NEW INSIGHTS INTO THE FAILURE OF MAGMA RESERVOIRS ON THE TERRESTRIAL PLANETS.

E. B. Grosfils, Geology Department, Pomona College, Claremont, CA 91711 (egrosfils@pomona.edu).

Introduction: Analytical and numerical models of pressurized ellipsoidal basaltic magma reservoirs in an elastic host have been used extensively to gain insight into magma plumbing systems, surface eruptions, and the formation of structural features [e.g., 1-15]. In spite of the 'simplicity' of the approach, however, the models often use/ignore different factors, and published results—even for identical conditions—can be contradictory. As more advanced numerical models (viscoelastic, etc.) are developed [e.g., 16-20], elastic models are often used to calibrate them. Until contradictions between existing elastic models are understood and resolved, however, calibrations using such models or results in the literature (which ones?) risk propagating errors into more advanced models. Caution is particularly critical for analysis of magma reservoirs on other planets, where model design and evaluation isn't guided by the plethora of datasets (e.g., gravity, seismicity, heat flow, surface tilt) often available to researchers studying magma systems on Earth.

In this study, using numerical models that explicitly incorporate all major geometrical and geological factors identified by previous authors (to my knowledge no other study has systematically examined them all together), I seek to resolve the contradictions between published elastic results. Here I present three interesting new insights, focusing on reservoirs subjected to physical conditions appropriate for Earth and (to help elucidate fundamental behaviors) Mars.

Model: Axisymmetric FEM analysis of a pressurized cavity isolated from edge effects in a uniform elastic host was performed using COMSOL Multiphysics. Variables examined include: reservoir shape (spherical, ellipsoidal), volume, depth and pressure; magma and host rock (uniform, smoothly increasing) density structure with depth; gravity; and, host rock stress state (lithostatic stress σ_L, uniaxial strain). Forces applied normal to the reservoir wall reflect a uniform pressure P resulting from infusion of new magma and a hydrostatic term $P_m = \rho_m g h$ with h the depth below the crest of the reservoir. For all models run, Poisson's ratio is 0.25, Young's Modulus is 60 GPa (failure location is insensitive to plausible variance), and the tensile strength of the rock is zero (failure location varies little if typical values of tensile strength, i.e. a few MPa, are instead used). Calibration of the numerical model against analytical test cases yields stress and displacement values that differ from analytical predictions in all cases by <0.03%.

In a previous effort [21], following common practice, P was treated as a dependent variable, defined in every model as a fixed multiple of the vertical stress at the depth where the reservoir was centered; the location (and orientation) of tensile failure was taken to be the spot where the greatest tensile stress value was produced. Subsequent assessment of this approach, however, indicates that failure location is sensitive to the value of P used, invalidating earlier results. Since the idea is to simulate injection of fresh material into the reservoir, in the current models P is treated as an independent variable explored, for any combination of geometric/magma/host conditions, until the value required to initiate (and characterize the location/alignment of) tensile failure is identified.

Three Key Results: Space limitations prohibit a full presentation here of all results. Instead, three important insights into elastic modeling of magma reservoirs are presented and discussed.

1. Surface Effect. Analytical models of pressurized spheres in an infinite medium [e.g., 22] have been adjusted to allow examination of reservoirs near the free surface of an infinite half-space [e.g., 3, 4, 10]. As an example, the failure criteria of [10]

$$P_m + P - P_L = 2\sigma_T/C \qquad [1]$$

($P_L = \rho_r g(h+D_T)$ with ρ_r a uniform host rock density, D_T the depth to the top of the reservoir; σ_T is the tensile strength) modifies a common tensile failure criterion [e.g., 6] by using the factor C to account for the free surface, where

$$C = [2h\,(R+D_T) + D_T^2] / (h+D_T)^2 \qquad [2]$$

(R is the reservoir radius). If magma and host rock density are set equal, then Eq. 1 becomes:

$$P - \rho_r g D_T = 2\sigma_T/C \qquad [3]$$

with all components except C constant for a given model. Plugging Eq. 2 into Eq. 3, for any given D_T and R the depth h beneath the crest at which the analytical model predicts failure can be calculated. Converting h to a failure location angle θ (θ=0 means failure at the crest, θ=90 at the midline, and θ=180 at the bottom), and repeating the analysis for a range of D_T, one can visualize the analytically predicted behavior of the free surface on the failure location. Fig. 1 reveals several key differences between analytical predictions and numerical models that incorporate the free surface explicitly. First, the analytical model predicts that the failure location will migrate toward θ=90 as D_T increases, while numerical results show this trend only briefly before failure rotates back toward θ=0. Second, numerical models show that the reservoir depth-to-center (DtC) at which the surface ceases to play a role is R/DtC=1/3. Models of failure that ignore free surface effects should limit their application to depths at or below this mark. Third, the gravity value used does not affect the failure location results. Net displacement vectors on Mars will be greater in magnitude than under Earth gravity, but the relative variation in magnitude as a function of θ is unchanged, and the location where failure first occurs as P increases thus remains unaltered.

