THICKNESS OF THE MAGNETIC CRUST OF MARS FROM MAGNETO-SPECTRAL ANALYSIS.

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Summary: Previous analysis of the magnetic spectrum of Mars showed only a crustal source field [1]. The observational spectrum was fairly well fitted by the spectrum expected from random dipolar sources scattered on a spherical shell about 46 ± 10 km below Mars' 3389.5 km mean radius. This de-correlation depth overestimates the typical depth of extended magnetized structures, and so was judged closer to mean source layer thickness than twice its value.

To better estimate the thickness of the magnetic crust of Mars, six different magnetic spectra were fitted with the theoretical spectrum expected from a novel, bimodal distribution of magnetic sources. This theoretical spectrum represents both compact and extended, laterally correlated sources, so source shell depth is doubled to obtain layer thickness. The typical magnetic crustal thickness is put at 47.8 ± 8.2 km. The extended sources are enormous, typically 650 km across, and account for over half the magnetic energy at low degrees. How did such vast regions form?

Observational Spectra: The magnetic spectrum of a planet is the mean square magnetic induction configured in spherical harmonics of degree n, averaged over a sphere of radius r containing the sources [2],

$$R_n(r) = (n+1)(a/r)^{2n+4} \sum_{m=0}^{n} [(g_n^m)^2 + (h_n^m)^2]. \qquad (1)$$

Here a denotes reference radius and (g_n^m, h_n^m) the Gauss coefficients of degree n and order m in a Schmidt-normal spherical harmonic expansion of the scalar potential V: $\boldsymbol{B} = -\nabla V$. Observational spectra are calculated from coefficients obtained via harmonic analysis of either measured data [3], binned data [4, 5], a map of such data [6], or fields from equivalent source models fitted to such data [7, 8]. These spectra differ, especially for $n > 50$, for each comes from a different analysis of variously selected MGS-MAG/ER measurements of the vector magnetic field around Mars.

Theoretical Spectra: Consider a thin crust with compact, effectively dipolar, sources. If we expect dipole positions to be uncorrelated, random samples of a uniform distribution on a spherical shell of radius $r_x < a$, and vector dipole moments to be vertical, uncorrelated, random samples of a zero mean distribution, then our expectation spectrum from an ensemble of such random radial dipoles on a shell is [1, 9]

$$\{R_n(a)\}^{ssr} = A\, n^2\, (n+1)\, (r_x/a)^{2n-2}. \qquad (2)$$

Amplitude A is proportional to the mean square moment of these sources. Spectrum (2) increases with n at low degrees, peaks near $n = 3/2ln(a/r_x)$, and falls off exponentially at high degrees. The cubic modulating polynomial is $n/(n + 1/2)$ times that expected from randomly oriented dipoles, which is important for $n = 1$.

More realistic theoretical spectra, which allow for crustal thickness, oblateness and magnetization by a planet centered paleo-dipole, have been derived and discussed; so have important spectral effects of laterally correlated sources [1, 9]. The latter were described via an ensemble of vertically and uniformly magnetized spherical caps. The main effect is to soften the expected spectrum at high degrees. We tend to overestimate source shell depth when this effect is omitted.

To include this effect simply, size and magnetization distribution functions for extended sources are recast as the characteristic half-angle ψ_0 (hence diameter) and mean square total moment $\{T^2\}$ for an ensemble of vertically and uniformly magnetized spherical caps on the shell of radius r_x. The resulting theoretical expectation spectrum is

$$\{R_n(a)\}^{sc} = B\, (n/2)\, [Z_n(\psi_0)]^2\, (r_x/a)^{2n-2}. \qquad (3)$$

Amplitude B is proportional to $\{T^2\}$ and, in terms of the Schmidt-normal associated Legendre polynomials $P_n^m(\cos\psi)$, $Z_n(\psi) = \sin\psi\, P_n^1(\cos\psi)/[1-\cos\psi]$.

Limited insight into spectrum (3) can be gained from its finite Taylor expansion in $\varepsilon = 1 - \cos\psi_0$, a quantity proportional to the characteristic area of small source regions. To first order in small ε,

$$\{R_n(a)\}^{sc} \approx B\, n^2\, (n+1)\, (r_x/a)^{2n-2} \times [1 - (\psi_0^2/4)n(n+1)]. \qquad (4)$$

For small caps and moderate degrees $n\psi_0 << 1$, the partial derivatives of the logarithm of spectrum (4) with respect to B, r_x, and ψ_0 are nearly proportional to 1, n, and $-n^2$, respectively. Separation of a small characteristic source size from amplitude and depth should thus be straightforward, unlike separation of amplitude from layer thickness. The negative sign of the partial w.r.t. ψ_0 describes a softening of the spectrum due to the small, but non-zero, area of extended sources.

The spectrum expected from a bimodal distribution of both compact and independent extended sources is given by the sum of spectra proportional to (2) and (3). Because $[Z_n(0)]^2 = 2n(n+1)$, this sum is just

$$\{R_n(a)\} = A\, n^2\, (n+1)\, (r_x/a)^{2n-2} \times (1 + [B/A][Z_n(\psi_0)/Z_n(0)]^2). \quad (5)$$

Method: Spectral parameters are estimated by a least squares fit of log-theoretical to log-observational spectra from degree n_{min} to n_{max}. In practice, log-observational spectra for Mars exhibit positive excess curvature for degrees 20-40. This is not because the dominant extended sources have 'negative areas', perhaps reflecting impact demagnetization, but because they are so large that small cap linearization (4) fails. Inclusion of higher order terms can help solve the non-linear inverse problem, yet with too small a trial value for ψ_0, the positive slope of the objective function near $\psi_0 = 0$ leads iterative linearized solutions astray. And values of $[Z_n(\psi_0)/Z_n(0)]^2$ depend strongly on ψ_0, so convergence from even a good guess of ψ_0 can be slow.

