

CONDITIONS IN AN INFALL-SUPPLIED PROTOPLANETARY DISK. R. M. Canup¹. ¹Southwest Research Institute; 1050 Walnut Street, Suite 300; Boulder, CO 80302; robin@boulder.swri.edu.

Introduction: The minimum mass solar nebula (MMSN) is the widely adopted starting condition for planet formation. The MMSN is constructed by augmenting the mass of the planets to solar composition across the region of their current orbits, leading to a disk containing ~ 0.02 solar masses (M_\odot). The MMSN concept assumes that the protoplanetary disk formed before the planets accreted, and that the disk initially contained a solar ratio of gas-to-solids, f , with $f \approx 240$ (50) interior (exterior) to the ice condensation radius. However both assumptions may be unrealistic. The disk is thought to be created by infalling gas and grains that have too much specific angular momentum to fall directly to the star, so that they instead flow into a circumstellar accretion disk. The infall could have persisted for up to $\sim 10^6$ yr [1-2], so that solid body accretion may have occurred while the infall was ongoing. A generally analogous situation likely existed within protosatellite disks around accreting gas giant planets [3-6]. Canup & Ward [3-5] argue that the gas-to-solids ratio in inflow-supplied protosatellite disks is reduced substantially compared to solar values. In such “gas-starved” conditions, objects can grow larger before being lost to inward Type I migration [3-4], which is driven by their interactions with the gas disk. Here I extend concepts developed in [3-4] to the circumstellar environment and consider an actively supplied protoplanetary disk as an alternative to the MMSN.

Quasi steady-state gas disk: Consider the collapse of a spherical molecular cloud core to form the Sun. As each shell of radius l and angular rotation rate $\omega(l)$ collapses, it will achieve force balance across the circumstellar disk within the so-called centrifugal radius, r_c , where $r_c = l^4 \omega^2 / [GM(t)]$, and $M(t)$ is the total accreted mass at time t [2]. I adopt a circumstellar disk gas viscosity $\nu = \alpha cH$, where α is a constant inferred to be $\sim 10^{-3}$ to 10^{-2} , H is the gas vertical scale height, and c is the gas sound speed. I consider a disk with an outer edge, r_d , beyond which material is removed due to photoevaporation, and assume an infall with a solar ratio of gas-to-solids, f . If the viscous spreading timescale at orbital radius r in the disk, $\tau_\nu \sim r^2/\nu$, is shorter than the timescale over which the infall changes, the disk gas surface density σ_G is in quasi steady-state. For an infall rate per unit area, F_{in} , that is uniform across the $r < r_c$ region, σ_G for $r < r_c$ is [3]

$$\sigma_G(r) \approx \left[4F_{in} r_c^2 / (15\nu) \right] \left[5/4 - (r_c/r_d)^{1/2} - (r/r_c)^2 / 4 \right]. \quad (1)$$

The Sun’s late infall rate was likely $\sim 10^{-6} M_\odot \text{ yr}^{-1}$ [1].

Build-up of disk solids: Small grains are delivered to the disk with the infalling gas. Once in orbit, grain accretion can allow particles to grow fast enough

to decouple from the gas [3,7], leading to a disk gas-to-solids surface density ratio, (σ_G/σ_S) , that differs from f of the infall. The gas drag orbital decay timescale of a particle with radius r_s , density ρ_s , and orbital frequency Ω is $\tau_{gd} \sim (8/3C_D)(\rho_s r_s / \sigma_G)(r/H)^3 \Omega^{-1}$, where C_D is the drag coefficient, while the accretion time due to binary collisions is $\tau_{acc} \sim (\rho_s r_s / \sigma_S) \Omega^{-1}$, so that $\tau_{acc}/\tau_{gd} \sim 10^{-2} [(\sigma_G/\sigma_S)/10^2][(H/r)/0.07]^3$ [3]. If $(\tau_{acc}/\tau_{gd}) \ll 1$, solids can accrete and build up across the $r < r_c$ region where they are deposited by the infall [3]. The remainder of the arguments here assume this case.

Growth of solid protoplanetary embryos: The oligarchic timescale to grow a mass M_p , radius R_p embryo from planetesimals having surface density σ_S , and the embryo’s Type I orbital decay timescale are

$$\tau_{olig} \sim \Omega^{-1} (1/F_g) (\rho_s R_p / \sigma_S) \quad (2)$$

$$\tau_1 \sim \Omega^{-1} (1/C_a) (M_\odot/M_p) (M_\odot/r^2 \sigma_G) (H/r)^2 \quad (3),$$

where $F_g \approx [bC_D(\sigma_G/\rho_s r_s)(r/H)]^{2/5} (\rho_s/\rho_\odot)^{1/3} (r/R_\odot)$ for planetesimals dynamically stirred by the protoplanet and damped by gas drag [8], where $b \sim 10$ is the protoplanet spacing in Hill radii, r_s is the planetesimal radius (with $r_s \sim 1$ to 10 km typically assumed), and R_\odot and ρ_\odot are the Sun’s radius and density. For full-strength Type I migration, $C_a \sim 3$ [3,9].

