THE DEFORMATION OF ASTEROIDS FROM YORP SPIN-UP . K. A. Holsapple[1]. [1]University of Washington 352400, Seattle, WA 98195, holsapple@aa.washington.edu.

Introduction: The Yarkovsky-O'Keefe-Radzievskii-Paddack, or "YORP" effect is a process that can modify the spin of small asteroids by the cyclical absorption and re-radiation of heat from the sun. That process has recently become a popular candidate for the formation of binary asteroids. Walsh et al. [1] reported a numerical study using an "N-body" code of the deformations of a rubble pile asteroid being slowly spun up. The N-body approach models the asteroid as a large number (~1000 in that case) of spherical particles with gravitational interactions, but no inter-particle bonding. They find that the increasing spin of both initially spherical and initially prolate bodies causes the bodies to adjust their shape and become oblate, leading then to mass shedding of particles form the equator, which can subsequently re-form into a bound satellite.

One would expect that the N-body analysis should approach a continuum analysis as the number of particles became infinite, so a continuum analysis of this same problem is warranted. Further, a successful analytical continuum approach has the advantage of determining the results as specific algebraic functions of the problem parameters, rather than tracking one solution at a time from a code calculation. Here I consider the YORP spin-up processes using a continuum model for a rubble-pile asteroid, and find the deformations experienced when the static spin limit is exceeded.

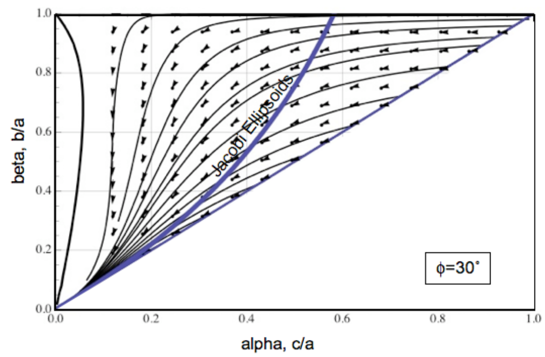
Assumptions: YORP is a process typically measured in millions or even hundreds of millions of years [2]. For example, Durech et al. [3] measured the rotational acceleration for the small 1.7 km asteroid 1862 Apollo as 5.5 10^{-8} rad/d^2 which is about 10^{-17} rad/s^2. For the ~300 m Itokawa asteroid, they measured a value about three times higher. The limit spin rates of asteroids are on the order of 10^{-3} rad/s, so that acceleration starting from a zero spin would achieve the limit spin in 3 million years. In such a process, the dynamics are quasi-static, and characterized by a very slowly changing spin or angular momentum. Since the YORP time scale varies as size R^2, larger asteroids spin up even more slowly.

So, adjustments of shape from YORP effects occur very slowly. That fact justifies ignoring all accelerations except those due to a slowly changing spin: i.e. using a quasi-static approach. That is the same steady-state assumption also used to determine the static limit spins of those bodies. Holsapple [4], [5] determined the spin limits of ellipsoidal asteroids using a continuum model for a homogeneous spinning, self-gravitating body. Those results for solid bodies generalize the classical studies presented in detail by Chandrasekhar [6] of the equilibrium limits for spinning fluid bodies - the well-known Maclaurin and Jacobi shapes for fluids. Holsapple used the well-known Mohr-Coulomb continuum failure models of soil mechanics to derive exact solutions for the envelopes of spin limits as a function of the ellipsoidal shape and the angle of friction of the body. Later Holsapple and Michel [7] used the closely related Drucker-Prager model and obtained essentially the same results for the limit spins.

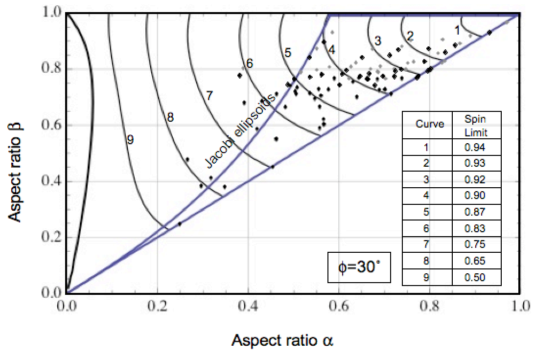
Here the continuum approach using the Drucker-Prager model is used to determine the deformations during the quasi-static, slow spin-up of asteroids. The results can be directly compared to the results of Wash et al. [1] for the rubble pile N-body approach. But, as presented below, the results are very different.

