Effect of Stochastic Charging on Cosmic Dust Aggregation

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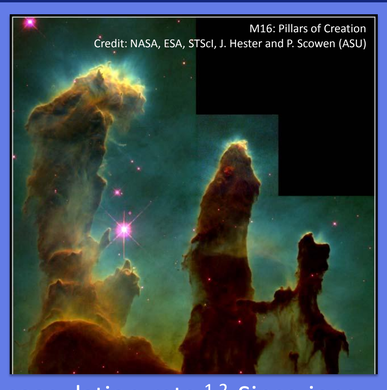
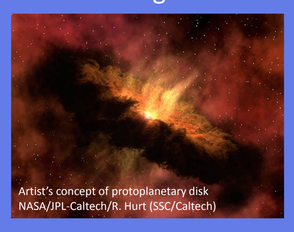
Introduction

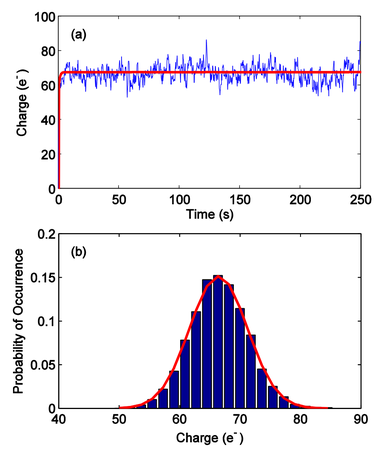


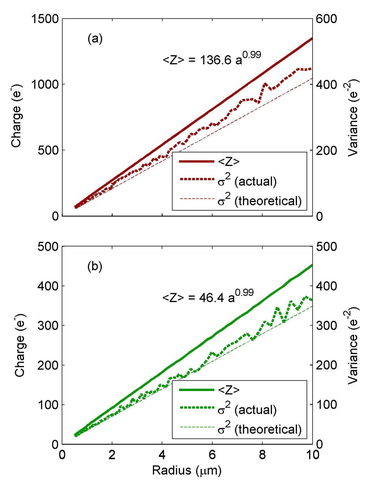
Coagulation of cosmic dust grains is a fundamental process which takes place in environments such as presolar nebulae, circumstellar disks and protoplanetary disks. Cosmic dust grains can become charged through interaction with their plasma environment, and the resultant electrostatic force among dust grains can strongly affect their coagulation rate.[1,2] Since ions and electrons are collected on the surface of the dust grain at random time intervals, the electrical charge of a dust grain experiences stochastic fluctuations. In this study, a set of stochastic differential equations is developed to model these fluctuations over the surface of an irregularly-shaped aggregate. The influence of the charge fluctuations on the coagulation process (driven by turbulent relative velocities)[3] and the physical characteristics of the aggregates formed is examined. It is shown that dust with small charges (due to the small size of the dust or a tenuous plasma environment) are affected most strongly.



Stochastic Charging of Monomers

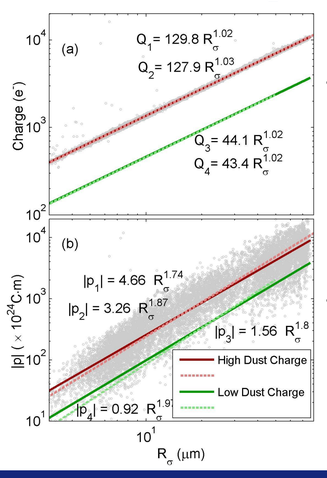
Below left, the charging time history for a single spherical grain (a). The charge has a normal distribution about the mean (b). Below right, the charge and variance are shown for spheres as a function of radius for hydrogen plasma with T = 900 K, and electron and ion densities (a) $n_e = n_i = 5 \times 10^8$ m^{-3}, (b) $n_e = 0.1 n_i$.

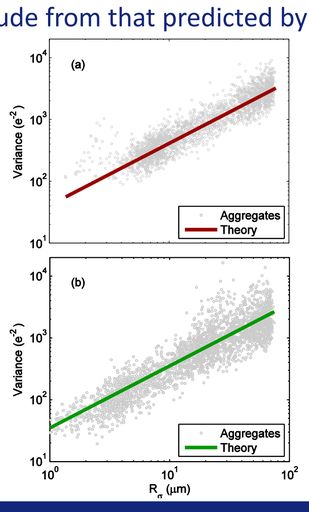




Stochastic Charging of Aggregates

Below left, the charge (a) and dipole moment (b) are compared for stochastic and non-stochastic charging conditions showing small difference. The variation in the charge (below right) can vary by almost an order of magnitude from that predicted by theory.





