ON THE FLOW AND FLUIDIZATION OF GRANULAR MATERIALS: APPLICATIONS TO LARGE LUNAR CRATERS, CLIFF COLLAPSES, AND ASTEROID SHAPES.

K. A. Holsapple (University of Washington 352400, Seattle, WA 98195, USA; holsapple@aa.washington.edu)

Introduction: There is a lingering belief that the standard continuum Mohr-Coulomb (MC) failure theory is not adequate for all aspects of granular flow problems, especially for the latter stages of the formation of large craters and of landslide collapse problems. Those beliefs are based on the fact that the final surface slopes in these problems are often well below the angle of repose of the material. Many researchers have used that fact to suggest a need for lubrication or fluidization mechanisms that create a reduced angle of friction.

Here I show those beliefs to be false. The reason for them is attributed to assumption that failures using the MC theory cannot occur with surface slopes θ less than the angle of repose. In fact, failure may occur at any surface slope less than the angle of friction ϕ. Furthermore, the angle of repose is a static concept, in dynamic problems the local acceleration must be combined with the local gravity vector in order to assess the slope angle α relative to the effective "down" direction. So the effective slope may be temporarily much different than the geometric slope.

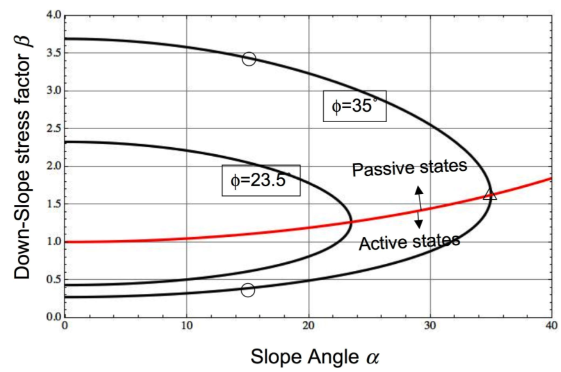
Failure Near Static Surface Slopes: A simple Mohr's circle yields valuable information about possible failure states according to the MC theory. Specifically, failure does not only depend on the angle of the slope, but also on the normal stress on a plane perpendicular to the slope (the "down-slope" stress).

Near a static free surface slope at angle α, it is convenient to use a local *x*-*y* coordinate system, with *y* measured perpendicular to the slope, and *x* in the down-slope direction. Then, using the equations of equilibrium, it is easy to find that the two-dimensional stresses near the surface are given as:

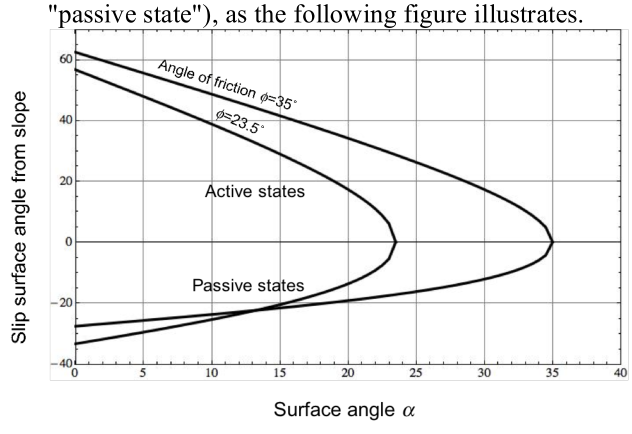
$$\sigma_x = \beta\rho g y,\ \ \tau_{xy} = -\rho g y Sin(\alpha),\ \ \sigma_y = \rho g y Cos(\alpha)$$

where the factor β defining the down-slope stress σ_y depends on other aspects of the problem. However, it is the value of that β and the slope angle α that determines the angle to the plane along which failure is possible, and *failures are possible for any slope angle less than the angle of friction* ϕ. However, among all possible β there is no possible equilibrium slope *greater* than the angle of friction. For example, for the angle of friction $\phi=35^\circ$, a slope angle $\alpha=35^\circ$ is only possible if the value of β=1.622, and then a sliding failure is possible exactly along the surface slope direction. That case illustrates the well-known *angle of repose* concept. But if the factor β is different than 1.622, then the maximum possible slope without failure is less, and may even be zero.

The following plot depicts, for two specific angles of friction ϕ=23.5°, ϕ=35°, the maximum slopes possible for different down-slope stress factors. The two values of β that allow failures for a zero slope (horizontal surface) at the left intersects are known in the

