TIDAL DISSIPATION IN THE ICE SHELLS OF ENCELADUS AND EUROPA. G. Mitri[1,2] and A. P. Showman[2], [1] Jet Propulsion Laboratory, California Institute of Technology (Giuseppe.Mitri@jpl.nasa.gov), [2] Department of Planetary Sciences and Lunar and Planetary Laboratory, University of Arizona.

Introduction: To explain the formation of surface features on Europa, Enceladus, and other satellites, many authors have postulated the spatial localization of tidal heating within convective plumes. The pits, domes and chaotic terrains observed on Europa [1,2,3,4] may have resulted from convective motions in the ice shell [1] or melt-thought of the ice shell [3,6,7], and in either case spatially concentrated tidal heating may be necessary to elevate the temperatures and induce partial melting [e.g., 9,10]. Likewise, an ascending diapir, perhaps aided by localized tidal heating, may help explain the active south polar regions of Enceladus [15].

However, the concept that enhanced tidal heating can occur within a convective plume has not been rigorously tested. Almost all models of this phenomenon adopt a tidal heating with a temperature-dependence derived for an incompressible, homogeneous (zero-dimensional) Maxwell material [9,10]. This zero-dimensional model generally predicts greater dissipation at warmer temperatures. However, it is unclear whether this formulation is relevant to the heterogeneous situation of a warm plume surrounded by cold ice.

To determine whether concentrated dissipation can occur in convective plumes, we develop a two-dimensional model to compute the volumetric dissipation rate for a Maxwellian viscoelastic compressible material. We apply the model to the Europa and Enceladus ice shells and we investigate the consequences for partial melting and resurfacing processes on these bodies.

Model: The model examines a two-dimensional horizontal section of a convective ice shell representing a cross section through a convective plume. In this geometry, an upwelling (or a downwelling) in the ice shell is represented as a disk. The disk is embedded in an unbounded material (matrix). At the interface between the disk and the surrounding matrix, the displacement and stress are continuous. We therefore reduce the problem to the classic inclusion problem, the elastic solution for which is well known [16].

For a viscoelastic body the stress-strain relation for a Newtonian rheology is given by

$$\dot{\sigma}_{ik} + (\mu/\eta)\sigma_{ik} = [2\mu\nu/(1-2\nu)]\dot{\varepsilon}_{jj}\delta_{ik} + 2\mu\dot{\varepsilon}_{ik} \quad (1)$$

where the dot denote the derivation in time t, σ_{ik} is the stress tensor, ε_{ik} is the strain tensor, δ_{ik} is the Kronecker delta, μ is the shear modulus ($4\cdot10^9$ Pa), and ν is the Poisson's ratio (0.314).

The ice shell is subject to periodic tidal flexing, represented as $\sigma_{ij} = \sigma_{ij}^0 e^{i\omega t}$ and $\varepsilon_{ij} = \varepsilon_{ij}^0 e^{i\omega t}$, where σ_{ij}^0 and ε_{ij}^0 are the complex amplitudes of the stress and strain tensors, respectively, ω is the orbital frequency, and in the exponent $i^2 = -1$. The stress-strain relation is then reduced to

$$\sigma_{ik}^0 = \frac{2\tilde{\mu}\nu}{1-2\nu}\varepsilon_{jj}^0\delta_{ik} + 2\tilde{\mu}\varepsilon_{ik}^0 \quad (2)$$

where $\tilde{\mu}$ is a complex shear modulus given by

$$\tilde{\mu} = \frac{\omega^2\mu\eta^2}{\mu^2+\omega^2\eta^2} + i\frac{\omega\mu^2\eta}{\mu^2+\omega^2\eta^2} \quad (3)$$

Eq. 2 is formally the stress-strain relation of a Hookean elastic body. Therefore, the solution of a viscoelastic problem for a body subjected to periodic stresses and strains corresponds to the solution of a Hookean elastic problem in the complex field with a shear modulus given by Eq. 3 [17,18]. The known solution to the elastic inclusion problem [16] therefore allows us to calculate σ_{ij}^0 and ε_{ij}^0 everywhere over the two-dimensional domain given assumptions about these quantities infinitely far from the plume.

We compute the volumetric dissipation rate of a Maxwellian compressible material subjected to a periodic tidal flexing as

$$W = \frac{\omega}{2\pi}\oint \mathrm{Re}\left[\sigma_{ik}\dot{\varepsilon}_{ik}\right] dt \quad (4)$$

where Re[Γ] is the real part of any complex function Γ. We compare the results obtained with Eq. 4 with the volumetric dissipation rate of an incompressible (ν = 0.5) zero-dimensional Maxwellian material that is obtained from Eq. 4 considering $\sigma_{ik}^0 = 2\tilde{\mu}\varepsilon_{ik}^0$

$$W = \omega^2\eta\left(\varepsilon_{ik}^0\right)^2 \Big/ \left[1 + \left(\omega^2\eta^2/\mu^2\right)\right] \quad (5)$$

or equivalently $W = \left(\sigma_{ik}^0\right)^2/4\eta$.

