

EFFECT OF STOCHASTIC CHARGING ON COSMIC DUST AGGREGATION. L. S. Matthews¹, B. Shotorban², and T. W. Hyde¹. ¹CASPER, Baylor University, Waco, Texas 76798, USA, ²Department of Mechanical and Aerospace Engineering, The University of Alabama in Huntsville, Huntsville, Alabama 35899, USA.

Introduction: The coagulation of cosmic dust grains is a fundamental process which takes place in many astrophysical environments, such as presolar nebulae, circumstellar and protoplanetary disks, and cometary tails. Since most cosmic dust grains are charged, the electrostatic force between dust grains can strongly affect the coagulation rate. Numerous experimental and numerical studies have examined charging processes and coagulation of dust grains [1-4].

Electrical charge of a dust grain experiences stochastic fluctuations since ions and electrons (plasma particles) are collected on the surface of the dust grain at random time intervals. This stochastic behavior is intrinsic noise [5] which occurs in systems with discrete nature. This study examines how the stochastic charge fluctuations alters the coagulation process and the physical characteristics of the aggregates formed.

Charging Model: The charge on an aggregate surface is found using orbital motion limited theory with a line of sight approximation, OML_LOS [1]. The surface of the aggregate is divided into many patches and the current density to each patch is calculated as a function of the electric potential at that patch, due to the charge on all of the patches. The line-of-sight approximation takes into account the fact that not all of the surface patches are open to the plasma environment, thus plasma particles incident from certain directions are blocked from impacting the grain surface.

Stochastic Charging Model. A master equation, needed for the description of the random charge fluctuations of the aggregates, can be developed as follows

$$\frac{dP(\mathbf{Z}, t)}{dt} = \sum_{p=1}^N I_{i,p}(\mathbf{Z} - \mathbf{e}_p) P(\mathbf{Z} - \mathbf{e}_p, t) + I_{e,p}(\mathbf{Z} + \mathbf{e}_p) P(\mathbf{Z} + \mathbf{e}_p, t) - [I_{i,p} + I_{e,p}] P(\mathbf{Z}, t),$$

where N is the total number of patches on the aggregate, $\mathbf{Z} = \{Z_1, Z_2, \dots, Z_N\} \in \mathbf{R}^N$ is the vector of the elementary charges collected on patches (e.g., Z_2 is the number of elementary charges collected on the patch number 2), $P(\mathbf{Z}, t)$ is the probability density function of a state at which the patch number 1 has Z_1 charges, the patch number 2 has Z_2 charges etc., $I_{i,p}$ and $I_{e,p}$ are the currents of ions and electrons, respectively, to the patch number p , and $\mathbf{e}_j \in \mathbf{R}^N$ is the unit vector, e.g., $\mathbf{e}_3 = \{0, 0, 1, 0, \dots, 0\}$. Eq. (1) is regarded as a gain-loss equation for the probabilities of separate states of the charges collected on patches. It is assumed that no charge is transferred from one patch to another.

One can obtain a Fokker-Planck equation from the master equation which is statistically equivalent to the following Langevin equation for the time evolution of the charge on the patch number p :

$$dZ_p(t) = [I_{i,p}(\mathbf{Z}) - I_{e,p}(\mathbf{Z})] dt + \sqrt{I_{e,p}(\mathbf{Z}) + I_{i,p}(\mathbf{Z})} dW_p(t),$$

where $W_p(t)$ is a Weiner process. The Langevin equation is solved numerically by Euler-Maruyama method:

$$Z_p^{(n+1)} = Z_p^{(n)} + [I_{i,p}(\mathbf{Z}^{(n)}) - I_{e,p}(\mathbf{Z}^{(n)})] \Delta t + \sqrt{I_{e,p}(\mathbf{Z}^{(n)}) + I_{i,p}(\mathbf{Z}^{(n)})} \sqrt{\Delta t} \xi_p,$$

where Δt is the time step, $Z_p^{(n)}$ is the charge of the patch number p at time step n , and ξ_p is a random number with a normal distribution.

Spherical monomers charged with a stochastic current tend to have a mean charge slightly more negative than when stochastic effects are not included. The effect is more pronounced for the smallest monomers, which carry less charge, and the relative deviation from the mean charge is also much larger for small monomers than it is for large monomers.

Coagulation Model: Coagulation was modeled using an initial dust population of silicate spheres with radii ranging from $0.5 \leq a \leq 10 \mu\text{m}$ with a powerlaw size distribution $n(a)da \propto a^{-3.5} da$. The hydrogen plasma environment had equal electron and ion temperature, $T_e = T_i = 900 \text{ K}$. Two different plasma densities were used, one in which the dust density is very low, so that a negligible percentage of the electrons reside on the dust grains, $n_e = n_i = 5 \times 10^8 \text{ m}^{-3}$, and one in which the dust is high enough that the electrons in the plasma are depleted, with $n_e = 0.1 n_i$ [1]. The charge on the dust is larger for the low dust density case. The relative velocities between two interacting dust grains was set assuming the grains were coupled to turbulent eddies in a protoplanetary disk [6].

