POROUS MATERIAL MODELS FOR IMPACT STUDIES. Keith A. Holsapple, University of Washington, 32400, Seattle, WA 98195, holsapple@aa.washington.edu.

Introduction: Recent research and measurements of small solar system bodies have pointed to a highly porous structure for many of them. For example, the Deep Impact event suggests a rocky/ice mixture material with a macroscopic mass density less than 0.5 g/cm^3. If so, the pore space could be as much as 75%. Other bodies also seem to have mass densities much lower than suspected a couple of decades ago. As a consequence, appropriate models for porous bodies are needed for scaling and code calculation studies. That porosity property is relatively little studied, yet most likely plays a major role in applications such as disruptive collisions, melt and vapor production, impacts into comets, crater scaling, ejecta from cratering, and so on.

Background: The mechanics of the response of porous materials to impacts is much different than nonporous materials: because of the large energy that is absorbed in the large crushing and volume change when subjected to pressure. That property of energy absorption has been a driver in the use and studies of porous materials for a long time. Thus, foams are used for packaging, and for components of the bumpers of automobiles. In the late 1960's there was considerable interest in the defense community about using porous materials to protect weapons systems from x-ray deposition from nuclear bombs. And that same crushing property probably serves to protect asteroids from collisions; including, unfortunately, some methods considered to deflect those threatening Earth.

As is the case for many aspects of physics, the developments in the planetary community followed on from earlier ones in the defense community. Seaman et al. [1] give a thorough review of models in 1974. All models are based on a picture of a solid material with a mass density ρ_s surrounded by voids giving a net macroscopic mass density of ρ. The ratio $\alpha=\rho_s/\rho$ is called the distension, the porosity is $\phi=1-1/\alpha$.

The most commonly used model is the "P-α" model of Herrmann [2], in which a crush behavior is specified in terms of a relation between the pressure P and the distension α. Other approaches include the Holt et al. [3] model which relates the strain ε to the distension α. Recently Wunnemann et al., [4] again present and study the strain-based approach, they call it an "$\varepsilon-\alpha$" model.

Thermodynamics of porous materials: The mechanics of porous materials is based on the dilatational response, which is governed by the equations of state. The equation of state fields include the pressure p, the internal energy per unit mass e, the mass density ρ, the entropy per unit mass η and the temperature θ. The porous material, a mixture of a solid component and voids, has equations of state of the form

$$p = p(\rho,\theta) \quad e = e(\rho,\theta) \quad \eta = \eta(\rho,\theta)$$

The solid component has its own thermodynamic state defined by not only by its mass density ρ_s, but also by its separate p_s, e_s, η_s, θ_s, all related by its equations of state:

$$p_S=p_S(\rho_S,\theta_S) \quad e_S=e_S(\rho_S,\theta_S) \quad \eta_S=\eta_S(\rho_S,\theta_S)$$

which define its constitution. But it is assumed that the voids have zero mass, zero internal energy, zero entropy and no defined temperature. Therefore the internal energy and entropy per unit mass for the mixture are the same as for the solid component, and there is but one temperature for both the mixture and the solid:

$$e=e_S(\rho_S,\theta) \quad \eta=\eta_S(\rho_S,\theta)$$

Therefore the free energy $\psi = e - \theta\eta$ is also the same for the porous material as for the solid

$$\psi = \psi_s(\rho_s,\theta)$$

But the relations

$$p=\rho^2\,\partial\psi(\rho,\theta)/\partial\rho \quad p_S=\rho_S^{\,2}\,\partial\psi_S(\rho_S,\theta)/\partial\rho_S$$

must hold. So, using $\rho_s = \alpha\rho$:

$$\frac{\partial\psi_S}{\partial\rho}=\frac{\partial\psi_S}{\partial\rho_S}\frac{\partial\rho_S}{\partial\rho}=\alpha\frac{\partial\psi_S}{\partial\rho_S}$$

which gives for the porous material pressure the relation

$$p = \left(p_S(\alpha\rho,\theta)\right)/\alpha$$

for the pressure of the porous material in terms of the pressure for the solid component.

This simple relation between the material pressure and the solid component pressure is an important part of all models. It can be proved in other ways, e.g. based on the average cross-section across any material with the given distension.

So, taken together, we have for the porous material the equations of state

$$e = e_s(\alpha\rho,\theta) \quad p = p_S(\alpha\rho,\theta)/\alpha \quad \eta = \eta_s(\alpha\rho,\theta)$$

and all are determined by the three EOS functions defining the solid component. Therefore any standard EOS forms for the solid part can be used to give the porous theory. These thermodynamic relations then determine all other aspects of the dilatational response.

