LIMITS TO NON-SYNCHRONOUS ROTATION FOR MAXWELL VISCOELASTIC BODIES

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Introduction: Synchronous rotation is commonly observed in the solar system, particularly among satellites which are close to their primary. As is well known, the tidal torque acting on a dissipative body in an eccentric orbit drives the rotation state close to synchronous. It has been suggested that some bodies, most notably Europa, have attained an equilibrium state which deviates slightly from synchronous rotation [1]. Evidence in support of that idea has been attributed to patterns of surface cracks in the ice shell [2, 3]. We suggest that this interpretation is incorrect. The observational basis can be explained otherwise, and the theoretical basis is incomplete.

Tidal stress patterns similar to those often attributed to non-synchronous rotation can be generated by small values of obliquity [4], as are expected for the Galilean satellites [5,6]. Though the forced obliquity of Europa is small, so too is the forced eccentricity against which it competes for control of the geometry of tidal stress.

Our main objective here is to examine the configuration in which the net torque, averaged over an orbital period, vanishes. We examine the particular case of homogeneous and radially stratified bodies with Maxwell visco-elastic rheology. While the tidal torque does indeed vanish at slightly super-synchronous rotation rates, there is an additional gravitational torque on any permanent departure from spherical symmetry.

In a Fourier analysis of the degree two tidal potential, the frequencies all take the form (p n $\pm$ 2 s), where p in an integer, n is the orbital mean motion, and s is the spin rate. For a body approaching synchronous rotation, some of these frequencies approach zero, but others do not. The nearly constant component of tidal potential produces a nearly constant bulge, on which the tide raising body exerts a "rigid" torque. This rigid torque opposes the dissipative tidal torque associated with the non-vanishing frequencies.

This balance between tidal and rigid body torques depends rather sensitively on the relevant parameters, which include the period and eccentricity of the orbit, and the density, rigidity, and viscosity structure of the extended body. Over much of the parameter space, rigid body torques prevail, and synchronous rotation is maintained, even though the mean tidal torque does not vanish.

Rigid body torque: The configuration of lowest rotational potential energy, for an extended body in proximity to a point mass, has the principal axis of least moment of inertia oriented toward the distant body. Any departure from that state will produce a gravitational torque which acts to return the system toward the state of minimum energy. The rigid body gravitational torque can be written quite generally as

$$T_{rb} = 3 \frac{GM}{r^3} (u \times J \cdot u) \tag{1}$$

where G is the gravitational constant, (M, r, u) are the mass, distance, and unit vector pointing to the distant body, and J is the inertia tensor of the extended body.

The spin angular momentum of a body in principal axis rotation is H = C , where C is the moment of inertia about the spin axis, and is the spin rate. In the absence of torques, the angular momentum remains constant. In an eccentric orbit, the torque varies with time and there are forced departures from uniform rotation, known as forced librations [7].

If the equator plane and orbit plane coincide, the rigid body torque simplifies to

$$T_{rb} = \frac{3}{2} n^2 \left(\frac{a}{r} \right)^3 \Delta J \ S \tag{2}$$

where n and a are the orbital mean motion and semi-major axis, the moment difference is J=B-A, with A<B the principal moments of inertia in the equatorial plane, and

$$S = \sin[2\gamma] \tag{3}$$

where is the angular separation between the distant body and the axis of least inertia of the extended body.

Tidal torque: The tidal torque is similar, in many regards, to the rigid body torque. The formulas above are applicable to both. However, in the tidal case, the moment difference J and lag angle function S are both produced as a response of the extended body to the imposed tidal potential of the distant body

Response to tidal potential: The imposed tidal potential causes deformation of the extended body, which produces an additional potential . At each harmonic degree j, the induced and imposed potentials are linearly related

$$\Psi_j = k_j \Phi_j \tag{4}$$

with the constant of proportionality k_j known as a Love number. In a homogeneous, incompressible elastic body, deformation is resisted by both gravitation and by elastic strength, and these two effects work in parallel. The degree two elastic Love number is [8,9]

$$k_2 = \frac{3}{2} \left(\frac{2\beta}{2\beta + 19\mu} \right) \tag{5}$$

where is the elastic rigidity, and = g R is an effective gravitational rigidity.

