ENERGY CONSERVATION AND PARTITION IN CTH IMPACT SIMULATIONS. D. G. Korycansky, *CODEP, Department of Earth and Planetary Sciences, University of California, Santa Cruz CA 95064* .

Introduction

The CTH hydrocode is designed to treat multi-dimensional problems of shock propagation and non-linear response in physical substances including geological materials [1], and is widely used in the planetary science community for a great variety of simulations. CTH solves the conservation equations for continua in a finite-volume Lagrangian formulation with subsequent rezoning steps to an Eulerian (fixed) grid. The code has many advanced characteristics that render it a good tool for the problems at hand, and which account for its widespread use in the planetary science community. Among these characteristics are adaptive mesh refinement (AMR) for efficient (and user-specifiable) allocation of computational resources to portions of a calculation where higher resolution is desirable, such as shock fronts and material interfaces. CTH incorporates the advanced equations of state SESAME and ANEOS, as well as simpler analytic models for numerous materials including many of geological interest. Likewise, a variety of material strength and damage models are incorporated into the code.

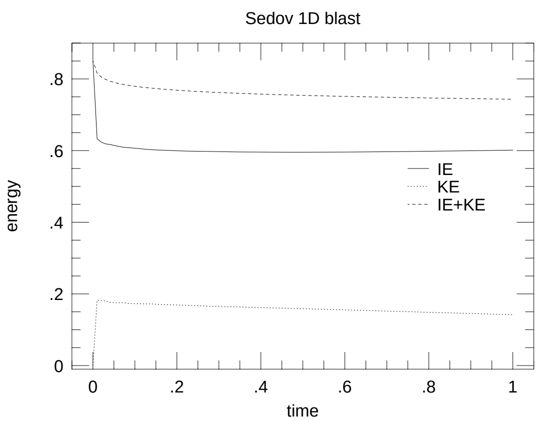

Figure 1: Energy vs. time for a 1-dimensional Sedov blast test. Plot shows internal energy (solid line), kinetic energy (dotted line) and the sum (dashed line). The grid is uniform with $\Delta r = 0.0005$ over the domain $r = 0$ to $r = 1.2$

In the work presented here we examine two aspects of the performance of CTH for hydrodynamical simulations. The first aspect is the overall energy conservation as exhibited by the code's actual performance. Notwithstanding the code's incorporation of the energy equation into the solution algorithm, numerical factors such as grid size, artificial viscosity, or other factors may affect the final results, and it is interesting and worthwhile to examine this.

The second aspect of the is the partition of energy from an impact into components–kinetic, thermal, or gravitational post-post-impact. This is less an examination of the code's performance, as we do not have an analytic result for the fractions of the initial kinetic energy of the impactor that go into the various forms as mentioned. We might expect that in the long term, the initial kinetic energy $1/2 m_i v_i^2$ of the impact of a body of mass m_i and velocity v_i would ultimately largely go into heating the target (i.e. thermal energy), but depending on the circumstances as significant fraction of the energy might go into escaping ejecta or into the atmosphere of any target body that had one, such as the Earth or Titan. For such bodies, impacts might drive significant temporary climate changes, and understanding energy partition post-impact is a basic part of understanding the effects of the process. Likewise, for extremely massive impacts, target erosion or catastrophic disruption is possible, and energy partition is a basic parameter for such an outcome.

Energy partition from impacts to the best of our knowledge has been explicitly studied in only a few papers by O'Keefe and Ahrens [3,4]; a figure from their first study is reproduced by Melosh [5]. Results are presented here in a similar format.

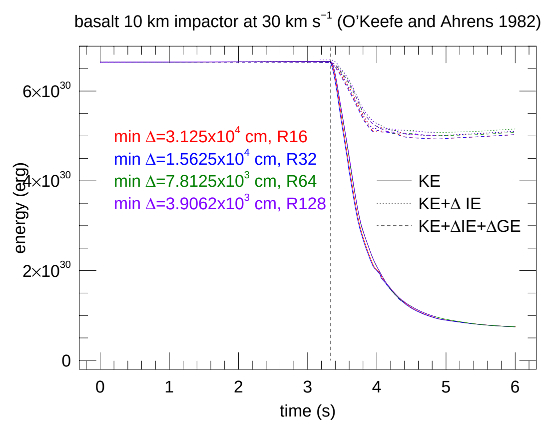

Figure 2: Energy vs. time for the initial stages of the impact of a 10-km basalt sphere into a basalt target at 30 km s^{-1}, after [3]. The solid curves are the kinetic energy, the dotted curves the sum of kinetic and internal energy, and the dashed curves the sum of kinetic, internal and gravitational potential energy. Several runs are shown with the indicated maximum mesh resolution Δ. (Rn is the resolution in terms of $n =$(impactor radius R)/Δ.)