$n_e = n_i = 5 \times 10^8$ m^{-3}

$n_e = 0.1\ n_i$

Calculating Charge on Aggregates

Stochastic Fluctuations

A master equation describes the random charge fluctuations on the aggregates[4,5]

$$\frac{dP(\boldsymbol{Z},t)}{dt} = \sum_{p=1}^{N} I_{i,p}(\boldsymbol{Z}-\boldsymbol{e}_p)\,P(\boldsymbol{Z}-\boldsymbol{e}_p,t) + I_{e,p}(\boldsymbol{Z}+\boldsymbol{e}_p)P(\boldsymbol{Z}+\boldsymbol{e}_p,t) - \left[I_{i,p}+I_{e,p}\right]P(\boldsymbol{Z},t),$$

N is the total number of patches on the aggregate, $\boldsymbol{Z} = \{Z_1, Z_2, \ldots, Z_N\} \in \mathbf{R}^N$ is the vector of the elementary charges collected on patches, $P(\boldsymbol{Z}, t)$ is the probability density function of a state at which the patch p_i has Z_i charges, etc., $I_{i,p}$ and $I_{e,p}$ are the currents of ions and electrons, respectively, to the patch p, and $\mathbf{e}_j \in \mathbf{R}^N$ is the unit vector, e.g., $\mathbf{e}_3 = \{0,0,1,0,\ldots,0\}$.

A Fokker-Planck equation may be obtained from the master equation[6] which is statistically equivalent to a stochastic differential equation for the charge of each patch

$$dZ_p(t) = \left[I_{i,p}(\boldsymbol{Z}(t)) - I_{e,p}(\boldsymbol{Z}(t))\right]dt + \sqrt{I_{e,p}(\boldsymbol{Z}(t)) + I_{i,p}(\boldsymbol{Z}(t))}\,dW_p(t),$$

where $dW_p(t)$ is a Wiener process. This equation is solved numerically by the Euler-Maruyama method:

$$Z_p^{(n+1)} = Z_p^{(n)} + \left[I_{i,p}(\boldsymbol{Z}^{(n)}) - I_{e,p}(\boldsymbol{Z}^{(n)})\right]\Delta t + \sqrt{I_{e,p}(\boldsymbol{Z}^{(n)}) + I_{i,p}(\boldsymbol{Z}^{(n)})}\sqrt{\Delta t}\,\xi_p,$$

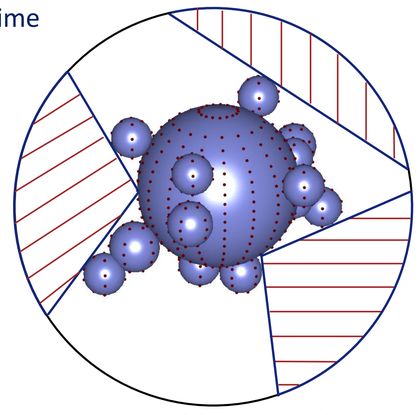
where Δt is the time step, $Z_p^{(n)}$ is the charge of the patch p at time step n, and ξ_p is a random number with a normal distribution. $Z_p^{(n)}$ is calculated using orbital motion limited theory (OML)[7] with a line of sight (LOS) approximation.[1]

OML_LOS

- Divide aggregate surface into many patches
- Determine open lines of sight to points at center of each patch (LOS_factor)
- Calculate charging current densities to each point, $J_e(\phi)$ and $J_i(\phi)$, using LOS_factor and potential ϕ due to charge on each patch
- Calculate charge collected on each patch with area A_p during time interval Δt:

$$Q_p = \Sigma\, J_\alpha\, A_p \Delta t$$

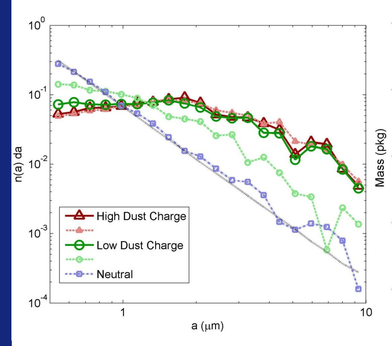
- Update charging currents and iterate in time



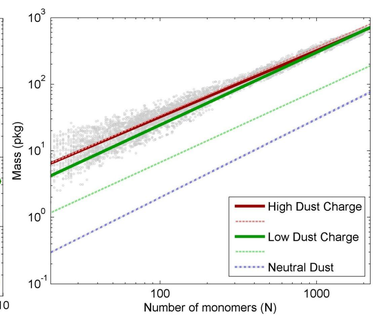
A 2D representation of the open lines of sight to three points on the surface of an aggregate.

Aggregate Characteristics

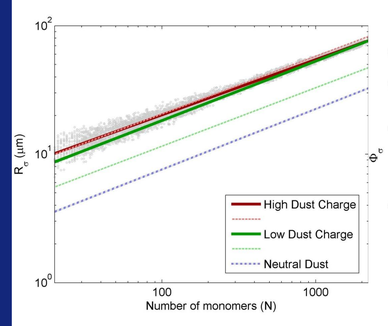
The morphology of the aggregates influences their coupling to the gas and resultant dynamics.



