2a$. Let $\lambda(m)$ represent the distance scale of resonance stirring to be discussed below. Then the fractional coverage due to N embryos is

$$C(r) \sim \sum_N \lambda(m) \frac{dm}{dr} \approx \frac{3}{2} \int m^2 \lambda(m) \frac{dN}{da} \frac{da}{a}, \quad (2)$$

where $m(r,a) = (2/3)a/(a-r)$ and $dN/da \sim 1/\Delta a$, Δa being a characteristic embryo spacing in semi-major axis. To ascertain a relevant distance scale associated with the resonant perturbations, we note that the first wavelength for self-gravity density waves in a particle disk of surface density σ_d is $\lambda_1 \sim 2r\sqrt{\pi\epsilon}$, where

$$\epsilon \equiv \frac{2\pi G\sigma_d}{r^2 dD/dr} \approx \frac{2/3}{m-1} \left(\frac{\pi\sigma_d r^2}{M_\odot} \right). \quad (3)$$

and $D \equiv \Omega^2 - m^2(\Omega - \Omega_e)^2$ is the frequency distance from resonance. For example, if we set $\pi\sigma_d r^2 \sim M_\oplus$, $r \sim 3AU$, $\lambda_1 \sim 2 \times 10^6 (m-1)^{-1/2} km \sim 0.01(m-1)^{-1/2} AU$. The wave propagates outward in the disk, toward the perturbing embryo. As it winds up, the amplitude increases to conserve the wave action, eventually becoming non-linear and breaking. The distance to non-linearity is

$$\lambda_{NL} = \sqrt{2\pi\epsilon} (G\sigma_d r^2 / \Psi_m) \quad (4)$$

where $\Psi_m \sim 1.6m(M_e/M_\odot)(a\Omega_e)^2$ is the so-called forcing function of an m^{th} order resonance. This reads $\lambda_{NL} \approx 1.2 \times 10^{-3} (r/m)(m-1)^{-1/2} (M_{ast}/M_e)$, which for $M_e \sim 10^{-1} M_\oplus$, $r \sim 3AU$, $m = 2$ is $\lambda_{NL} \sim 3 \times 10^6 km$. Substituting λ_{NL} into eqn. (2), setting $M_e \sim 2\pi a \sigma_j \Delta a$, (where σ_j is the surface density of solids in the Jovian zone) and integrating gives, $C \approx 3 \times 10^{-3} (M_{ast}/M_e)(M_j/M_e) \mathcal{F}$, where $\mathcal{F} \approx 0.69 - \ln[\sqrt{a_o/r} + \sqrt{a_o/r - 1}]$, is a slowly varying function of order unity, a_o is the inner boundary of the embryo swarm, and $M_j \equiv \pi\sigma_j a_o r$. Coverage is complete, *i.e.*, $C \geq 1$, if $M_e \lesssim 0.03 \sqrt{M_{ast} M_j}$. For $M_{ast} \sim M_\oplus$, $M_j \sim 30M_\oplus$, $M_e \lesssim 0.2M_\oplus$ this requires $N \geq 150$. Diffusion of embryos, higher-order resonances, and secular resonances will increase coverage and decrease the required N . Although larger N increases coverage of the resonance “forest”, the implied smaller embryo size

decreases their perturbing power. *The key question is whether at any time, during the growth and winnowing of their number, the stirring is both strong and complete enough to erode the belt population.*

The torque density for a single embryo is $dT/dr \approx -2.5r\sigma_d G^2 M_e^2 / \Omega^2 (a-r)^4$ and the accompanying excitation of the belt is given by $(\Omega - \Omega_e)(-dT/dr)$. Summing over the ensemble yields,

$$de^2/dt \sim 3.7 \frac{\Omega}{\pi} \left(\frac{M_e}{M_\star} \right) \left(\frac{M_J}{M_\star} \right) \left(\frac{r}{a_o - r} \right)^2 \quad (5)$$