The governing equations are those for momentum balance, both linear and angular, and the constitutive equations for the assumed material. A fundamental assumption here, and in the classical fluid studies reported in [6], is that the motions are homogeneous: the velocity gradient is the same throughout the body. In that case the partial differential equations that govern the motions of a continuum reduce to a set of ordinary differential equations. So the analysis here closely follows that of Chandrasekhar [6] given for the fluid bodies. But, for a solid, in addition to the pressure, the complete stress tensor must be retained; so there are 5 more stress variables. And in addition, the incompressible assumption for a fluid is replaced by the flow rules for a plastic body. Since the flow rule allows for the dilation characteristic of granular materials the mass density is also a variable to be found. And finally, the constant of proportionality of the flow rule is an unknown variable. Thus, the 10 equations and unknowns of the fluid theory (see [6], pg 71, eq. 57) become 17 equations and unknowns for the solid case. The common assumption of motions about the shortest principle axis reduces the set to 11 equations and unknowns, which can be integrated using numerical means. And, within the accuracy of the numerical integrations, the result are exact. The complete equations in various forms and comparisons to others in the literature will be given in a forthcoming paper.

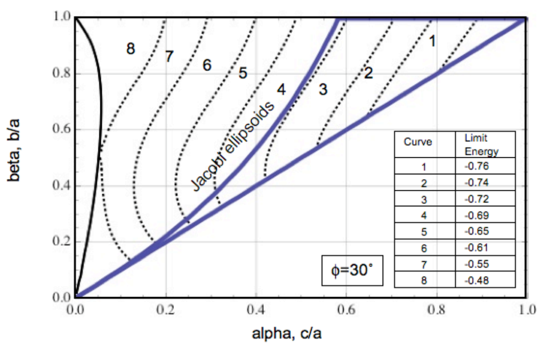
Some Results: The shape of the ellipsoidal asteroid is measured by the ratios of its axes, $\alpha=c/a$ and $\beta=b/a$ where the axes semi lengths satisfy $a\geq b\geq c$. In a plot in the $\alpha-\beta$shape space, assuming a friction angle of 30˚, the deformations during slow spin-up follow the arrowed paths shown:



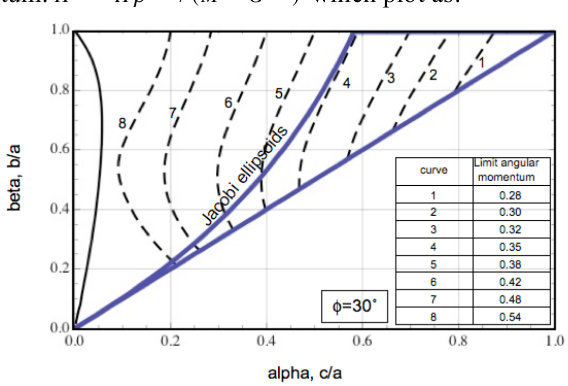
These spin-up trajectory curves cross the curves of constant limit scaled spin $\Omega = \omega \big/ \sqrt{\pi \rho G}$ given as:



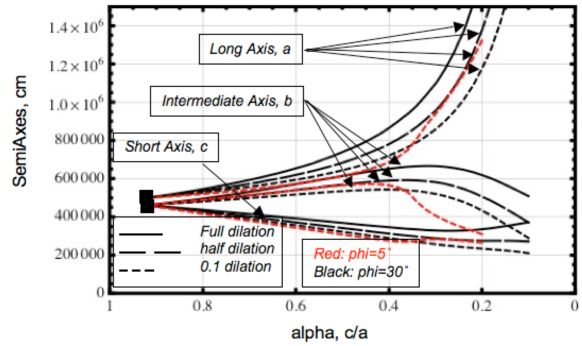
and the curves of constant scaled energy $E^* = E / (\rho^{1/3} M^{5/3} G)$ shown as



and the curves of constant scaled angular momentum: $H^* = H\rho^{1/6} / (M^{5/3} G^{1/2})$ which plot as:



An overlay of these plots will show that, during spin up, the energy and angular momentum are increasing, but that the spin is actually <u>decreasing</u>. That fact is a result of the body's shape changes: it becomes longer and thinner, as shown:



which increases it angular moment.

Summary: Here it is found that an ellipsoidal body will adjust its shape smoothly and in a well-defined way during increasing angular momentum. With imposed increasing angular momentum such as might be donw by the YORP effect, initially oblate bodies (including spherical ones) tend to remain oblate and become more flattened, but bodies beginning as a general ellipsoidal or prolate never become oblate, as depicted on the first figure above. And, interestingly, it is found that the term "spin-up" is not even appropriate. For spins below the limit spins, where the body remains rigid, the YORP effect can indeed increase the spin rate; but at the <u>limit</u> spin states, where deformations are possible, the deformation is an elongation which increases the rotary inertia so that an increasing angular momentum results in a *decreasing* spin rate. The spins never approach the spins necessary for mass loss from the equator.

Those results are very different than the Walsh et al. [1] results using the N-body calculations. Some reasons will be discussed.

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