The closest fits of bimodal spectrum (5) to log-observational spectra are instead found by sweeping through trial values of both relative amplitude B/A and ψ_0; for each pair, a linear system is solved for optimal $\log A$ and $\log(r_x/a)^2$. Four sweeps of ever finer resolution are usually enough to locate the minimum sum of square residuals per degree of freedom, denoted S_4^2, to six digits, and optimal B/A and ψ_0 to three digits.

Results: Results are presented from analyses of spectra denoted FSU from [3], MG2 from [4], MG4 from [5], JEC from [6] (courtesy of J. Arkani-Hamed), P87 from [7], and LPM from [8]. Analysis of a non-observational spectrum from a constrained magnetization model [10] revealed much about model assumptions, but nothing about the magnetic crust of Mars. Care is needed to account for different reference radii.

The four spectral parameters estimated give: source shell depth $z = (3389.5 - r_x)$ km, regarded as half the typical thickness of Mars' magnetic crust; cap half-angle ψ_0, hence typical breadth of extended sources; amplitude A for compact sources; and relative amplitude B/A for extended sources. Table 1 lists results

Table 1: Results from Analysis of FSU Spectrum [3]

n	decorr. depth	S_2^2 %	S_4^2 %	z km	B/A -	ψ_0 deg.
1-90	42.9	13.74	9.01	26.2	1.96	6.50
2-90	40.1	7.83	4.84	25.5	1.48	5.78
3-90	38.8	6.83	4.42	25.4	1.33	5.57
6-90	37.1	6.28	4.43	25.4	1.24	5.46
1-50	70.4	16.36	15.01	36.5	1.48	6.70
2-50	62.3	8.36	7.48	26.3	1.51	5.85
3-50	59.3	7.48	6.73	19.4	1.67	5.49

results from analysis of the FSU spectrum [3]. The first column shows range of degrees fitted. The second and third give de-correlation depth and the sum of squared residuals per degree of freedom, S_2^2, from the 2 parameter fit of spectrum (2). The remaining columns give S_4^2, z in km, B/A, and ψ_0 in degrees from the 4 parameter fit of spectrum (5). Omission of R_1 halves the sum squared residuals. This is in part due to non-radial sources, though external fields are a concern [4].

For all observational spectra and all degree ranges analyzed, bimodal spectrum (5) gives a slightly better fit than does spectrum (2) for radial dipoles alone; in short, $S_4^2 < S_2^2$. Source depth z is positive definite and is indeed about half the de-correlation depth. Moreover, $\psi_0 \approx 5.5°$. Extended sources are enormous, typically 650 km across, and account for over half the magnetic energy in the spectrum at low degrees.

Considering all six observational spectra in plausible degree ranges recommended by the various authors, the overall mean source shell depth is 22 ± 9 km; however, most of the scatter comes from spectrum MG4, which suggests a shell depth of but 7 km instead of 22 km. It can be argued that the correlative technique used to obtain that model [5] retains signal from broad scale sources, but filters out under-sampled signal from compact sources ($B/A \approx 2.3$ instead of 1.7). If so, then broad scale sources are shallower than compact sources. Exclusion of 1σ outliers nonetheless yields a typical magnetic crustal thickness of 47.8 ± 8.2 km. The typical area of extended sources remains 330,000 km^2.. This poses a fundamental question: how did such vast regions of roughly uniformly magnetized crust form? Recent magnetic maps [11, 12] help distinguish among some very different answers.

References: [1] Voorhies, C. V, T. J. Sabaka and M. Purucker (2002) *JGR, 107,* E6, doi:10.129/2001 JE001534, June. [2] Lowes, F. J. (1966) *JGR, 71*, 2179. [3] Cain, J., B. Ferguson and D. Mozzoni (2003) *JGR, 108,* E2, doi:10.1029/2000JE001487, Feb.. [4] Arkani-Hamed, J., (2002) *JGR, 107,* E10, doi:10.1029/2001 JE1835, Oct.. [5] Arkani-Hamed, J. (2004) *JGR, 109,* E09005, doi:10.1029/2004JE00 2265, Sept.. [6] Connerney, J.E.P., et al. (2001) *GRL, 28,* 4015-4018. [7] Purucker, M., et al. (2000) *GRL, 27,* 2449-2452. [8] Langlais, B., M. E. Purucker and M. Mandea (2004) *JGR, 109,* E02008, doi:10.1029/ 2003JE002048, Feb.. [9] Voorhies, C. V. (1998) *NASA Technical Paper* 1998-208608, 38pp, Dec.. [10] Whaler, K. A., and M. E. Purucker (2005) *JGR, 110,* E09001, doi:10.1029/2004 JE002392, Sept.. [11] Hood, L. L. et al. (2005) *Icarus, 177*, 144-173. [12] Connerney, J. E. P., et al. (2005) *Proc. Nat. Acad. Sci., 102,* doi:10.1073/pnas.05 070469102, Oct..