STRUCTURAL MODELING OF RUBBLE PILES IN TWO AND THREE DIMENSIONS. D. G. Korycansky, *CODEP, Department of Earth and Planetary Sciences, University of California, Santa Cruz CA 95064* .

Introduction

One of the puzzles about the structure of asteroids and other small bodies of the solar system is amount of internal void space, as deduced from measurements of their bulk densities compared to the mineral grain densities of their surfaces. In many cases, void fractions up to 50% or more are inferred (Britt and Consolmagno [1]). The question arises as to whether this large void fraction is the result of large-scale internal structure ("macroporosity"), or small-scale grain-density effects ("microporosity"). Here we present preliminary work on making structural models of rubble piles in two and three dimensions.

Methodology

We use a penalty method for structural modeling: given a configuration of (potentially overlapping) polygons (in 2D) or polyhedra (in 3D), the positions and depths of intersections are found using Minkowski summation and the configuration is modified to minimize or eliminate overlaps. In effect a “penalty force" is applied to overlapping objects in order to separate them. The method is equivalent to “first order dynamics" (i.e. non-inertial motion, ‘or $F = mv$) (cf. Erleben et al. 2005 [2]). Additionally, the center of the intersection is needed for rotational displacements; this information must be found separately, as the Minkowski sum does not provide it.

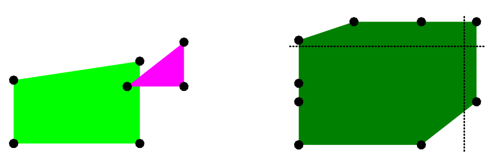

Figure 1: Left: 2 polygons P (green) and Q (red). Right: Minkowski sum of P and $-Q$; because P and Q intersect, the Minkowski sum includes the origin.

Collision detection is done using by a method using the Minkowski sum operation. (In this case we actually want the “difference”, as opposed to the sum, i.e. the sum of one object and the negative of the other.) The Minkowski sum $P \oplus -Q$ of an object P and another object Q is the pairwise difference of all points (i, j) in P_i and Q_j. Two polyhedra intersect if their Minkowski sum (difference) encloses the origin. Minkowski sums have two useful properties: 1) the Minkowski sum of two convex polyhedra is itself convex, and 2) the minimum distance of the sum polyhedron from the origin is the displacement vector needed to separate the original polyhedra. The first property implies that the Minkowski sum of two convex polyhedra can be constructed using only the vertices of the polyhedra; in that case a convex hull algorithm is required to complete the construction of the sum.

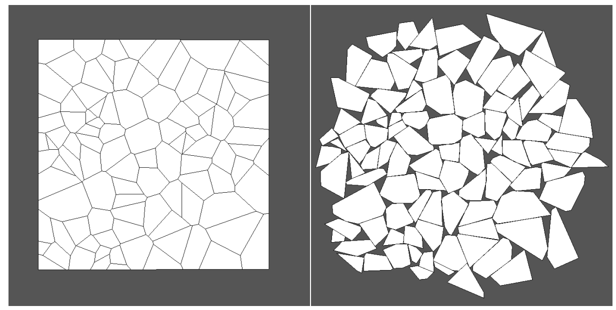

Figure 2: 2D results: left, a Voronoi decomposition of a square region. Right: Result of randomly perturbed configuration relaxation to a non-penetrating set.

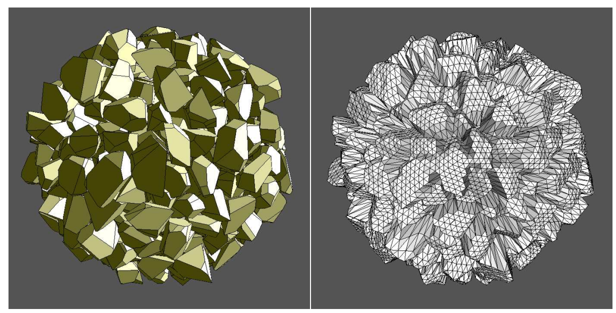

Figure 3: 3D results: left, the result of a trial run with 1000 blocks of a Voronoi decomposition. Right: The ”shrink-wrapped” polyhedron used to find the volume enclosed by the blocks.

As noted, overlapping pairs are determined from Minkowski sums that enclose the origin. For pairs that do overlap, displacements and rotations are calculated that are intended to resolve the overlap; if a given object is involved in multiple intersections, the sum of the displacements/rotations is accumulated. The displacements and rotations are then applied and the procedure is repeated, until (hopefully) an equilibrium is reached. A central potential is also applied to bring the blocks into contact.