Results: Sedov calculation

The problem of a point explosion in an infinite medium is a classic problem of hydrodynamics. In the case where the explosion (taken to occur at the origin of coordinates $r = 0$) occurs in a radially stratified medium (density $\rho \propto r^{-q}$, the "Sedov problem" analytic or quadrature solutions are available. We follow the detailed recent discussion by Kamm and Timmes [2], and present results for the simple case of a uniform medium. We made calculations for a 1-dimensional problem, with a gammalaw perfect gas ($\gamma = 1.4$ in this case). We used dimensionless scaling for the total initial energy $E_0 = 0.851072$ yielding the analytic shock location at $r = 1$ for $t = 1$. As usually required for numerical treatments of this problem, the initial energy was spread over a small region near the origin ($r = 0$ to $r = 0.05$ in this case). We made calculations for grid zoning from 120 to 2400 gridpoints in the region $r = 0$ to 1.2.

Some results are shown in Fig. 1, which shows energy versus time for the kinetic, internal, and total energies for our highest-resolution calculation. It will be noted that the total energy decays noticeably, especially at the initial phase of the calculation. The overall decrease at the end of the calculation is $\Delta E/E = 12.7\%$ of the total initial energy E_0. Changes in grid resolution from coarsest to finest improved the overall energy conservation marginally (from an $\Delta E/E = 16.9\%$), and resetting the von Neumann artificial viscosity parameter from the nominal $q = 2.0$ to $q = 0.5$ actually decreased the accuracy of the energy conservation.

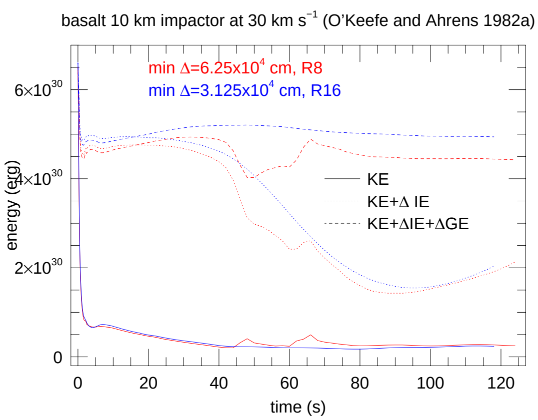

Figure 3: Energy vs. time for an extended calculation of the impact of a 10-km basalt sphere into a basalt target at 30 km s^{-1}, after [3]. The solid curves are the kinetic energy, the dotted curves the sum of kinetic and internal energy, and the dashed curves the sum of kinetic, internal and gravitational potential energy. Runs are shown with the indicated maximum mesh resolution.

Results: Energy partition

For the energy partition calculations we first made simulations of a problem duplicating that of [3]: the vertical impact of a $d_i =$ 10 km diameter basalt sphere into a basalt half-space target at $v_i = 30$ km s^{-1}. We have also looked at impact velocities of 10 and 20 km s^{-1}, but we present only the highest-velocity results here. Additional calculations have been made for ice impactors into ice targets, suitable for outer-planet satellites. The equation of state is the SESAME library from Los Alamos National Laboratories. O'Keefe and Ahrens carried out the calculation (as shown in Fig. 10 from [3]) out to a dimensionless timescale $\tau = 50 d_i/v_i$ or physical time $t \approx 17$ s; we present results of both short calculations to examine the initial energy conservation (Fig. 2) and longer ones up to the crater formation timescale ($t \sim 120$ s). The computational domain is 200 km in radius and ± 200 km above and below the impact target surface. For these calculations we make use of CTH's adaptive mesh, testing maximum grid resolution down to $\sim 3.9 \times 10^3$ cm. In order to simplify the energy accounting, the boundaries are reflecting, so that pressure waves below the surface reflect from the outer radius, and ejecta are forced to remain in domain. For short-time calculations we start with the impactor at an altitude of 100 km above the surface, to clarify the effects of advection of material through the domain as opposed to the actual impact.

Typical results of short-time calculations are shown in Fig. 2, which shows an impact in which the impactor starts at an altitude of 100 km above the target, and strikes the surface at $t = 3.33$ s, as indicted by the vertical dashed line. While the advection of the impactor through the grid does not appear to cause significant energy errors, differences of $\sim 15\%$ appear during the first "contact and compression" phase, in which the impactor strikes the surface and the initial compression shock wave develops. The amount of energy error is not strongly dependent on resolution, nor does depend greatly on run parameters such as artificial viscosity or other parameters that are available to improve code performance.

Figure 3 shows results of an extended-time calculation, lasting through the expected crater-formation time ($t \sim 120$ s, [6]). Only two runs are shown, with somewhat lower maximum resolutions than presented in Fig. 2. It will be seen that apart from the initial contact-and-compression phase, that overall energy conservation (kinetic+internal+gravitational) is fairly good, considering the complexity of the process. Higher resolution runs (carried out to $t = 30$ s) again show only modest improvements in energy conservation compared to the ones shown in Fig. 3. There is a substantial change in internal energy that is largely balanced by gravitational energy: we attribute this to the expansion of a low-velocity rising plume of hot material. Consideration of the amount of energy change shows that the mass involved is $\sim 10^{-2}$ of the target mass in the grid, or approximately 100 times the impactor mass, showing the rather delicate balance of energy that is involved in the problem.

Acknowledgments

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References

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