A GENERAL ANALYTIC MODEL FOR THE THERMAL CONDUCTIVITY OF LOOSE, INDURATED OR ICY PLANETARY REGOLITH. Stephen E. Wood, Dept. of Earth and Space Sciences, Univ of Washington, Box 351310, Seattle, WA, 98195-1310, sewood@ess.washington.edu.

Introduction: Thermal conductivity is one of the most highly variable properties of planetary surfaces, with values ranging over more than three orders of magnitude from ~0.001 W/mK for dust on airless bodies to 2-5 W/mK for exposed bedrock or solid ice. The effective thermal conductivity (k_{eff}) of porous, particulate material increases with the degree of intergranular cementation and is a function of atmospheric pressure when the pore size is comparable to the mean free path. It is the most variable component of thermal inertia (I=sqrt[$k\rho c$]) - the parameter that controls the surface temperature response to diurnal and seasonal insolation cycles; and below the depth of the seasonal thermal wave (a few meters), for a given interior heat flow Q, k_{eff} also determines the geothermal gradient ($dT/dz = Q/k$). These effects on surface and subsurface temperatures make it an important factor in determining the thermal stability of ices or hydrated minerals within or beneath the regolith.

The effective thermal conductivity of porous planetary regolith involves contributions from three heat transfer mechanisms: heat conduction through the solid particles and their physical contacts k_s, heat conduction by the gas in the pore space k_g, and radiative heat transfer between the solid particles k_r. For loose particulate materials on Earth, Mars, Venus, and Titan, k_g is often the dominant component. The thermal conductivity of an ideal gas is independent of pressure, but as the pressure (or density) decreases in a porous medium, the mean free path of gas molecules (λ) begins to approach the pore size (d_{pore}) and this reduces the efficiency of gaseous heat transfer. When the Knudsen number (Kn = λ / d_{pore}) becomes >>1, then k_g reaches a minimum and k_{eff} becomes controlled by k_r, k_s and the degree of interparticle contact.

Previous Work. In the planetary literature, Mellon et al. [1,2] have described models for k_{eff} of loose particulates and icy regolith - both based on the parallel/series conductor approach. Neither has been quantitatively compared to measured values to evaluate their accuracy, but the loose particulate model has been used to analyze thermal property data from the TECP soil probe on the Phoenix Mars Lander [3]. Another recent and detailed model for loose particulates on planetary surfaces is the finite-element numeric model of Piqueux and Christensen [4,5]. They showed the model can closely match laboratory measurements of k_{eff} for glass beads over wide range of pressures, although they used one of those data sets to derive an empirical expression for k_g(Kn). In contrast, the model described here uses a theoretical expression from kinetic theory. Their model also does not include radiative heat transfer. More detailed comparisons to other previous models in the general physics literature are given in Wood (2011) [6].

Model Description: The model is based on the Maxwell-Eucken theoretical expressions for the upper and lower bounds for the conductivity of heterogeneous, isotropic material with one continuous phase [7,8,9]. These equations provide tighter bounds than the parallel and series approximations often used to estimate k_{eff} for porous media. The effect of interparticle contact and/or cementation is represented by an empirical factor f_{sc} which represents the fractional continuity of the solid phase ($0 \le f_{sc} \le 1$). The effect of radiative heat transfer is included, as well as the dependence of gas conductivity on temperature and Knudsen number. For unconsolidated particulates, a method is presented for estimating the porosity as a function of particle size, density, and gravitational force. The only free parameters in the model for loose regolith are f_{sc} and a pore size factor which are determined by least-squares fits to laboratory measurements of k_{eff} for glass beads over a wide range of particle size and pore gas pressure [10,11].

For partially cemented particles, such as sandstone or icy regolith, f_{sc} is calculated as a function of the relative number and size of the interparticle bonds with two adjustable parameters. Due to a lack of suitable conductivity data for partially cemented particles, those parameters are estimated based on a comparison with a more detailed, finite-element numeric model for regular packings of spheres connected by pendular rings of cement [5].

The conductive and radiative components of the effective thermal conductivity are taken to act in parallel, so that $k_{eff} = k_c + k_r$. The general form for the conductive component in the model is

$$k_c = k_{c,min} + f_{sc}\,(k_{c,max} - k_{c,min})$$

$$k_{c,min} = \frac{k_g\phi + k_s v_s \dfrac{3k_g}{(2k_g + k_s)} + k_{cem}\chi \dfrac{3k_g}{(2k_g + k_{cem})}}{\phi + v_s \dfrac{3k_g}{(2k_g + k_s)} + \chi \dfrac{3k_g}{(2k_g + k_{cem})}}$$

$$k_{c,max} = \frac{k_{cem}\chi + k_s v_s \dfrac{3k_{cem}}{(2k_{cem} + k_s)} + k_g\phi \dfrac{3k_{cem}}{(2k_{cem} + k_g)}}{\chi + v_s \dfrac{3k_{cem}}{(2k_{cem} + k_s)} + \phi \dfrac{3k_{cem}}{(2k_{cem} + k_g)}}$$

where ϕ is the porosity, v_s = 1-ϕ, and χ is the volume fraction of cement (or ice).

