

COMPARISON OF DISCRETE ELEMENT METHODS FOR SIMULATING LOW-SPEED RUBBLE PILE COLLISIONS: FIRST RESULTS. Derek C. Richardson¹, Sarah K. Munyan², Stephen R. Schwartz^{1,3}, and Patrick Michel³, ¹University of Maryland (Department of Astronomy, College Park, MD 20742-2421, USA, dcr@astro.umd.edu), ²Thomas Jefferson High School for Science and Technology (6560 Braddock Rd, Alexandria, VA 22312, USA), ³University of Nice-Sophia Antipolis, CNRS, Côte d'Azur Observatory (UMR Lagrange, B.P. 4229, 06304 Nice Cedex 4, France).

Introduction: Much of the evolution of small solar system bodies is dominated by collisions, whether from the initial build-up of planetesimals to form planetary embryos [1], or the subsequent impacts between remnant bodies that exist today [e.g., 2]. N -body simulations of low-speed collisions between small bodies typically treat the outcomes using "billiard-ball" physics, for which energy loss and momentum transfer are parameterized by restitution coefficients that approximate the underlying mechanical processes. Moreover, since many small bodies may have low tensile/cohesive strength [3], the collisions can often be treated as impacts between rubble piles, the outcomes of which are dictated by the dissipation parameters and gravity [4].

With recent interest in simulating granular processes at much smaller scales, such as regolith on asteroid surfaces, new simulation techniques that treat the collisional physics using a "soft-sphere" approach (the so-called Soft-Sphere Discrete Element Method, or SSDEM, as opposed to the billiard-ball hard-sphere approach, HSDEM) are being developed [5, 6]. In SSDEM, particles experience a mostly spring-like repulsive force when they interact, parameterized at minimum by normal and tangential spring constants, but usually also with damping coefficients, and optionally including various friction forces that oppose relative sliding, rolling, and twisting motions. The main advantages of SSDEM over HSDEM include a more realistic treatment of contact forces and a balanced force-only approach that eliminates the computational bottleneck of time-ordering the HSDEM events. A disadvantage is that much smaller timesteps are needed to resolve the particle interactions with SSDEM than with HSDEM, but this can be offset by the fact that true parallelism can be achieved with SSDEM, permitting simulations with millions of particles [6, 7].

Here we present our first results of comparing directly our implementations of HSDEM and SSDEM in the N -body code PKDGRAV [8, 9] as applied to low-speed rubble pile collisions, both as a consistency check but also as a validation of the earlier hard-sphere approach. Our SSDEM implementation is based on [10], with added features [6]. For these first tests, we did not include any frictional forces.

Method: We constructed rubble piles of 3 different sizes (radii ~ 1.0 km, 0.66 km, and 0.42 km, consisting of 1939, 625, and 183 identical particles, respectively), each with bulk density ~ 2 g/cc, in order to have 3 impactor mass ratios, 1:1, 3:1, and 10:1. We carried out moderate-speed (20 m/s), low-speed (1 m/s), and glancing impacts (speeds ~ 1 m/s, displacements ~ 1 km) for these 3 cases, for a total of 9 runs per test. Runs were carried out first with HSDEM, then with SSDEM using identical starting conditions. The normal coefficient of restitution was fixed at 0.8 in all cases, with no tangential dissipation. For the SSDEM runs, a Hooke's law spring constant of 1.3×10^{11} kg/s² was used, corresponding to a maximum penetration of $\sim 1\%$ of the particle radius for the highest-speed collisions. As is standard, the tangential spring constant was set to 2/7 times this value, which makes the tangential and normal springs oscillate at the same frequency [6]. The timestep was fixed at ~ 5 s for the HSDEM runs and ~ 5 ms for SSDEM, in both cases chosen for optimal sampling of the dominant forces.