The characteristic stirring time scale is $\tau_{stir} \sim e^2/(de^2/dt)$. Setting this equal to the damping time scale due to collisions, $\tau_{col} \sim 2.7\Omega^{-1}(Rp_p/\sigma_d)$, and/or gas drag, $\tau_{drag} \approx e^{-1}(C_D\Omega)^{-1}(Rp_p/\sigma_{gas})(c/r\Omega)$, determines the equilibrium velocity of the asteroid population. Figure 1 compares the velocities for each damping mechanism to the particles' escape velocities as a function of particle radius R . Also shown are the threshold velocities for erosion for two assumed values of particle strength. For $N \sim 100$, and ~ 30 Earth masses total in pre-jovian core embryos (some of which may have eventually been ejected), we find that the asteroid velocities will exceed their escape velocities for asteroid radii less than $\sim 10^3$ km. This suggests that the asteroids would fragment and/or erode on their collision time scale τ_{col} , which becomes ever shorter as asteroid sizes decrease. Smaller R results in a smaller dispersion velocity, but also a smaller escape velocity. The former is always larger than the latter, and the collisional cascade continues until particle strength can resist further erosion. This occurs at a few meters in size, but this size is very mobile due to gas drag. We suggest that once runaway growth starts in the Jovian zone, oligarchic objects quickly appear and begin to excite the asteroid zone. This aborts accretion there and initiates a fragmentation cascade. As the size decreases, the cascade accelerates until the orbital decay time due to gas drag becomes less than the oligarchic growth time. As a result, it is possible that the majority of the primordial asteroid belt eroded and drifted into the terrestrial zone *before* the formation of the Jovian core.

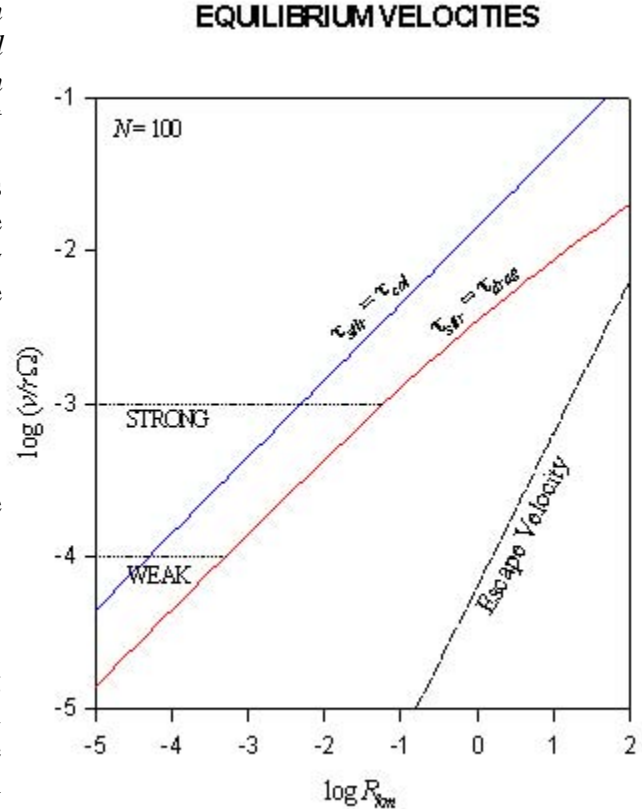


Figure 1. Asteroid dispersion velocities due to stirring by pre-Jovian embryos. Equilibrium velocities for gas drag and collisional damping are shown. Also shown is the asteroid escape velocity as a function of mass and the velocities corresponding to specific strengths of strong and weak bodies, i.e., 3×10^6 , 3×10^4 ergs/cm³.

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REFERENCES

- [1] Bottke, W. F., *et al.* 2002. In *Asteroids III* 395.
- [2] Ward, W. R. 1980. *Lunar & Planet Sci.* **11**, 1199. [3] Heppenheimer, T. A. 1980. *Icarus* **41**, 76.
- [4] Nagasawa, *et al.* 2000. *Astron. J.* **119**, 1480.
- [5] Safronov 1979, In *Asteroids II* 975. [6] Wetherill, G. W. 1992. *Icarus* **100**, 307. [7] Wetherill, G. W. and J. Chambers 2000. *M&PSA* **35**, 168. [8] Morbidelli, A., J. *et al.* 2000. *M&PS* **35**, 1309. [9] Petit, J.-M., *et al.* 2002. In *Asteroids III*, 711. [10] Thommes, E. M. J. Duncan, H. F. Levison 2003. *Icarus* **161**, 431.