In an actively-supplied disk, σ_S is affected by the supply of infalling solids. Consider a protoplanet accreting planetesimals across an annular feeding zone of area $A = 2\pi r \delta r$. The planetesimal surface density will reflect a balance between the infall-supplied solids and loss due to accretion onto the protoplanet, with $d\sigma_S/dt \sim (F_{in}/f) - (dM_p/dt)/A$. If the first term dominates, $d\sigma_S/dt \sim (F_{in}/f)$, implying $\sigma_S(t) \sim (F_{in}/f)t$. Setting $t \sim \tau_{olig}$ and substituting $\sigma_S(t) = (F_{in}/f)\tau_{olig}$ into eqn. (2) gives $\tau_{acc,olig} \sim [\rho_s R_p f / (F_g F_{in} \Omega)]^{1/2}$. Thus when the infall supply rate is fast compared to the sweep-up rate by the protoplanet (“regime 1”), the protoplanet grows with timescale $\tau_{acc,olig}$, which increases more slowly with R_p than the standard expression in (2). Alternatively, if the sweep-up rate by the protoplanet is rapid compared to the rate of infall supply (“regime 2”), the growth rate is limited by the supply rate. In this case, σ_S is maintained at the value for which $(F_{in}/f) \sim (dM_p/dt)/A$, so that $\tau_{acc,in} \equiv M_p (dM_p/dt)^{-1} \sim M_p f / (F_{in} A)$. A protoplanet’s growth will be limited by the longer of the two timescales $\tau_{acc,olig}$ and $\tau_{acc,in}$. In protosatellite disks, $\tau_{acc,in} \gg \tau_{acc,olig}$ and satellites grow in regime 2 [3]. In the circumsolar disk, the terrestrial region is inflow-regulated (regime 2), while the outer disk will likely be limited by the oligarchic accretion time (regime 1).

Critical embryo mass: A protoplanet accreting within an infall-supplied disk and subject to Type I migration will not have its final mass set by the typical isolation mass defined for a static disk. Instead, the embryo continues growing by accreting infalling solids until it reaches a critical mass, M_{crit} , for which the characteristic time for its further growth is comparable to its Type I decay timescale [4], with M_{crit} found by setting $\tau_1 = \max(\tau_{acc,in}, \tau_{acc,olig})$. A protoplanet cannot grow much larger than M_{crit} before it is lost to collision with the Sun, because Type I migration becomes more rapid as the protoplanet's mass increases [3-4,10]. If the infall lasts longer than the time needed to form mass M_{crit} objects, the system will produce (and lose) multiple generations of objects with this characteristic mass [4]. Such losses may have been substantial since the total mass processed through the disk was likely larger than that represented by the current planets [1].

Figure 1 compares $\tau_{acc,in}$, $\tau_{acc,olig}$, and the Type I decay timescale as a function of embryo mass at $r = 1$ AU (a) and 5 AU (b), for an infall rate of $10^{-6} M_{\oplus} \text{ yr}^{-1}$, with σ_G given by eqn. (1), $r_c = 20$ AU, $(H/r) = 0.07(r/1 \text{ AU})^{1/4}$, $\alpha = 0.005$, $f(1 \text{ AU}) = 240$ (rock solids) and $f(5 \text{ AU}) = 50$ (ice + rock). I use F_g from [8] and $\tau_{acc,in}$ as derived in [4], in which $\delta r \sim 2re$, with embryo eccentricity, e , set by a balance between scattering and density wave damping. I assume full strength Type I migration [9], the most restrictive case for growth.

At 1 AU (Fig. 1a), growth is inflow-limited (regime 2) for $M_p > 0.01 M_{\oplus}$, the maximum protoplanet mass is $M_{crit} \sim 0.2 M_{\oplus}$, and the time to form a mass M_{crit} object is $\sim 8 \times 10^5$ yr. At 5 AU (Fig. 1b), growth is in regime 1, $M_{crit} \sim 5 M_{\oplus}$, and both the time to form and lose a mass M_{crit} object is $\sim 2 \times 10^5$ yr. The predicted σ_S at 5 AU is substantially higher than that of the MMSN at 5 AU, which shortens the accretion time. The Type I migration timescale is lengthened at both 1 and 5 AU due to a reduced gas-to-solids ratio in the disk compared to the MMSN.

