LONG-TERM STRENGTH OF ICY VS SILICATE PLANETARY BODIES . H. J. Melosh[1] and F. Nimmo[2], [1]Department of Earth and Atmospheric Sciences, Purdue University (550 Stadium Mall Drive, West Lafayette, IN 47907, jmelosh@purdue.edu), [2]Department of Earth and Planetary Sciences, UC Santa Cruz, Santa Cruz CA 95064.

Introduction: Without strength, topography is impossible. Strengthless, self-gravitating bodies are either oblate spheroids, if rotating in free space, or triaxial spheroids, if tidally locked to a larger primary. Long-term topographic deviations from either of these rather uninteresting shapes requires internal strength. This fact evidently fascinated (one might even say, obsessed) geophysicist Harold Jeffreys to the extent that he devoted many papers and multiple chapters to the topic in his famous book *The Earth* [1].

Jeffreys defined the general conditions needed to support a given topographic load. He showed that, to support a topographic feature of height h, that the planet must be able to sustain a stress difference of order ½ to 1/3 $\rho g h$ somewhere in the vicinity of the load, where ρ is the density of the crust and g is the surface acceleration of gravity. This stress magnitude is independent of the rheology of the material, its loading history, or any considerations of isostasy or lithospheric structure. This result is so general that, in a recently submitted book manuscript, Melosh [2] has elevated this to the status of a theorem, Jeffrey's Theorem.

Jeffreys also showed that, under much less general conditions, the depth at which loads are supported is comparable to the breadth of the load. This conclusion can be modified by isostasy and lithospheric flexure, but only at the cost of introducing stresses that may greatly exceed the minimum required by Jeffrey's Theorem.

Planetary Strength: The nature of the resistance a planet can offer to stresses that tend to deform it depends upon its composition. Strength is a complex subject because the resistance of solids to a differential stress depends on many factors, such as the nature of chemical bonding, defect structures, deformation history, temperature, pressure, load duration, presence or absence of corrosive fluids and so on. Most of our understanding of strength comes from experiment, not deduction from fundamental principles.

Many years of experimentation on planetary materials, principally silicates (because that is what the Earth is composed of) show that solids near their melting points slowly creep and so do not possess long-term strength (indeed, we now understand that no material can sustain differential stresses forever, but when the deformation timescale exceeds the age of the Universe by large factors, there is no practical limit to its duration). However, materials that are well below their melting points may be able to support stress differences for geologic ages.

One simple model of strength suggest that fractured materials support stress differences by means of friction, so that the maximum sustainable stress difference τ is a function of pressure p,

$$\tau = p \tan\phi$$

where ϕ is called the "angle of internal friction" and parameterizes the fricional strength of the material. Most silicate materials (including water ice [3]) have angles near 30°.

At high pressures the failure envelope is observed to flatten out and become nearly constant, equal to a plastic yield stress Y that ranges from a few tims 0.1 to around 1 GPa.

$$\tau = Y \ \text{for}\ p \gg 0$$

These strength models can be combined with Jeffrey's Theorem and a simple model for stress differences on a small body of mean radius R (Fig. 1) to create an expression for the maximum topographic ranges (above the geoid for rotating or tidally distorted bodies) on planetary bodies.

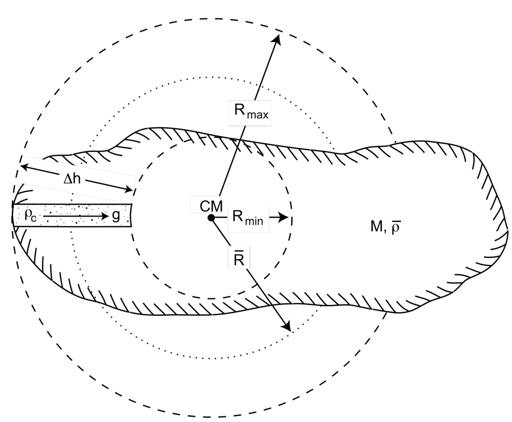

Fig. 1 Geometrical model of the stresses developed by topography on an irregular planetary body. The irregularity is greatly exaggerated for most bodies.

Thus, when friction is important, but the internal pressure is not too high, the maximum topographic height is limited by

$$\frac{h}{R} \leq \tan\phi$$

When pressure is high enough to reach the strength plateau, the topographic height is limited by

$$\frac{h}{R} \leq \frac{3}{2\pi} \frac{Y}{G\rho\overline{\rho}} \frac{1}{R^2}$$

where G is Newton's gravitational constant and $\overline{\rho}$ is the mean density of the planet. Note the dependence of *h/R* on $1/R^2$, which is characteristic of plastic yielding.

These expressions can be used to perform a crude, empirical *in situ* estimate of the strength parameters of actual planetary bodies and compare the results to laboratory measurements of the strength of materials. This is a way of checking whether the strength of materials has an important dependence on the size scale of the load, among other factors, because laboratory measurements are limited to specimens that are typically only a few cm in size, whereas planetary scales range up to 10^4 km.

The results of a comparison of these models with the maximum topographic deviations observed on planetary bodies is shown in Figure 2. The lines in this figure represent the limits of the strength laws: The sloping lines are the high pressure limits and are labeled with the values assumed for the yield strength *Y*.

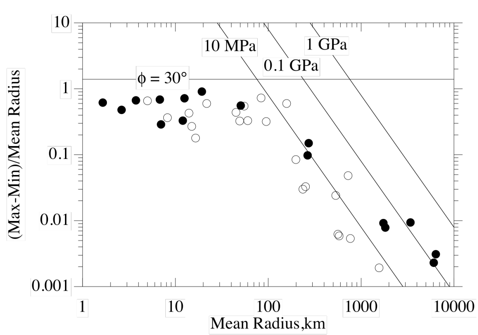

Fig. 2 Maximum topographic deviations vs. size for a suite of solar system bodies, including all of the terrestrial planets, the Moon and the largest asteroids. Silicate bodies are shown with filled circles and icy bodies are open circles.

Conclusions: Figure 2 indicates that topography on small bodies, less than about 100 km in diameter, is consistent with support by frictional strength, as expected, and that icy bodies exhibit about the same degree of frictional strength as silicate bodies, also as expected from measurements of the friction of cold ice [3].

Larger planetary bodies' topography seems to be limited by the $1/R^2$ limit characteristic of plastic yielding. However, Figure 2 suggests that the ultimate, long-term strength of icy bodies is about a factor of 10 smaller than that of silicate bodies. Furthermore, the effective strength of about 0.1 GPa observed for silicate bodies is itself about a factor of 10 smaller than is typically observed for the strength plateau of cold rocks [4]. It is uclear at the present time whether this is an effect of temperature, scale or time.

The major conclusion of this work is that, even on planetary scales, cold ice and rock exhibit similar frictional strength, but when plastic yielding becomes important, ice is about ten times weaker than rock. This observation has important implications for the topographic extremes that might be expected on extrasolar planets, both for small bodies and for superEarths circling around other star systems.

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References: [1] Jeffreys, H. (1962*) The Earth*, 4th Ed. Cambridge. [2] Melosh, H. J. (in press, 2011) *Planetary Surface Processes*, Cambridge. [3] Beeman et al. (1988) JGR 93, 7625. [4] Handin, J. (1966) in Clark, S. P. *Handbook of Physical Constants*, GSA, p. 223.