One very important part of those is the resulting expression for the wave speeds, given as

$$c / c_0 = \left(1-K_S\alpha_P/\alpha^2\right)^{-1/2}$$ in terms of the slope α_P of the response curves, the bulk modulus K_0 and

wave speed c_0 of the solid component. The wave speed for the elastic portions are typically much slower than the bulk speed of the solid component, and the slope of the elastic curves is a very important detail for calculations of ejecta, disruptions, and for seismology.

Because there is a new state variable (the distension α) in these EOS relations, additional relations are needed to relate the distension to any two of the pressure, density energy, entropy or temperature.

Deformations of porous materials: Fig. 1 shows a generic plot of the response of a porous material to loading at constant temperature, as the distension versus the pressure.

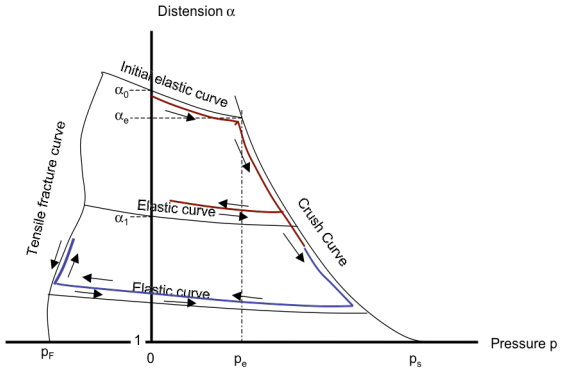

Fig. 1. Loading and unloading curves for a typical porous material at constant temperature.

Important elements of this plot include (1) the crush curve defining the compressive pressure required to crush the material (the rightmost curve), (2) the elastic unloading curves from the crush curve (nearly horizontal), and (3) the curve depicting fracture of the materials at some prescribed tensile pressure (the left curve) where voids can be reintroduced into the material. A comprehensive model needs to describe those three essential features.

A model will use some functional fits for the crush curve and for the unloading curves. Herrmann [2] shows how the unloading curves can be determined by knowledge of elastic wave speeds. Wunnemann et al. [4] fit a response in the strain-distension space (that form is not directly measured in material tests), and assume the unloading is at constant distension. (That assumption makes the porous wave speeds the same as the solid wave speeds.) Neither model addresses the fracture curve at the left, or the dependence of the crush on the other independent thermodynamic variable, such as the temperature.

A common way to test and validate a given model of choice is to compare code calculations using the model to certain experimental results. The Herrmann [2] and the Wunnemann et al. [4] models were both compared to an impact into a porous aluminum specimen, and both did well. But that experiment exercises only the crush curve representation of a material with small porosity, and not the unloading and/or tensile spall at all. The crush curve can easily be adjusted to fit that experiment. And thousands of time steps are easily accommodated in the code calculation of that experiment.

In contrast, hypervelocity impacts into small bodies are at velocities of tens of km/s and the loading is very rapid. In a code calculation, the entire pressure pulse and crush is often over a few time steps only. So a robust method is desired that captures the correct response in a few time steps, without excessive sub cycling. Studies of impacts using the Herrmann model as implemented in the CTH code will be shown, and the results can be very poor. So an alternate approach is needed.

A new approach: I will outline a new approach to this problem, and a new way to implement it in code calculations. I begin with the same P-α form as the Herrmann model, but use a log-based empirical fit: the nature of that fit can be seen in the following figure 2 where it is compared to static crush measurements of an 83% porous material.

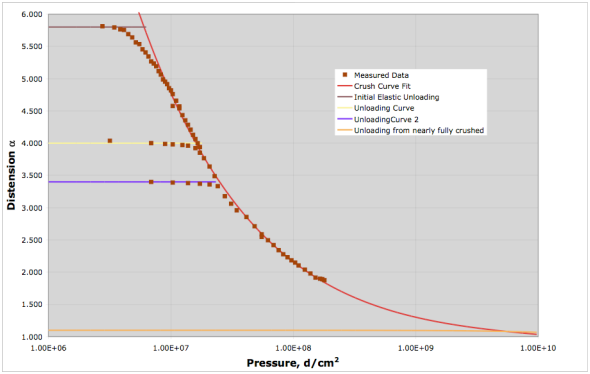

Fig 2. Measured data for a material with initial porosity of 83%, for loading and for unloading.

I shall present the forms of these fits. The elastic curves are determined by data for elastic wave speeds. I will present a new robust direct calculation method that obviates the need for sub cycling and gives good results with crush over only a few time steps. Finally, I will present calculations of impacts, comparing the Herrmann, Wunnemann et al., and this new approach.

References:

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[4]. Wunnemann, K., Collins, G. S., Melosh H. J., Icarus 180, 514-527, 2006.