DERIVATION OF LUNAR GRAVITY ANOMALIES AND ITS INDISPENSABLE EFFECTS IN LUNAR INTERIOR STRUCTURE RESEARCH AND FUTURE PLANS.   H. W. Yang[1], W. J. Zhao[2], Z. H. Wu[3],

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Introduction: Here are three parts below, including derivation of the spherical coefficients of lunar gravity model from orbit tracking data, acquisition of lunar gravity anomaly, and advantages of the more detailed gravity model to future plans, through which we will realize satellite gravity measurements bring us a further knowledge of the structure and evolution of the moon.

We will build in brief the relationship between orbital elements and spherical coefficients. Simply, the motion of a satellite is described by Kepler's three law, which is formulated by six orbital elements. But because of the irregularities of lunar gravity field and other disturbing forces, the six elements have been changed with time. The change rate of the six orbital parameters can be expressed below [1] (Fig. 1),

$$\frac{da}{at} = \frac{2a^2}{b}\sqrt{\frac{a}{GM}}\left(eS\sin\upsilon + \frac{p}{r}T\right) \quad \frac{di}{at} = \frac{r}{b}\sqrt{\frac{a}{GM}}W\cos(\omega+\upsilon)$$

$$\frac{de}{at} = \frac{b}{a}\sqrt{\frac{a}{GM}}\left[S\sin\upsilon + \left(\frac{r+p}{r}\cos\upsilon + \frac{er}{p}\right)T\right] \quad \frac{d\Omega}{at} = \frac{r}{b}\sqrt{\frac{a}{GM}}W\frac{\sin(\omega+\upsilon)}{\sin i}$$

$$\frac{d\omega}{at} = \frac{b}{a}\sqrt{\frac{a}{GM}}\left[-\frac{1}{e}S\cos\upsilon + \frac{r+p}{ep}T\sin\upsilon - \frac{r}{p}W\sin(\omega+\upsilon)\cot i\right] \quad (1)$$

They are well-known Newton's Equations. In my opinion, the expressions are not hard to understand as they look like. It just depends on Newton' second law, $F = m \cdot a$. Here, F is the total of all perturbing forces, including the variations of the moon, attractions of the sun and earth, the radiation pressure of the sun, and earth tides (not drag of atmosphere on the moon). The acceleration, a can be described by derivatives of satellite velocity or location. Further, the location and velocity of a satellite can be expressed with the six orbital elements. Only considering lunar disturbing gravitation, three components S, T, W of perturbing forces along can be expressed with spherical coefficients. So, we realize the correlation between orbital elements and spherical coefficients.

In analytical geometry, coordinates of satellite can be computed by the six orbital elements [2].

$$X = r[\cos\Omega\cos(\omega+\upsilon) - \sin\Omega\sin(\omega+\upsilon)\cos i]$$

$$Y = r[\sin\Omega\cos(\omega+\upsilon) + \cos\Omega\sin(\omega+\upsilon)\cos i]$$

$$Z = r\sin(\omega+\upsilon)\sin i, \quad r = a(1-e^2)/(1+e\cos\upsilon) \quad (2)$$

So, depending on the equations, the change of position, ΔX, ΔY, ΔZ, can be approximated by $\Delta_t a$, $\Delta_t e$, …, in the form of   Taylor expansion.

$$\Delta X = \frac{\partial X_0}{\partial a}\Delta_t a \cdot \frac{\partial X_0}{\partial e}\Delta_t e + \frac{\partial X_0}{\partial i}\Delta_t i + \frac{\partial X_0}{\partial \Omega}\Delta_t\Omega + \frac{\partial X_0}{\partial \omega}\Delta_t\omega + \frac{\partial X_0}{\partial \upsilon}\Delta_t\upsilon ,$$

$$\Delta_t a = \int_{t_0}^{t}\frac{da}{dt}dt, \quad \Delta_t e = \int_{t_0}^{t}\frac{de}{dt}dt, \ldots, (\Delta Y, \Delta Z \text{ is similar}) \quad (3)$$

Further, the change of coordinates of satellite can be expressed as a summation of initial position and the changes of position.

On the other side, the coordinates(X_p, Y_p, Z_p) of satellite in terrestrial system is a vector summation of the position of observing station in the same system and the coordinates(α, δ, s) of satellite in observing station coordinate system, which is acquired by Doppler tracking. Finally, we know the correlation between the Doppler orbit data and spherical coefficients. We give the expressions in the form below

$$\alpha = \alpha(X_p, Y_p, Z_p; t; a_0, e_0, i_0, \Omega_0, \omega_0, T_0; C_{nm}, S_{nm})$$

$$\delta = \delta(X_p, Y_p, Z_p; t; a_0, e_0, i_0, \Omega_0, \omega_0, T_0; C_{nm}, S_{nm})$$

$$s = s(X_p, Y_p, Z_p; t; a_0, e_0, i_0, \Omega_0, \omega_0, T_0; C_{nm}, S_{nm}). \quad (4)$$

The explicit formulations are referred to Kaula [3]. The expressions are used for solving the spherical coefficients and satellite orbit determination. In practice, the equations will be solved by means of a least-squares collocation. To get a strong solution, more observations should be better. The theories above are the fundamental principals of GEODYN/SOLVE.

