INTERIOR STRUCTURE OF TITAN. G. Mitri\textsuperscript{1}, L. Iess\textsuperscript{2}, N. J. Rappaport\textsuperscript{1}, \textsuperscript{1}Jet Propulsion Laboratory, California Institute of Technology (Giuseppe.Mitri@jpl.nasa.gov), \textsuperscript{2}Dipartimento di Ingegneria Aerospaziale ed Astronautica, Universitá La Sapienza.

Introduction: Cassini radio science data indicates that Titan is nearly in hydrostatic equilibrium [1]. We have used thermal models [2] and hydrostatic equilibrium theory [3] to derive interior models of Titan. We show as the measurement of the principal quadrupole gravitational coefficient $C_{22}$ from the Radio Doppler data from Cassini spacecraft will improve our knowledge of the interior structure of Titan.

Moment of inertia: The axial moment of inertia $(C)$ gives us an indication of the degree of the interior differentiation of Titan. For an undifferentiated interior structure

$$C = \frac{2}{5}MR^2$$

(1)

where $M$ (1.346-10\textsuperscript{23} kg) is the total mass and $R$ (2,575 km) is the radius. Assuming an interior structure of Titan differentiated in $n$ layers of constant densities, we can write the axial moment of inertia as

$$C = \frac{8\pi}{15} \sum_{j=1}^{n} \rho_j \left( R_j^5 - R_{j-1}^5 \right)$$

(2)

where $R_j$ and $R_{j-1}$ are the outer and inner radius of the $j^{th}$ shell with density $\rho_j$, respectively. The axial moment of inertia constrains the mass distribution in the interior.

The measured gravity field of Titan is dominated by a near hydrostatic quadrupole component [1]. Therefore we focus our analysis to models of interior structure consistent with hydrostatic equilibrium theory [3,4]. The near hydrostatic equilibrium allows us to adopt the Radau approximation. Therefore, we can write the moment of inertia normalized to $MR^2$ as [3]

$$\frac{C}{MR^2} = \frac{2}{3} \left[ 1 - \frac{2}{5} \left( \frac{4 - k_f}{1 + k_f} \right)^{3/2} \right]$$

(3)

where $k_f$ is the fluid Love number.

The rotational distortion parameter is given by [3]

$$q_r = \frac{\omega^2 R^3}{GM} = 4.0 \cdot 10^{-5}$$

(4)

where $G$ is the gravitational constant, and $\omega$ (4.56-10\textsuperscript{-6} s\textsuperscript{-1}) is the rotational angular velocity of Titan, equal to the mean motion.

The principal quadrupole gravitational coefficient $C_{22}$ is given by [3]

$$C_{22} = \frac{B - A}{4MR^2}$$

(5)

where $A$, $B$ and $C$ are the ellipsoidal principal moment of inertia $(C > B > A)$. In the condition of rotational and tidal equilibrium, $C_{22}$ is related to $q_r$ by

$$C_{22} = \frac{1}{4} k_f q_r$$

(6)

Here we adopt a range of values for the principal quadrupole gravitational coefficient $C_{22}$. For a fluid Love number $k_f = 3/2$, $C_{22} = 1.5 \cdot 10^{-5}$, the axial normalized moment of inertia $C/\!\!MR^2$ is 0.4, and Titan is a undifferentiated body. For $k_f = 1$, $C_{22} = 10^{-5}$, and $C/\!\!MR^2 = 0.34$. These values indicate a partial differentiation of the interior of Titan. For $k_f \sim 0.8$, $C_{22} = 8 \cdot 10^{-6}$, and $C/\!\!MR^2 \sim 0.31$. These values indicate a full differentiation of the interior. For comparison, $C/\!\!MR^2$ is 0.346 for Europa, 0.3115 for Ganymede and 0.3549 for Callisto [4].

Thermal model: Here we describe our method to determine the interior structure of Titan [2]. We consider that the interior structure of Titan is differentiated in $n$ layers. The outer layers of Titan are composed mainly of water in liquid and solid phases. The water-ice can be present in the interior of Titan in high-pressure polymorph layers with phases I, III, V, and VI. We explore different scenarios for the deeper interior of Titan in the presence of a metallic core and a silicate mantle, or of a mixture of rock, metals and ice.

The pressure $(P)$ dependence of the ice I, III, V, VI and the liquid water is determined as

$$\rho(P) = \rho_0 \left[ \frac{K_0'}{K_0} \right]^{\frac{n}{K_0'} + 1}$$

(7)

where $K_0'$ and $K_0$ are the pressure derivative of the bulk modulus and the bulk modulus, respectively. The values of $K_0$ and $K_0'$ are as in [5,6]. The reference density $(\rho_0)$ of the ice I is 920 kg m\textsuperscript{-3}, ice III is 1,140 kg m\textsuperscript{-3}, ice V is 1,235 kg m\textsuperscript{-3}, ice VI is 1,320 kg m\textsuperscript{-3}, and liquid water is 1,000 kg m\textsuperscript{-3} [5].

For an initial ammonia-water concentration of the subsurface ocean and ice I grain size, using scaling law of thermal convection [7], we determine the thickness and thermal state (conductive and convective states) of the ice I shell. The temperature of the interface between the ice I shell and the subsurface ocean can be found from the phase diagram of the water for a given
ammonia-water concentration [8,9]. The basal temperature of the ocean is determined considering an adiabatic ocean and equaling the temperature at the base of the ocean at the phase diagram of the water. The temperature and pressure at the base of the ocean determines the phases of the ice high pressure layer.

Assuming that the present radiogenic and tidal heat flux (F) from the interior is 0.007 W m$^{-2}$ [7] for a radius of 2,575 km, and further that the mass of Titan is constant during its thermal evolution, and considering that the triple point pressure of the ice III is 209.5 MPa, ice V is 355.5 MPa and ice VI is 618.4 MPa [9], we compute the volume, mass and thickness of each layer (ice I, III, V, VI, and subsurface liquid layers) in fraction of the heat from the interior.

Figure 1 shows an example of our calculations. This model is for an initial ammonia-water concentration in the liquid layer of 5 per cent and an ice I grain size of 0.1 mm. The interior of Titan is differentiated into a rocky interior, a high pressure ice layer (with phases III, V and VI), a subsurface liquid layer, and an outer ice I shell. The water ice-liquid outer layer is ~600 km thick. For this model, the rocky interior density is 2,900 kg m$^{-3}$. Such low density requires that the silicates in the rocky interior are likely hydratates. Consistent with the scenario of model illustrated in Fig. 1, the Cassini RADAR determination of Titan’s spin rate offset from synchronous rotation suggest that the ice I shell is decoupled from the interior by a subsurface ocean [10].

Using Eq. 2, the moment of inertia computed for the model of Fig. 1 is $C/MR^2 = 0.34$ for $F = 0.007$ W m$^{-2}$. The interior structure model proposed is consistent with the Cassini gravimetric data.

Fig. 1 shows also that, under a range of conditions, thermal convection can occur in the ice I shell (see also [7]). For $F = 0.007$ W m$^{-2}$ the convection is in a subcritical status.

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