PLANETARY SURFACE DATING FROM CRATER SIZE-FREQUENCY DISTRIBUTION MEASUREMENTS: DIFFERENTIAL FORMS OF PRODUCTION FUNCTION POLYNOMIALS. G. Michael, G. Neukum, Freie Universitaet Berlin, Germany. (gregory.michael-at-fu-berlin.de)

Crater size-frequency counts are presented in different forms: most commonly either reverse-cumulative or with a root-2 incremental binning [1]. While it is straightforward to plot data points in either presentation for comparative purposes, the transformation of the production functions between the two is less so. Here we present the expressions [2] for the production functions polynomials of the reverse-cumulative presentation in the Hartmann (root-2 incremental) and R-plot presentations.

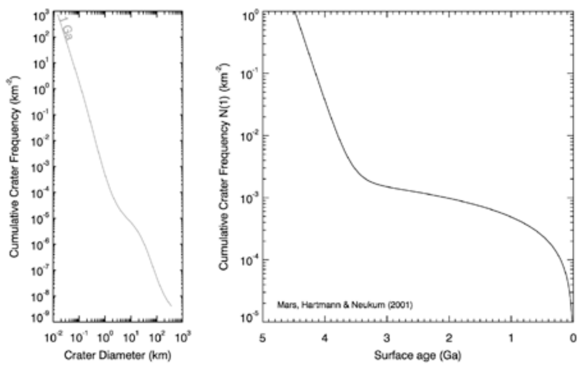

Figure 1: Left: Polynomial derived from piecewise-normalised reverse-cumulative production function [3,4]. Right: Mars chronology function [4,5,6].

Since the production function is a simple curve on a log N_{cum} vs. log D plot, it was found convenient to approximate it with a polynomial function [3,6],

$$\log N_{cum} = p, \text{ where } p = \sum_{i=0}^{n} a_i x^i \text{ and } x = \log D$$

This can be expressed in a differential form,

$$F = -\frac{dN}{dD} = -\frac{dN}{dp}.\frac{dp}{dx}.\frac{dx}{dD}$$
$$= -10^p \ln 10 \,. \sum_{j=1}^{n} j a_j x^{j-1}.\frac{1}{D \ln 10}$$
$$= -\frac{1}{D}.10^{\sum_{i=0}^{n} a_i x^i}.\sum_{j=1}^{n} j a_j x^{j-1}$$

so as to be able to make a direct comparison with the isochrons used in the Hartmann presentation, or for considerations in connection with transforming the function between the Moon and other bodies in an R-plot.

Hartmann presentation

In the Hartmann presentation, which is non-cumulative, the data are split into bins of width $D(\sqrt{2}-1)$, so the production function takes the form:

$$H(D) = FD(\sqrt{2}-1)$$
$$= -(\sqrt{2}-1)10^{\sum_{i=0}^{n} a_i x^i}.\sum_{j=1}^{n} j a_j x^{j-1}$$

Figure 2 compares the Hartmann isochrons for Mars (2002 iteration, unpublished) with their equivalents derived from the Ivanov (2001) production function [4] and the Hartmann and Neukum (2001) chronology [5]. The Hartmann values are essentially the same for diameters below 1 km; they exceed those of the Ivanov production function by around a factor of 3 in the diameter range of 2-8 km; agree again over the range 16-64 km; and are greater by a factor of ~2 at the 256 km bin.

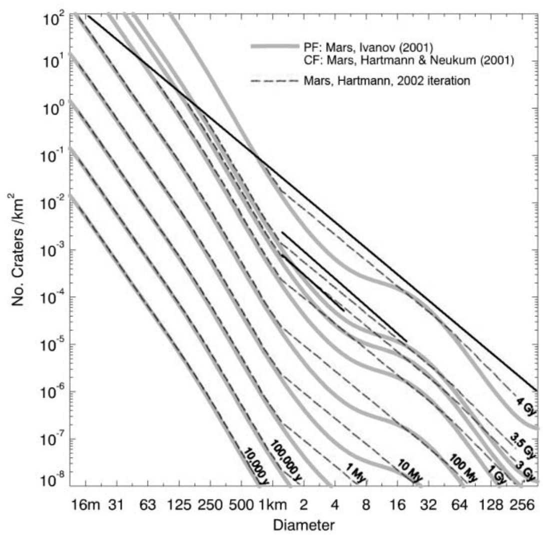

Figure 2: Hartmann isochrons for Mars (2002 iteration, unpublished) with their equivalents derived from the Ivanov (2001) production function [4] and the Hartmann and Neukum (2001) chronology [5]. Solid black lines mark the saturation equilibrium, and Noachian—Hesperian and Hesperian—Amazonian boundaries.

R-plot presentation

The R-plot [1] is the size-frequency distribution relative to a standard function, $S=D^{-3}$. The standard function S is a first order approximation to the observed crater populations, and this relative form permits the more subtle deviations from this distribution to be emphasised. R is defined as

$$R(D) \equiv F(D)/S(D)$$

so can be plotted from the polynomial production function as

$$R(D) = -D^2 10^{\sum_{i=0}^{n} a_i x^i} . \sum_{j=1}^{n} j a_j x^{j-1}$$

The R-curves for the lunar [3] and Mars [4] production functions are given in Figure 3.

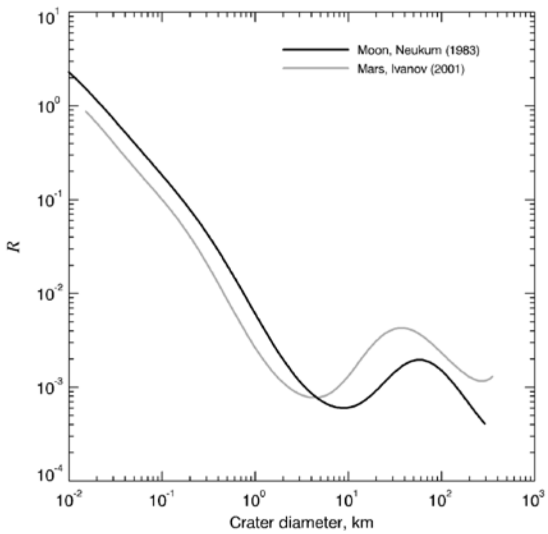

Figure 3: R-curves for the lunar [3] and Mars [4] production functions, for t = 2 Ga. Note the shift of the extrema towards lower crater diameters when going from the Moon to Mars, which is largely a consequence of the lesser impact velocities at Mars, as well as the shift towards higher surface density, resulting from the increased impactor flux at Mars.

Software

A software tool named craterstats, developed by the first author to derive ages from crater counts and using the methods shown here, is available for others to use from http: //hrscview.fu-berlin.de/software.html.

Acknowledgement

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