2. Magma Pressure. Many magma reservoir models [e.g., 8,12] use at most a few P values, but systematic analysis here shows that the value of P employed can strongly affect if/where failure occurs. In addition, the value of P required to cause initial failure in both uniform and smoothly varying host density models is a linear function of the ratio between the radius of the reservoir and its *DtC* (Fig. 2), even when the free surface effect is felt (i.e., R/DtC>0.33). At any given *DtC*, proportionately greater P is required to induce failure of a smaller reservoir, with the maximum P required to induce failure approaching $3\sigma_L$ as R/DtC approaches zero. One implication is that, if some process (e.g. neutral buoyancy) promotes the collection of magma into a small equant body at depth, it will remain trapped at depth until thermally terminated or until considerable P builds up, promoting failure at the crest (Fig. 1) and vertical dike propagation. A second implication is that many published analytical elastic models, which commonly use Eq. 1 or a variant for their failure criteria, incorrectly use a P of $\sim\sigma_L$ to determine when a magma reservoir wall will fail. For internal self-consistency, unless special failure circumstances are invoked (e.g., to propagate a new dike into pre-fractured host rock requires overcoming viscous resistance to magma injection, not creation of a new fracture [18,23]), pressures less than those shown in Fig. 2 are insufficient to induce failure of a reservoir in an elastic half-space model—even though a consequence is that elastic models can require geologically unreasonable P values. This underscores that more sophisticated approaches may be required to improve our understanding of reservoir failure.

3. Uniaxial Strain. Some published models [e.g., 9] examine pressurized reservoirs (often with P = σ_L) in a host under uniaxial strain. Exploring the P required for failure quickly reveals, however, that failure occurs at/near the crest of the reservoir even when P = 0. Large magma reservoirs won't form in an elastic host subjected to uniaxial strain, and this calls into question models that assume the presence of a large reservoir under these host conditions.

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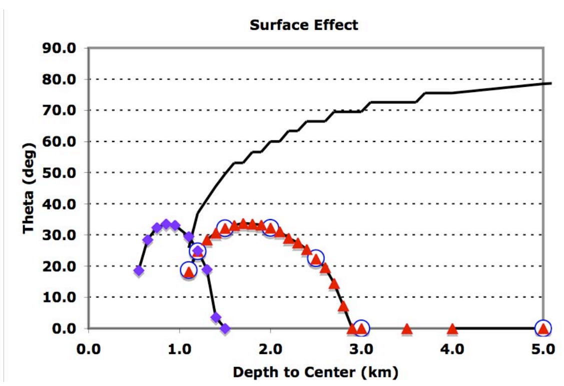

Figure 1: *Surface effect on failure location, ρ_m = ρ_r. Solid line is analytical prediction (see text) for R=1 km. Red triangles (Mars) and open circles (Earth) show numerical failure results for 1 km radius reservoir; purple diamonds show numerical results for a 0.5 km radius Mars reservoir.*

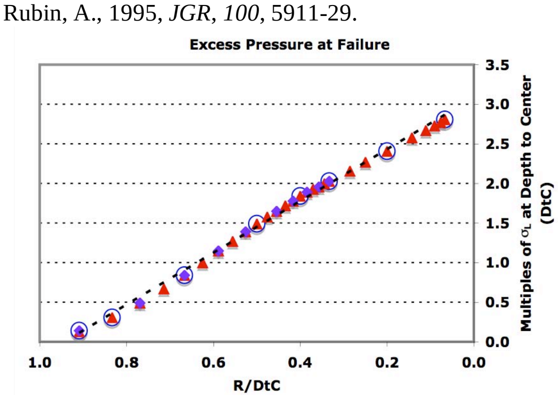

Figure 2: *Uniform pressure required for first tensile failure at the walls of the magma reservoir for a uniform host rock density; if host density varies with depth the result is nearly identical. Triangles, diamonds and circles as in Fig. 1, but here the dashed line is the linear best fit, with R^2>0.99.*