Fig. 1b implies that every $\sim \text{few} \times 10^5$ yr, a several Earth mass object formed beyond the snowline migrates inward through the terrestrial zone due to Type I decay. During each such passage, many, if not most, of the large objects in the terrestrial region

would likely be removed by accretion or by being resonantly driven into the Sun. Thus once r_c extends beyond the snowline, objects in the terrestrial region might not actually reach the M_{crit} mass before being removed by inwardly migrating cores. Terrestrial embryos would then be limited to the size of object that could grow between such core passages. For the Fig. 1 conditions, this implies $\sim 0.05 M_{\oplus}$ objects near 1 AU, and for embryo spacings estimated in [4], this gives $\sigma_S \sim 7 \text{ g/cm}^2$ at 1 AU, equivalent to σ_S in the inner MMSN disk.

As the infall rate decreases, M_{crit} at 5 AU increases (e.g., to $\sim 10 M_{\oplus}$ for a $10^{-7} M_{\oplus} \text{ yr}^{-1}$ infall for the case in Fig. 1). Cores might thus grow large enough to undergo gas accretion (requiring $M_p \sim 5$ to $10 M_{\oplus}$, depending on opacity, [11]) on a timescale comparable to their Type I decay timescale.

Discussion: Because $\tau_{olig}/\tau_1 \propto (\sigma_G/\sigma_S)(1/\sigma_G)^{2/5}$, accretion can win out over migration if (σ_G/σ_S) is reduced. Prior works [12,13] have reduced (σ_G/σ_S) by starting with a MMSN-type disk and waiting to form protoplanetary cores until the nebula has partially dissipated. Here I find that an infall-supplied disk can be gas-poor even as it is forming, due to the decoupling of infalling solids from the gas. Preliminary estimates here suggest that this can produce generally favorable conditions for the formation and survival of giant planet cores (even in the most restrictive case of full strength Type I migration) while also producing a solid surface density in the inner disk compatible with the later accretion of the terrestrial planets.

References: [1] Ward-Thompson D. (1996) *Astrophys. Space Sci.* 239, 151-170; [2] Hueso R. & Guillot T. (2005) *Astron. Astrophys.* 442, 703-725; [3] Canup R. M. & Ward W. R. (2002) *Astron. J.* 124, 3404-3423; [4] Canup R. M. & Ward W. R. (2006) *Nature* 441, 834-839; [5] Canup R. M. & Ward W. R. (2009) In *Europa*, Univ. Az. Press, 59-84; [6] Ward W. R. & Canup R. M. (2010) *Astron. J.* 140, 1168-1193; [7] Makalkin A. B. *et al.* (1999) *Sol. Sys. Res.* 33, 456-462; [8] Thommes E. W. *et al.* (2003) *Icarus* 161, 431-455; [9] Tanaka H. *et al.* (2002) *Astrophys. J.* 565, 1257-1274; [10] Ward W. R. (1997) *Astrophys. J.* 482, L211-L214; [11] Ikoma M. *et al.* (2000) *Astrophys. J.* 537, 1013-1025; [12] Ward W. R. *et al.* (2001) In *Astrophysical Ages and Time Scales*, Astron. Soc. Pacific Press, 111-119; [13] Thommes E. W. & Murray N. (2006) *Astrophys. J.* 644, 1214-1222.

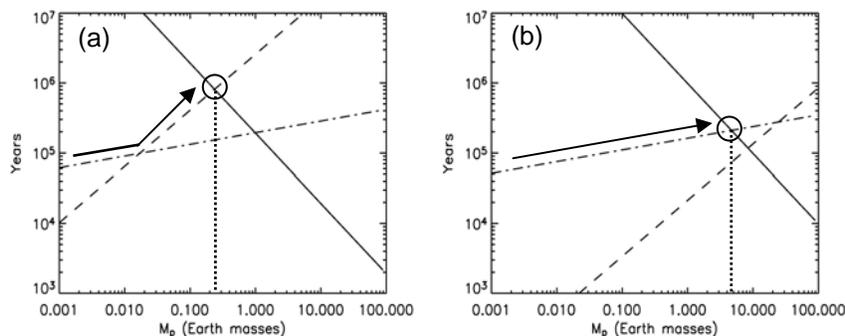


Fig. 1: Timescales $\tau_{acc,in}$ (dashed), $\tau_{acc,olig}$ (dot-dashed), and τ_1 (solid), shown at 1 AU (a) and 5 AU (b) as a function of protoplanet mass. The growth timescale is set by the longer of $\tau_{acc,in}$ and $\tau_{acc,olig}$ (arrows). The critical mass for which the growth timescale equals the Type I decay timescale is indicated by the circles and vertical lines.