In a Maxwell visco-elastic rheology, elastic and viscous effects are combined in series. At each frequency ω, the effective rigidity is a complex quantity

$$\bar{\mu}[\omega] = \mu \left(\frac{(\omega\tau)^2 + i\omega\tau}{1 + (\omega\tau)^2} \right) \tag{6}$$

where the relaxation time is = / , the ratio of the viscosity and elastic rigidity. Using this in the expression for the elastic Love number, we obtain a complex quantity relating imposed and induced potentials. The real part is in phase with the forcing, and contributes nothing to the tidal torque. The imaginary part is in quadrature, and yields the torque.

The four parameters in the complex Love number for a homogeneous Maxwell body only occur in two dimensionless combinations,

$$m = \frac{19\,\mu}{2\,\beta} \quad \text{and} \quad w = \omega\,\tau \tag{7}$$

In terms of these reduced parameters, the result is

$$k_2[m,w] = \; = \frac{3}{2}\left(\frac{1 + (1+m)^2 w^2 - i\,m\,w}{1 + (1+m)^2 w^2} \right) \tag{8}$$

The imaginary part vanishes in both high and low frequency limits. The frequencies of interest here are harmonics of the orbital mean motion: $w_p = p\,n$.

Visco-elastic tidal torque: Using this form of the Love number, we can now estimate the moment difference and phase lag associated with a tidal torque. The moment difference induced by a tidal potential, when averaged over an orbital period, is a sum of terms like

$$\Delta J[e,m,w_p] = \left| k_2[m,w_p] \right| C_p^{-3,0}[e] \left(\frac{n^2 R^5}{G} \right) \tag{9}$$

where R is the mean radius of the extended body, and $C^{-3,0}[e]$ is a Cayley coefficient in the Fourier expansion of $(a/r)^3$. [7]. A lower limit to the moment difference for rigid body torque is the secular limit (w 0) of this.expression. Any non-tidal asymmetries are expected to reorient themselves to align with the tidal bulge, in much the same way as polar motion on Earth or Mars, for example, is expected to move positive mass anomalies into the equator plane [9,10,11]

The factor in the tidal torque associated with the phase lag between the imposed and induced potentials has explicit form

$$\begin{aligned} S[m,w] &= \sin[\,2\tan^{-1}[k_2[m,w]]] \\ &= \frac{-2\,m\,w}{1+w^2}\left(\frac{1+(1+m)\,w^2}{1+(1+m)^2\,w^2} \right) \end{aligned} \tag{10}$$

Combining these results, we can now write an expression for the tidal torque admittance function

$$F[e,m,w_p] = \Delta J[e,m,w_p]\; S[m,w_p] \tag{11}$$

This converts imposed potential into torque.

Final torque balance: If the rigid body torque acted alone, and the rotation rate were initially synchronized with the orbital rate, the average orientation of the long axis of the extended body would be toward the point mass, and the only departures from steady rotation would be small forced librations about that mean orientation. The deviations in both angle and angular rate from the configuration of minimum potential energy would both average to zero. If a sufficiently small steady torque is added to the rigid body torque, the equilibrium state is still steady libration, but the center of libration is offset in angle, but not in rate, from the unperturbed state. The angular offset is just that required to yield a mean rigid body torque equal and opposite to the applied perturbation. The Moon occupies such a state [12].

For synchronous rotation to be achieved by a visco-elastic satellite, all that is required is that the maximum rigid body torque should exceed the tidal torque at the synchronous rate. The least favorable case is that in which the only permanent moment difference is that due to the secular tidal effect. In that situation, the ratio of the tidal torque to the maximum value of the rigid body torque is just

$$W[e,m,n,\tau] = \frac{2}{3}\sum_p \left| k_2[m,w_p] \right| F_p[e]\, S[m,w_p]]] \tag{12}$$

with

$$F_p[e] = C_p^{-3,0}[e] \,/\, C_0^{-3,0}[e] \tag{13}$$

Any satellite for which this ratio is less than one is virtually certain to achieve synchronous rotation. The present analysis is only strictly applicable to homogeneous bodies. If there is a thin elastic lithosphere, decoupled from the deep interior by a fluid layer, as has been argued for Mercury [13], the rigid torque is very nearly the same, but the tidal torque is appreciably reduced, making resonant rotation even more likely.

References:

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