

LIMITATIONS OF SAMPLE SIZE IN METEORITE THIN SECTION AND SPECTROSCOPIC STUDIES: IMPLICATIONS FOR THE HEDS AND VESTA. A. W. Beck¹, C. E. Viviano², and T. J. McCoy¹,
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Introduction: Sampling error introduced through the limitation of sample size has significant and often overlooked implications for meteorite studies. Here we examine the errors associated with sample size as a function of grain size and abundance of a sample. We utilize equations used to determine the sample size necessary to accurately represent a sample to a significant degree of confidence. As we continue to explore planetary bodies that are the parent source for groups of meteorites in the collection (*i.e.*, Mars, the Moon, and Vesta), the anticipated surface mineralogy and spectroscopic variability of the parent body is necessarily informed by the available meteorite samples. Since meteorite studies help to resolve geologic interpretation on a global scale, rigorously quantifying the limitations related to a meteorite sample size is important for understanding remote sensing results from missions to these parent bodies. The equations utilized here are applicable to any suite of meteorites, including the HEDs (Howardites, Eucrites, and Diogenites) thought to come from the parent body 4Vesta [1]. The interpretation of the spectral data from the Dawn mission to asteroid Vesta is dependent upon a collective dataset from the HED suite. Therefore, the quantitative analysis of the sample set is fundamental to understanding the limitations of our interpretation of the Vestan surface.

In classic petrographic analysis studies, *e.g.* [2], the total analytical error (E_a) is defined as the sum of the measurement error (E_m) and the error due to the size of the sample taken from the rock, or the sampling error (E_s). The measurement error is intrinsic to the instrument or techniques being utilized to analyze the thin section (*e.g.*, the instrument measurement error from a microprobe analysis). Since E_m is constant between sections studied, we are more interested in characterizing the variability of sampling error for meteorite analysis, as biases produced from this error vary between samples. The sample variance (E_s^2) can be estimated through empirical relationships related to grain size, sample area/volume, and mineral distribution (*i.e.*, degree of heterogeneity). At the endmember example, either the mineral crystals are stochastically independent (homogenous distribution) or the composition of any one crystal is related to the composition of adjacent crystals (heterogeneous distribution) [*e.g.*, 3].

It would be useful to address the following questions: 1) What error is introduced when a meteorite section or powdered sample of given mass, M_s , is taken from the original sample?, and 2) How much mass of a meteorite sample is necessary so that the sampling

error will not exceed a specified variance (traditionally 90, 95, or 99% confidence)? These questions relate to what is known as the Fundamental Error (σ_{FE}^2), which represents the relative bias between the actual parameter (*e.g.*, the mean) of the meteorite and the estimated mean obtained from the meteorite section or powdered sample. This error σ_{FE}^2 is a function of compositional heterogeneity, grain size distribution, and M_s :

$$\sigma_{FE}^2 = \frac{K d_N^3}{M_s},$$

where K is a constant relating to factors such as shape, granularity, mineralogic composition (*e.g.*, density), etc. [4]. If a homogenous distribution and mineralogic characteristics of a given phase is assumed, this calculation may be simplified to only assess the errors associated with sample size.

A particular sample size can be calculated to minimize sampling error, based on the probability of sampling a particular phase of interest, p . For the HED case study, we assume p is the probably (or actual percentage) of olivine in the sample. The number of particles of sample necessary for providing the desired standard deviation for sampling, s_{samp} , is the following:

$$n = \left(\frac{1-p}{p}\right) \left(\frac{1}{(s_{\text{samp}})^2}\right)$$

where n is the number of particles necessary. For instance, we could calculate the percentage of olivine with a relative standard deviation for sampling of 1%. If the olivine abundance is 23% (see Table 1 [5], average), this would require a sample of $\sim 3.3 \times 10^4$ particles. The greater the number of particles sampled, the more accurately the error may be estimated. With this in mind, a meteorite thin section with the smallest grain size is most accurately representative of the actual population distribution of the meteorite. Assuming a particle size of 0.14 mm (sample MIL 07001), the required thin section size to have $\sim 3.3 \times 10^4$ particles of olivine would be 1.4×10^3 mm² or 14 cm². A more reasonable thin section size of 2 cm² would correspond to $\sim 2.3 \times 10^3$ particles or a standard deviation of a 20%. For the samples representing the harzburgite diogenites (Table 1) this is the best case for the lowest sampling error E_s . Thus the total analytical error, E_a is greater than 20%.

If the olivine abundance is 42% (see Table 1, sample A 881548), this would require a sample of $\sim 1.3 \times 10^4$ particles. Assuming a particle size of 10 mm (sample A 881548), the required thin section size to have $\sim 1.3 \times 10^4$ particles of olivine would be 3.3×10^3

mm² or 328 m², which is far greater than any sample size of a meteorite available.

Assuming a grain size of 10 mm (sample A 881548), an abundance of 42% olivine, and a sample size of 2 cm², this would equate to a sample size of <1 particle and demonstrates how sensitive sampling error is to the grain size of the phase of interest.

Conclusions: Although the HEDs are a voluminous group of achondrites in the existing meteorite collection, the samples provided for thin-section and analytical studies are necessarily small. Even at higher abundances in a thin section sample (*e.g.* A 881548), the dominance of the grain size factor on sampling error will cause a sample with large grain sizes to have a much larger sampling bias. This study confirms that samples of fine-grained harzburgitic diogenites are statistically the most representative of their parent meteorite. This work supports the conclusions of *Beck et al.*, [6] that spectral data from samples MIL 07001 and GRA 98108 are more likely representative of the exposed olivine-bearing surfaces of Vesta than the spectra of samples that are coarser grained.

References: [1] McSween H.Y. et al. (2011) *Space Sci. Rev.*, 163:141-174. [2] Chayes (1956) *John Wiley & Sons, Inc., New York*. [3] Neilson & Brockman (1977) [4] Minnitt et al. (2007) *J. South. Afr. Inst. Min. & Metal.*, 505-511. [5] Beck et al. (submitted, 2012) *MAPS*.

Table 1. Olivine abundances and grain sizes in the Antarctic harzburgitic diogenites

Sample & Total Mass	Section #	Normalized Mode		Average OL grain size (mm)	
		OPX	OL		REF
A 881548 110.2 g	31-1	54	46	10	(1)
	51-2	20	80	10	(1)
	23 (split)	100 ^a	0 ^a	-	(2)
	AVG (±1s)	58	42(40)	10	
ALH 77256 676 g	99	64	36	1.35	(3)
	116	100	0	-	(4)
	115	99	1	-	(4)
	128	89	11	1.90	(5)
	5	97	3	0.95	(6)
	AVG	90	10(15)	1.40(.5)	
GRA 98108 12.7 g	16	81	19	0.35	(5)
	2	68	32	0.30	(6)
	15	70 ^a	30 ^a	0.45	(7)
	AVG	73	27(7)	0.36(.1)	
MIL 07001 924 g	2	91	9	0.15	(6)
	6	87 ^b	13 ^b	0.10	(6)
	16 (slab)	90 ^a	10 ^a	-	(6)
	40	87	13	0.10	(6)
	55	91	9	0.15	(6)
	54	90	10	0.20	(6)
	AVG	89	11(2)	0.14(.1)	
	Group AVG	77	23(15)		

^aEstimated modes, all others calculated from BSE images

^b13% olivine is a more accurate abundance for this section.

REFS: (1) Yamaguchi et al. 2011, (2) Barrat et al. 2008, (3) Sack et al. 1991, (4) Bowmen et al. 1997, (5) Beck & McSween 2010, (6) This study, (7) Righter 2001