As the interaction time between two colliding grains is very short compared to the charging time, the charge on each grain was assumed to be constant throughout the particles' interaction. Upon a successful collision, the new aggregate was charged using OML_LOS and saved to a library. Aggregates were built in three generations, with the first generation grown to a size $N = 20$ monomers through collisions with single monomers, and the second generation

grown to a size $N \sim 200$ through collisions between monomers (60% probability) or first generation aggregates (40% probability). The third generation aggregates were grown to a size $N \sim 2000$ through collisions with monomers (50%), first generation aggregates (30%) or second generation aggregates (20%).

Results: Aggregates are characterized using their equivalent radius, the radius of a circle with area equal to the projected cross-section averaged over many orientations, and compactness factor, the ratio between the volume of all the constituent monomers and the volume of a sphere with radius equal to the equivalent radius. While there is little difference between the aggregates formed from the highly charged dust, the inclusion of stochastic charging effects has a marked effect on the aggregates formed from dust with a low charge. In this case, the aggregates formed with the stochastic charging effects are similar to the highly charged dust, having a lower compactness factor (Figure 1) and incorporating a larger percentage of large monomers into the aggregates (Figure 2). However, differences are seen for both dust populations for the average size of the successfully colliding aggregates (Figure 3) and the collision probability (Figure 4). Stochastic effects increase the average size of the aggregates which successfully collide for the mid-sized (generation 2) aggregates, but reduce the average aggregate size incorporated in the largest (generation 3) aggregates. Stochastic charging effects also markedly increase the collision probability for all aggregate sizes.

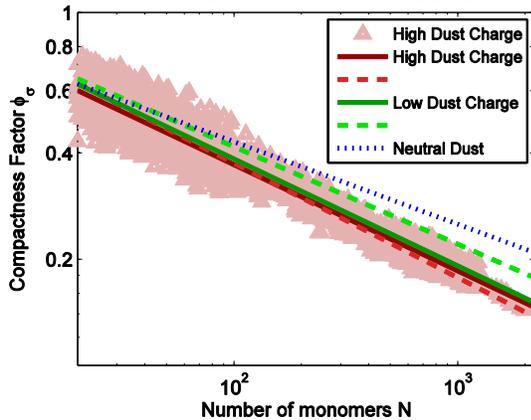


Figure 1. Compactness factor as a function of aggregate size. Linear log-log fits to all the data sets are shown, with the solid lines denoting the stochastic charging and dashed lines the non-stochastic charging. Data points are shown for the high dust charge, stochastic charging case.

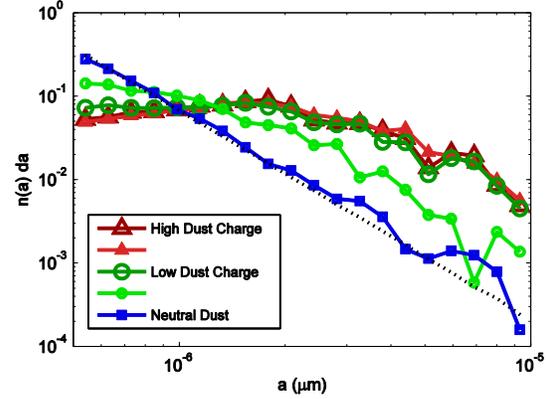


Figure 2. Distribution of monomer sizes contained in the third generation aggregates. The smaller, lighter symbols denote charging without stochastic effects.

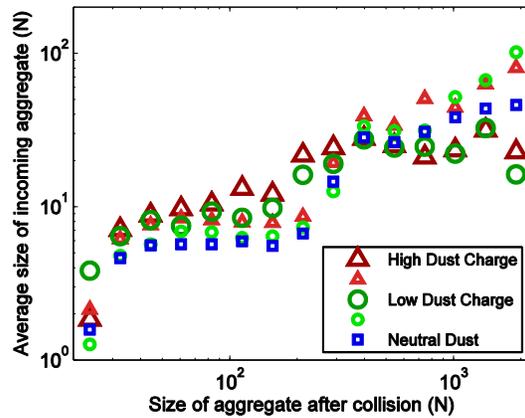


Figure 3. Average size of colliding aggregate versus the size of the resultant aggregate.

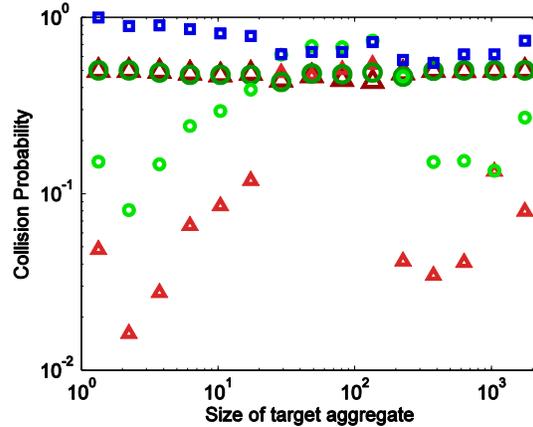


Figure 4. Collision probability versus size of target aggregate.

References: [1] L. S. Matthews, V. Land, T. W. Hyde (2012) *ApJ* 744:8. [2] S. Okuzumi (2009) *ApJ* 698. [3] B. Shortorban (2011) *Phys Rev E* **83**, 066403. [4] L. S. Matthews et al. (2011) *AIP Conf. Proc.* 1397, 397. [5] N. G. Van Kampen, *Stoch. Proc. in Phys. and Chem.* (Elsevier, Amsterdam, 2007). [6] C. W. Ormel and J. N. Cuzzi (2007) *A&A*, 466.