#	M_H	M_S	e_H	e_S	P_H	P_S	ρ_H	ρ_S
1	< 1%	< 1%	0.50	0.52	23	5.3	3.4	3.2
2	99.3%	99.6%	0.11	0.09	1100	1000	2.0	1.9
3	99.8%	100%	0.47	0.43	5.0	5.1	2.2	2.1
4	1%	< 1%	0.12	0.52	7.7	5.6	1.4	3.2
5	98.1%	98.9%	0.04	0.09	480	180	2.1	1.9
6	75.2%	75.9%	0.09	0.13	12	11	2.0	1.9
7	< 1%	$\sim 1\%$	0.25	0.26	8.3	5.3	2.6	1.8
8	98.2%	97.9%	0.04	0.05	180	86	2.0	2.1
9	90.9%	88.4%	0.11	0.13	20	20	1.8	2.0

Table 1: Summary of simulation results, showing largest remnant mass fraction (M), ellipticity (e), rotation period (P) in h, and bulk density (ρ) in g/cc, for the hard-sphere (subscript H) and soft-sphere (subscript S) runs. See main text for run descriptions. Ellipticity is $e = 1 - (q_2 + q_3)/2$, where q_2 and q_3 are the intermediate- and short-to-long-axis ratios, respectively (so $e = 0$ is spherical, $e = 1$ is elongated).

Results: Table 1 shows the results of the tests, where runs 1–3 are the 1:1 mass ratio cases, 4–6 are 3:1, and 7–9 are 10:1; runs 1, 4, and 7 are 20 m/s head-on, 2, 5, and 8 are 1 m/s head-on, and 3, 6, and 9 are glancing. The columns give characteristics of the largest remnant after sufficient time has elapsed for the values to no longer change significantly, separated between the hard-sphere and soft-sphere cases (see

table caption). Results should be compared pair-wise for each run. In the moderate-speed runs the largest remnants consist of only a few particles, so the results have understandable large variations in shape, spin, and density. In the remaining cases, results between HSDEM and SSDEM are quite similar, with SSDEM often, but not in all cases, showing a somewhat higher final ellipticity, suggesting a higher shear strength that may arise from the more careful treatment of contact forces (recall there is no friction).

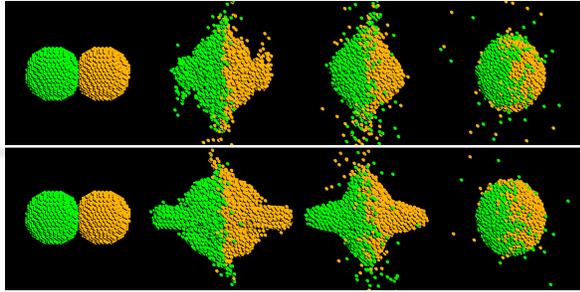


Figure 1: Snapshots from run 2 using HSDEM (top) and SSDEM (bottom). This case is a slow-speed head-on collision between equal-mass rubble piles. Due to the nature of the hexagonal close-packing of the progenitors, they are slightly skewed in these images.

Figure 1 shows snapshots from run 2 for the HSDEM and SSDEM cases. The biggest difference between the two cases occurs in the middle frames, during the peak of the impact shock. Due to the symmetric nature of SSDEM (meaning, reaction forces are calculated for all particles at the same time each step, whereas in HSDEM the impact calculations take place one after another, so multiple simultaneous contacts cannot be modeled), there is much more structure in the impact splash shown in the lower panel. However, in this case (and the other cases considered), the details do not particularly affect the final outcome.

Conclusions: We carried out a small suite of rubble pile collision simulations using both HSDEM and SSDEM, spanning a representative parameter space of mass ratio and impact speed and angle. We find that the collision outcomes are qualitatively similar between HSDEM and SSDEM. A notable difference is that the SSDEM outcomes, such as the ejecta splash, retain a higher degree of structure and symmetry than their HSDEM counterparts. We also find that the largest remnants in the SSDEM simulations typically have slightly higher shape ellipticities than their HSDEM counterparts. The differences in outcome, however, are small in all cases at this scale. This is not true at smaller scales, such as for impacts on small agglomerates consisting of spherical particles glued together by

sintering, where SSDEM is required to achieve a good match with laboratory outcomes [11].

We plan to carry out a more comprehensive comparison study in the future of low-speed rubble pile collisions that includes friction to see whether more significant differences arise in that case.

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