

**GRAIN SIZE-DEPENDENT VISCOSITY AND OCEANS IN ICY SATELLITES.** E. S. G. Rainey and D. J. Stevenson, California Institute of Technology, Pasadena, CA, *emma@gps.caltech.edu*.

**Introduction:** To model thermal evolution of large icy satellites it is necessary to model convection in the outer ice layer. The major uncertainty involved in such models is the viscosity of ice, which is not well known at stresses appropriate for planetary interiors. Extrapolation of laboratory results to planetary conditions gives a viscosity that is grain size-dependent [1].

Most models of thermal convection in icy satellites have used a Newtonian viscosity, since the stresses involved are very low and the stress dependence of ice viscosity is expected to be weak. However, the grain size dependence of ice viscosity is significant, and since grain size is stress-dependent the viscosity of ice in the interior of large icy satellites is effectively non-Newtonian. Here we derive scaling laws for grain size in a convective icy satellite, and with scaling laws for non-Newtonian stagnant lid convection, apply them to the question of whether subsurface oceans exist in large icy satellites.

**Grain size:** The viscosity of ice is a function of temperature, stress, and grain size [2]

$$\eta = Ad^p \sigma^{(1-n)} \exp\left(\frac{E^*}{RT}\right), \quad (1)$$

where  $\eta$  is viscosity,  $d$  is grain size,  $\sigma$  is differential stress, and the constants  $A$ ,  $p$ ,  $n$ , and the activation enthalpy  $E^*$  are experimentally determined. For icy satellites  $p = 1.4$ ,  $n = 1.8$ , and  $E^* = 49$  kJ/mol [1]. In the interiors of icy satellites, grain size can be affected by growth, dynamic recrystallization, and the nucleation of new phases as convecting ice crosses a phase boundary. Which processes are important depends on whether the icy satellite has a subsurface ocean or not (see Figure 1).

*Icy satellites with oceans.* If an ocean exists, the outer layer of ice above the ocean must be entirely ice I. In this case grain size is determined by growth and dynamic recrystallization. Models for mean recrystallized grain size generally take the form [2]

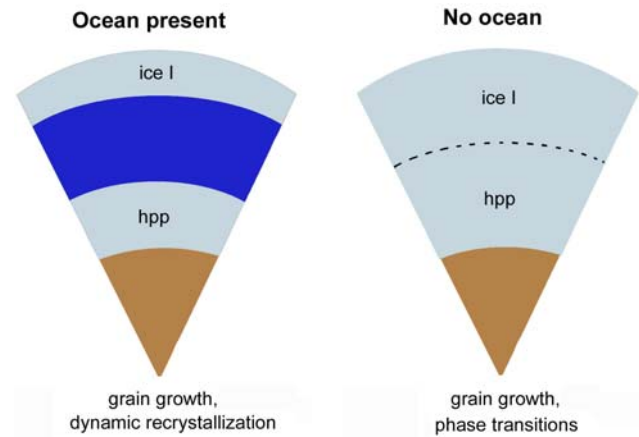
$$d = K\sigma^{-m}, \quad (2)$$

where  $K$  is a constant which can be temperature-dependent, and  $m$  is  $\sim 1.25$  for ice at low stresses [3]. The value for  $K$  can be fitted from ice core data [2].

*Icy satellites without oceans.* If there is no ocean, grain size is determined by growth and phase transitions (if high pressure phases are present, which is true for most icy satellites large enough to convect). The grain growth law for ice can be written as

$$d^2 = C_0 \exp\left(\frac{-Q_{gr}}{RT}\right) t, \quad (3)$$

where  $C_0$  is an empirically determined growth rate,  $Q_{gr}$  is an activation energy, and  $t$  is the time available for grain growth. Assuming the ice shell is convecting as a single layer,  $t$  scales as  $t \sim D/u$ , where  $D$  is the thickness of the convecting layer and  $u$  is the velocity [4].



**Figure 2: Factors affecting grain size.** Cartoon (not to scale) of interiors of icy satellites with and without internal oceans. When an ocean exists, the outermost ice layer is entirely ice I, whereas if there is no ocean the convecting ice can cross a phase boundary between ice I and high pressure phases.

**Convective scaling laws:** Since radioactive heating rates were high in the early solar system, it is likely that most large icy satellites had some liquid water early on. Therefore, to answer the question of whether oceans exist today, it is appropriate to use the grain size scaling for icy satellites with oceans, and determine whether melting temperatures can be reached in the interior at current expected heat production rates. If not, any previous ocean that existed is expected to have frozen.