However, the most important problem of lunar gravity models is a lack of direct observations on the farside. The satellite-to-satellite Doppler tracking in SELENE makes compensations for the loss and also improve the quantity of the spherical coefficients [4].

Additionally, an effective method of gravity reduction is essential for researching the structure and evolution of the moon. So far, Bouguer anomalies map have been calculated globally with Clementine data [5] by a simple method [6], which is not adequate.

Because of high-frequency features of the topography and isostatic effects, downward continuation of the satellite gravity signal is rather difficult. For modeling the topographic and isostatic masses, they must be divided into mass elements of simple geometrical shape. Depending on the location of the computation point Q, different mass elements methods will be applied [8]. To describe the effect in the near zone, the tesseroid (Fig. 2) can yield sufficient results. It is determined in the form of summation of triple integral of each volume element,

$$V(Q) = G\sum_i \rho_i \int\!\!\iint_{\Omega_i} \frac{d\Omega}{\ell} \quad (6)$$

In the immediate vicinity, the prism with a rectangular shape is provided. If in the far zone, the effect of the distant masses can be approximated by point mass, mass layer or mass line [8]. It is reasonable that proper mass elements can be used in the relevant area [8].

According to the theories above, we know precise satellite gravity measurements and effective gravity reductions can produce gravity anomalies that are reliable to interpret the internal structure and evolution of the moon. In practice, combination of gravity anomalies and topography will lead to many important scientific results. The discovery of the mascons is a good example. It is characterized by high positive gravity anomaly and low elevation surface. The five principal mascons were commonly identified, Imbrium, Serenitatis, Crisium, Nectaris, and Humorum, which can be easily visible in the figure 3. However, the mechanism to support he positive gravity anomaly of the nearside mascons remains controversial. In other word, we need more geophysical data from other deployments like lunar seismometers, or principals like the theory of meteorite impact. In the new gravity model from SELENE mission, new masons can be identified. Moreover, the model reveals that negative anomaly rings appear on the farside [4]. This kind of diversity phenomenon appears everywhere on the global moon. Therefore, division of the moon depending on geophysical and other results is indispensable.

Efficiency of the gravity model and determination of satellite orbit are interactive to each other. Precisely satellite tracking orbit is determined by gravity field model with high quality. Inversely, gravity model with high precision is directly influenced by satellite orbit parameters. Although satellite-to-satellite tracking in SELENE mission, satellite-to-satellite tracking like GRACE, or a gravity gradiometer like GOCE have not been performed. They should be a high priority for future lunar science [9].

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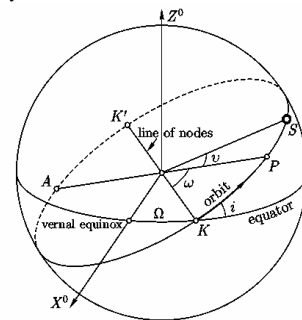

Fig. 1 *Satellite ellipse orbit: P the perigee, A the apogee, K the ascending node, K' the desending note, S the instantaneous position of satellite*
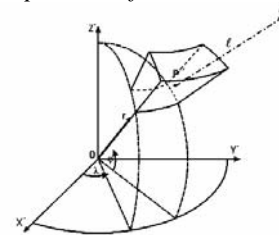

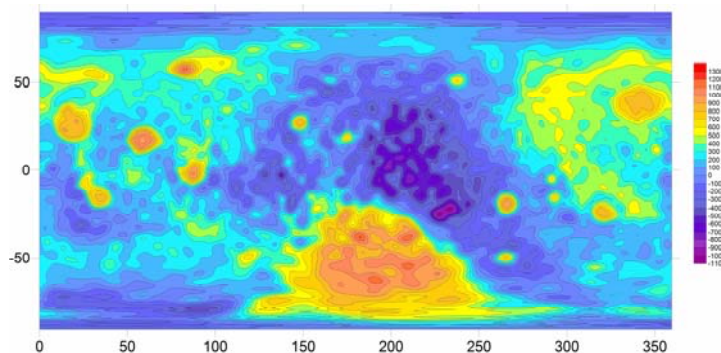**Fig. 2** *Geometry of the tesseroid*

Fig. 3 *A 50th degree resolution gravity anomaly on the lunar surface*
（Programmed by Hongwei Yang, data from LP150Q and LALT-STM359）