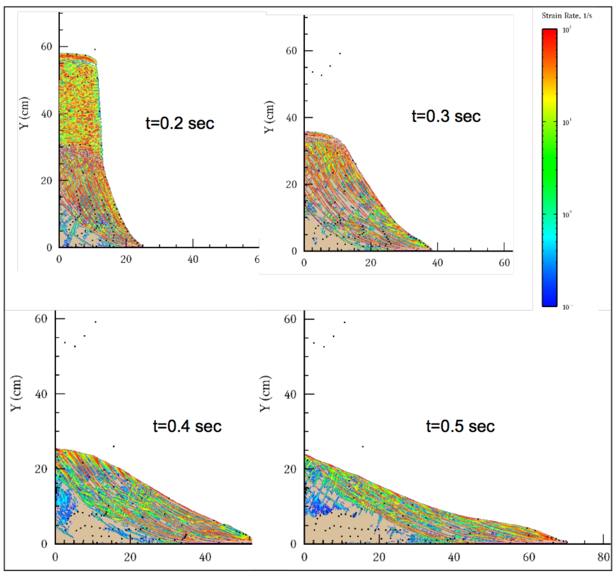


study of soil retaining walls as the *Rankine* states [1]. In that case the factor β relates to the so-called "coefficient of active stress" [1]. For other slope values there are also two down-slope stress values that admit failures, given by the upper and lower curves (except for the furthest point to the right which is for a failure with the slope at the angle of friction). And in those intermediate cases, the failure is not along the slope direction, but is on a plane either steeper than (the "active" stress state), or shallower than the slope angle (the "passive state"), as the following figure illustrates.

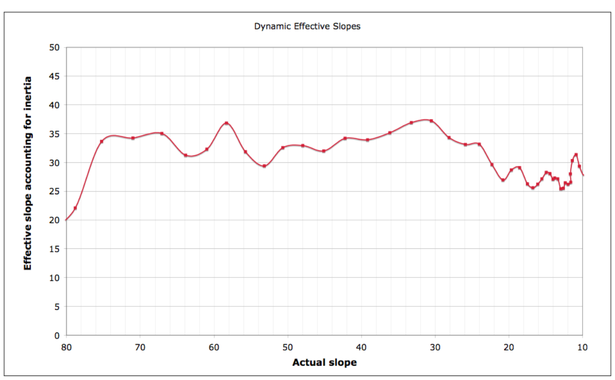


Dynamic processes: In the case of dynamic processes, the analysis of the failure near a free surface at slope θ can be repeated just as for the static case, but the consideration of equilibrium includes a non-zero acceleration: $div[\sigma] + \rho\mathbf{g} - \rho\mathbf{a} = 0$. Thus, the gravity vector $\boldsymbol{g}$ must be replaced by the effective gravity vector ***g-a***, according to D'Alembert's principle, to give an equivalent static analysis. Then any surface location will have an effective slope relative to the direction of this effective gravity greater or less than the geometric slope, depending on the direction and magnitude of the acceleration.

The Cliff Collapse Problem: An exemplary example that illustrates these concepts is that of the collapse of a vertical surface (a cliff) of a granular material, for which experiments and code calculations are available in the literature [2]. The results of a code calculation by the author are presented in the next figure [3], using a friction angle of 35˚. The results of that code calculation are "spot-on" with the experimental results. The contours illustrate the plastic strain rate



as the failure progresses. Note how the failures are along planes steeper than the slope at the steeper slope locations and less than the surface slope near the more shallow surface slopes, creating a "circular arc" failure surface, as is often assumed *a-priori* for the prediction of landslides [1].



From this solution, the acceleration of each point near the surface can be determined, and the effective slope using both the downward gravity and local acceleration can be found. For a single location at the surface the effective slope versus the geometric slope, as time progresses to the right, is shown above.

At the early times at the left, the geometric slopes are much larger than the angle of friction ϕ=35˚, but a down and rightward acceleration is created so that the *effective* slopes are always less than 35˚. And for the latter times of the run out of this problem, the geometric slopes range as low as 10˚, but the lateral acceleration (slowing) is sufficient to make the effective slopes on the order of 25-30˚.

The results of a detailed analysis of this calculation show that 1.) The effective slope must account for the local acceleration. 2.) No effective slope may be greater than the angle of friction. 3.) At the steeper surface locations, the stress state is in the "active" condition, and the failures are along a slope steeper than the surface. 4.) At the shallow slope locations, the stress state is in the "passive" condition and failures are along surfaces with a slope shallower than the geometric slope. These latter two results are responsible for the "circular-arc" failure surface in the problem.

Thus, the MC theory correctly predicts slope collapse from the initial vertical 90˚ to a final slope less than 10˚. No fluidization mechanisms need be invoked to obtain the extensive and shallow slope run outs.

Asteroid shapes: The same idea, that failure can only occur with local slopes at the angle of repose, has led to efforts to find asteroid shapes which have at all surface locations a slope equal to the angle of repose [4,5]. In fact there are many shapes that have imminent failure at all points but whose local surface slopes are much less than the angle of repose, and whose overall shape is a smooth ellipsoid. The reader is referred to publications such as [6] of the author many such examples.

Large flat craters: Again, there is no need to invoke late stage fluidization mechanisms to explain the transition at large scales of terrestrial and lunar craters to increasingly broad and shallow shapes. The fact of shallow final wall slopes does not imply such a need. The MC theory allows for continuous collapse to a final broad flat shape. Results of code calculations of large impacts will be shown at the conference to illustrate this fact.

References: [1] Lambe and Whitman (1979). *Soil Mechanics*, Wiley, New York. [2] Lajeunesse et al., (2005) Phys Fluids 17, 103302-103302-15. [3] Holsapple, K. A. (2011) submitted. [4] Withers 31st LPSC, Abstract #1270. [5] Harris, A. K., et al. (2009) Icarus 199, 2, 310-318. [6] Holsapple. K. A. (2004) Icarus 172, 272 303.