Results: For the case of loose particulates, comparisons to laboratory data for glass beads over a wide range of pressure and particle size show that f_{sc}=0.01 can provide accurate fits to all of the data (**Fig. 1**). For the case of partially cemented particles, an analytic expression for f_{sc} as a function of porosity, cement/ice volume fraction, and particle coordination number was found which produces values very close to those of Piqueux and Christensen's finite-element model [5] (**Fig. 2**).

References: [1] Mellon, M.T. et al (1997) *JGR 102*, 19357-69. [2] Mellon, M.T. et al. (2008) *The Martian Surface*, ed. J. Bell III, Cambridge Univ. Press, pp. 399-427. [3] Zent, A.P. et al. (2010) *JGR, 115*, E00E14. [4] Piqueux, S. and P.R. Christensen (2009a) *JGR, 114*, E09005. [5] Piqueux, S. and P.R. Christensen (2009b) *JGR, 114*, E09006. [6] Wood, S.E. (2011) submitted to *Icarus*. [7] Maxwell, J.C. (1892) *A Treatise on Electricity and Magnetism*, Vol. 1, Clarendon Press, Oxford. [8] Hashin, Z. and S. Shtrikman (1962) *J. Appl. Phys., 33*, 3125-3131. [9] Brailsford, A. D. and K. G. Major (1964) *Brit. J. Appl. Physics, 15*, 313-319. [10] Presley, M.A. and P.R. Christensen (1997) *JGR, 102*, 6551-6566. [11] Huetter E.S. et al. (2008) *JGR, 113*, E12004.

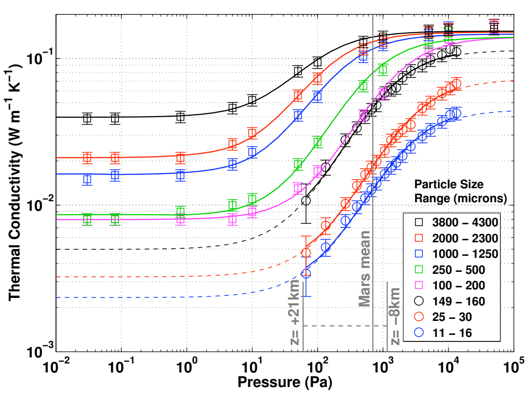

Figure 1. Comparison of the analytic model fits (thick solid lines) to laboratory measurements (symbols) of the thermal conductivity of glass beads as a function of particle size and pore gas pressure. Squares indicate data obtained in N2 gas at 300-360. and circles are data obtained in CO2 gas at 300-450 K [10]. Both studies used a line heat source which heated the samples more at lower conductivities. The dashed lines indicate extrapolations of the model fits to lower and higher pressures but for a constant temperature of 300K. The vertical gray lines indicate Mars' global average surface pressure and the values at the extremes of surface elevation.

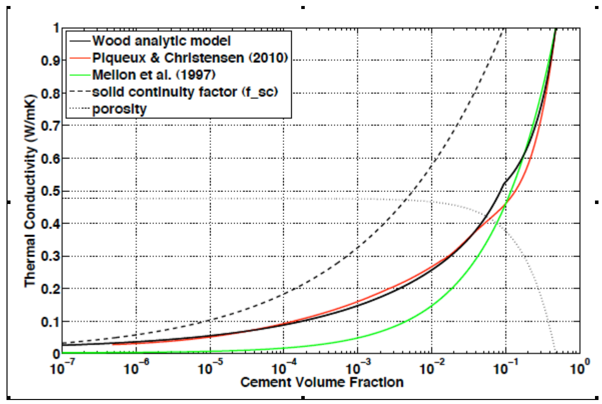

Figure 2. Comparison of theoretical values for the thermal conductivity of a cemented particulate medium (solid lines) as a function of cement volume fraction: one from a finite element model described [5] (red), one from an analytic model by Mellon [1] (green), and one from the analytic model described in this work (black). The dotted line indicates the corresponding porosity and the dashed line is the value for f_sc used in the author's analytic model. For each model, the medium is made of uniformly-sized spheres arranged in a simple cubic packing.