The Newtonian viscosity of the ice is given by

$$\eta = \eta_0 \exp\left(A\left(T_{ref}/T - 1\right)\right) \quad (6)$$

where η_0 is the ice viscosity at the reference temperature T_{ref} = 273 K (10^{13} Pa s ≤ η_0 ≤ 10^{15} Pa s), A = 26 is a constant corresponding to an activation energy for the water ice of 60 kJ mol^{-1}, and T is the temperature.

Results and Discussion: The tidal heating is strongly temperature dependent in a thermal plume and concentrated dissipation can occur in convective plumes (Fig. 1). As example, we present the computation of the volumetric dissipation rate for the

Europa ice shell. Fig. 1 shows the volumetric dissipation rate W as a function of the distance r from the center of the thermal plume. This model assumes a tidal stress of 0.1 MPa. The radius of the plume is 1 km and $\eta_0 = 10^{14}$ Pa s. The grey colored area in the plot represents a thermal plume in the ice shell with a temperature of 250 K, 260 K, and 270 K. The white area shows the ice surrounding the thermal plume with a fixed temperature 250 K. As expected, the tidal dissipation far from the plume is independent of the plume temperature. The existence of the plume causes a slight perturbation in the background tidal-heating rate within several plume radii of the plume edge. Most interestingly, however, the tidal-heating rate *inside* the plume depends strongly on plume temperature and is six times greater for a plume with temperature 270 K than a plume with temperature 250 K. Our calculation provides the first quantitative verification of the hypothesis by Sotin et al. [9] and others that the tidal dissipation rate is a strong function of temperature inside a convective plume.

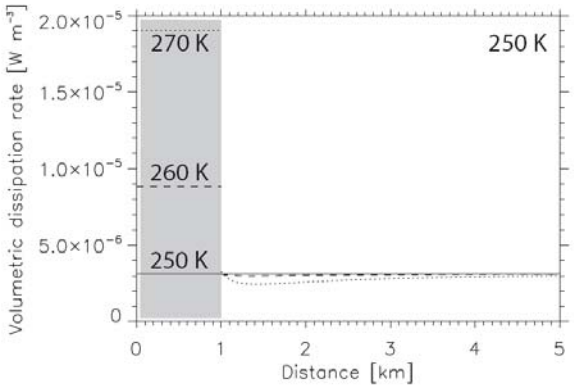

Figure 1. *Volumetric dissipation rate of the Europa ice shell as a function of distance from the center of a thermal plume.*

Fig. 2 shows the volumetric dissipation rate W in a plume versus plume temperature when the background temperature is held constant at either 200 K or 250 K (solid lines). Here, we assume that $\eta_0 = 10^{13}$ Pa s. This is compared with the volumetric dissipation rate of an incompressible zero-dimensional Maxwellian material for a fixed stress (0.1 MPa) (dotted line), for a fixed strain rate ($2 \cdot 10^{-5}$) (dashed line), and with a two-dimensional compressible Maxwellian model (dotted-dashed line). Just as occurs with the zero-dimensional Maxwell model at constant strain amplitude (dashed line), our results (solid lines) show that the tidal-dissipation rate in a plume increases strongly with temperature at low temperatures and decreases with temperature at high temperatures, reaching a maximum at intermediate temperatures. Interestingly, however, the temperature at the maximum in our solution exceeds that in the zero-dimensional Maxwell model. This difference arises from the fact that the stress and strain within the plume are affected by the deformation of the surrounding matrix. Note that if $\eta_0 > 10^{14}$ Pa s then the maxima occur at temperatures above 270 K; in this case, the plume dissipation rate increases monotonically with temperature up to the melting point.

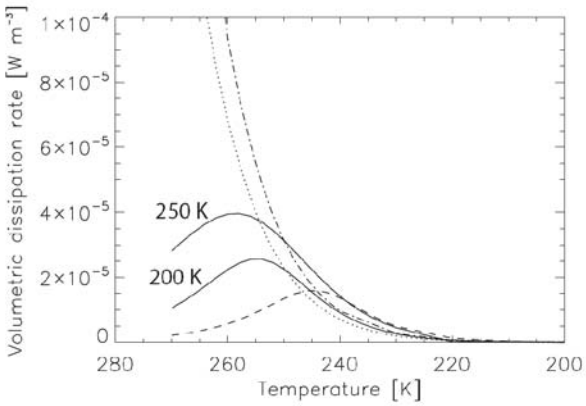

Figure 2. *Dissipation rate as a function of plume temperature when the temperature of the surrounding matrix is held constant (solid lines). Europa parameters are assumed.*

Our results (Figs. 1-2) imply that the tidal heating is strongly temperature dependent in a convective ice plume and could produce elevated temperatures and local partial melting in the ice shells of Europa and Enceladus. On Europa, such localized heating could help allow the formation of domes and chaos terrains by convection. On Enceladus, localized tidal heating in a thermal plume could explain the concentrated activity at the south pole and its associated heat transport of 2–7 GW

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