Using equation (2) for recrystallized grain size with the viscosity equation (1), we have a viscosity that is temperature-dependent and effectively non-Newtonian with stress exponent  $n' = 3.55$ . To calculate the temperature of the convective interior, we can use the scaling laws of Solomatov [5] for heat flux in a non-Newtonian fluid convecting in the stagnant lid regime. The scaling laws are derived by combining the scaling of the boundary layer thickness with internal Rayleigh number for a non-Newtonian fluid,

$$\delta = DRa_n^{-n'/(n'+2)}, \quad (4)$$

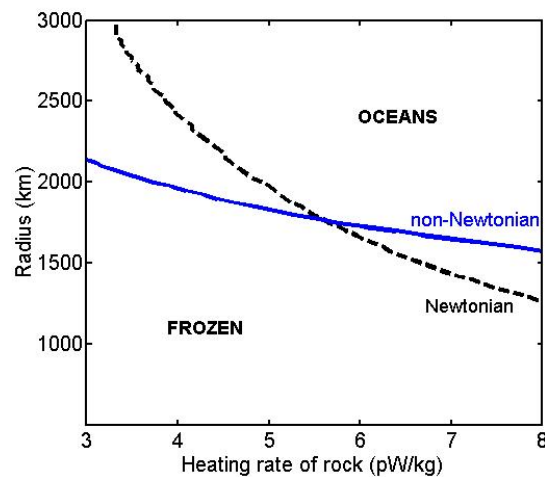
with Fourier's law of heat conduction across the thermal boundary layer. Here  $\delta$  is the thickness of the thermal boundary layer.  $Ra_n$  is similar to the usual Rayleigh number [5]:

$$Ra_n = \frac{\rho \alpha g \Delta T D^{(n'+2)/n'}}{\kappa^{1/n'} b^{1/n'} \exp(E^*/n'RT)} \quad (5)$$

Combining equations (1), (2), (4), and (5) gives an expression that can be solved for interior temperature  $T$  for a given heat flux.

**Implications for internal oceans:** In this simple approximation the internal temperature is independent of the thickness of the ice layer, and depends only on the heat flux. By assuming convective heat flux is in equilibrium with internal heating rates, we can calculate the temperature of the convecting interior. If the temperature at the base of the ice layer reaches the minimum melting temperature of ice I ( $\sim 251$  K at 207 MPa) under present-day heat fluxes, then the subsurface ocean still exists.

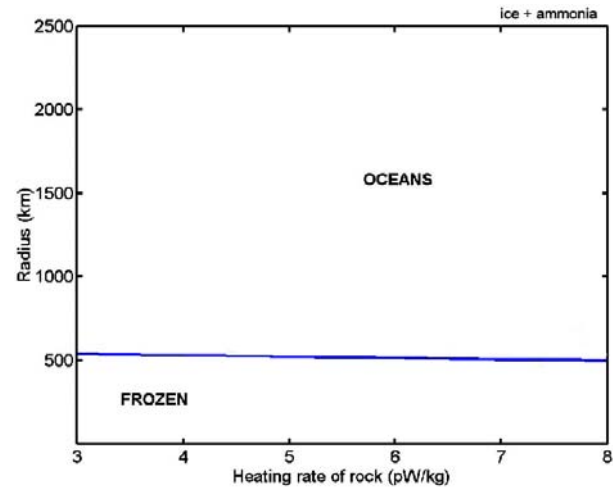
**Comparison to Newtonian model.** Comparing with a typical Newtonian viscosity model, the temperature in the grain size-dependent model is increased at likely heat production rates. This works in favor of retaining oceans (see Figure 2). For heating rates of interest, the curve separating the oceans and frozen regime is relatively flat, implying that size is a more important factor than heating in determining whether an icy satellite currently has an ocean.



**Figure 2: Oceans phase diagram.** Phase diagram showing the region of stability of liquid water in icy satellites of varying radii and heating rates, assuming 60% rock by mass. The blue line is for the grain size-dependent model, with rheological constants from [1]. The dotted line is for a Newtonian model with melting point viscosity  $10^{14}$  Pas and activation energy 53 kJ/mol. Expected heating rates due to radioactive decay are between 4 and 6 pW/kg.

**Other important factors.** Composition is another key factor. If the ice layer is not pure  $H_2O$ , the melting

temperature of the ice decreases. This effect is most dramatic for ammonia. With only a small amount (less than a few percent) of ammonia present, the minimum melting temperature of the ice-ammonia system drops dramatically, to  $\sim 176$  K. This has major implications for the existence of liquid oceans (see figure 3). If ammonia is present, any icy satellite large enough to convect can have an ocean.



**Figure 3: Effects of ammonia.** Diagram showing the region of stability of a liquid water layer in icy satellites of varying radii and heating rates, when ammonia is present. The viscosity is grain size-dependent with constants taken from [1].

**Discussion:** Internal oceans should be common among large icy satellites in the solar system. We are not surprised to find evidence for oceans in Callisto and Ganymede, expect that an ocean probably exists in Titan, and depending on the composition of the ice, might also find oceans in Triton, Pluto, and other large Kuiper Belt objects. The effect of a grain size-dependent viscosity is significant for heat transport in icy satellites, slowing cooling compared with Newtonian models. It should be noted that the scaling laws used are not applicable to icy satellites in which external tidal stresses exceed the convective stresses, such as in Europa. In this case the viscosity is determined by the external stress (and is lower due to the negative dependence of viscosity with stress).

**References:** [1] Goldsby D. L. and Kohlstedt D. L. (2001) *JGR*, 106, 11017-11030. [2] Durham W. B. and Stern L. A. (2001) *Ann. Rev. Earth Planet. Sci.*, 29, 295-330. [3] Shimizu I. (1998) *GRL*, 25, 4237-4240. [4] Solomatov V. S. (2001) *Earth Planet. Sci. Lett.*, 191, 203-212. [5] Solomatov V. S. (1995) *Phys. Fluids*, 7, 266-274.