Results

Sample 2D results are shown in Fig 2: initial close-fitting Voronoi decomposition polygonal sets are perturbed and relaxed to a configuration as shown. For two dimensional problems, the procedure appears to work well.

Some 3D results are shown in Fig 3, where the left panel shows a rubble pile with 1000 blocks. The right panel shows the "shrink-wrapped" pile: a 10242-face polyhedron has been fitted in order to estimate the enclosed volume of the pile.

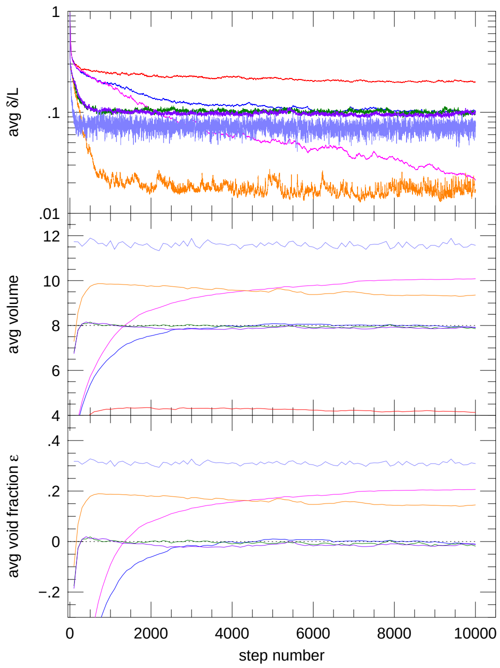

Figure 4: 3D results: Several 100-body runs. Shown are average interpenetration distance δ/L, total enclosed volume, and void fraction ε for various trials as a function of step number in the calculations.

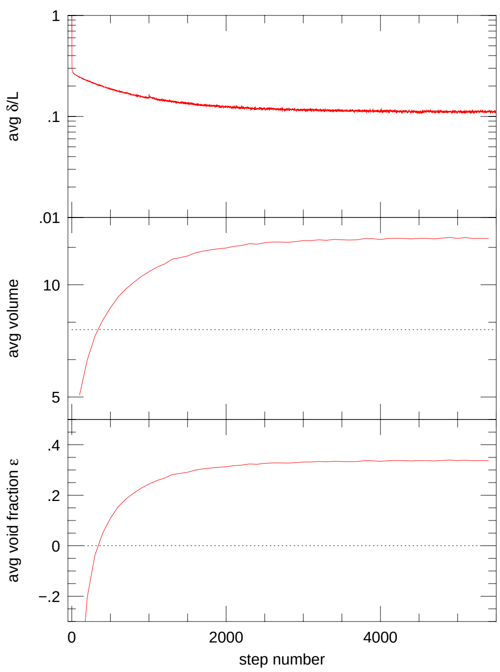

Figure 5: 3D results: similar results for the 1000-body run shown in Fig 3.

Figs 4 and 5 show the average interpenetration distance δ/L, total volume (as measured by the shrink-wrap procedure, and resulting void fractions, as a function of iteration step number n, for several 100 and 1000-block models. In these cases we are interested primarily in the final configuration, as opposed the pseudo-dynamics of the intermediate steps. In Fig 4, we varied run parameters such as the degree of correction applied per step (e.g. a "relaxation" parameter $\ll 1$ similar to those often used in numerical algorithms for elliptic partial differential equations, or similarly the amount of external forcing potential used to draw the blocks into contact). Results were somewhat mixed for 3D calculations. In general void fractions ranging from 0 to 30% result from the calculation indicating that in some cases a good deal of residual interpenetration remains despite application of the procedure. We did not find a set of simulation parameters that would guarantee minimal interpenetration of the blocks at the end of the calculation.

Conclusion

Overall we have found that preliminary work is encouraging, but ultimately not completely satisfactory at present. More development is needed for three-dimensional modeling to provide definite results. In particular, a more robust scheme would need to be developed for 3D calculations, to deliver configurations that have little interpenetration (i.e. block overlap), and do so more efficiently. Further work is required, perhaps along the lines of studies done of packing fractions of polyhedra such as the work discussed by Torquato and Jiao [3].

Acknowledgments

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References

[1] Britt and Consolmagno 2001, *Icarus* **152**, 134. [2] Erleben, K., *et al.* 2005. *Physics-Based Animation*, Charles River Media, Inc., Hingham, Mass. [3] Torquato and Jiao 2012, *Phys rev. E* **86** 01102.