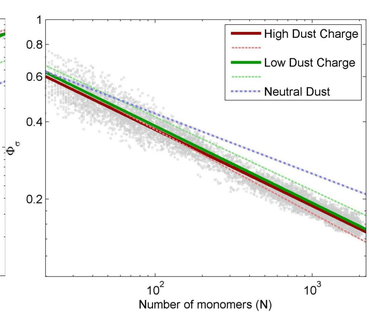
Distribution of monomer sizes in aggregates. The dotted line is the initial distribution. Highly charged and/or stochastically charged dust incorporate a greater number of large monomers into an aggregate.



Aggregate mass. The greater number of large monomers makes the highly / stochastically charged aggregates more massive for a given number of monomers. Data points are shown for the high dust charge, stochastic case.



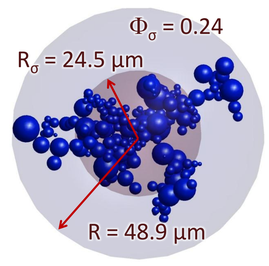
Equivalent radius. The more highly charged and stochastically charged aggregates grow to larger equivalent radii for the same number of monomers due to the inclusion of larger monomers and increased fluffiness of the aggregates.



Compactness factor. The stochastically charged aggregates trend to the same compactness factor, which is less than that for the dust with small charges, but greater than that for the highly charged dust. Data points are shown for the high dust charge, stochastic case.

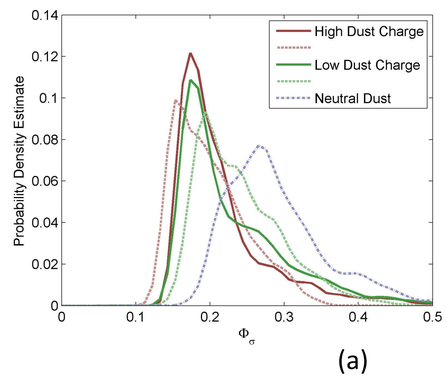
Compactness Factor

Fluffier aggregates, characterized by lower compactness factors[8], are more strongly coupled to the gas and have greater collision cross sections.



$$\Phi_\sigma = \frac{\sum r_i^3}{R_\sigma^3}$$

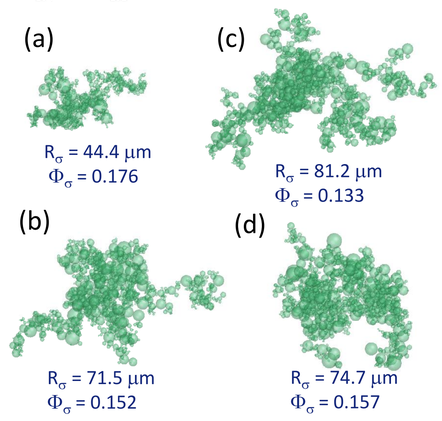
r_i = radius of ith monomer
R_σ = Radius of average projected cross section



Charged aggregates have lower compactness factors and are therefore fluffier than neutral aggregates. The distributions for the stochastically charged populations are more strongly peaked and intermediate to those for the non-stochastically charged dust.

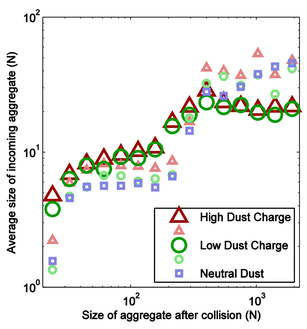
Aggregates grown under different charging conditions have different physical characteristics. Here, four aggregates with N≈2000 are shown to scale:

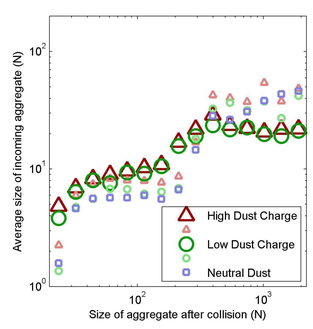
a) Low charge, non-stochastic,
b) Low-charge, stochastic,
c) High-charge, non-stochastic
d) High charge, stochastic.

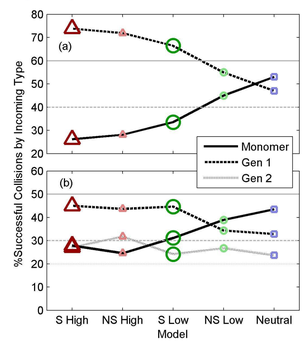


Collision Statistics

At larger N, stochastic charging increases the collision probability for the highly charged dust, but decreases that for the dust with small charges (left). The average size of an aggregate incorporated into a larger one tends to approach a constant size (middle) while the average size for the non-stochastically charged aggregates continues to increase. The small ($N \leq 20$) aggregates tend to more efficient at colliding with the larger grains than do monomers (right).







Smaller, lighter symbols denote non-stochastic charging

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