AN IMPROVED LUNAR MOMENT OF INERTIA DETERMINATION: A PROPOSED STRATEGY; M. P. Ananda, A. J. Ferrari, W. L. Sjogren, Jet Propulsion Laboratory, Pasadena, CA 91103

Currently the uncertainties in the low degree harmonic coefficients and libration parameters map into an uncertainty of 0.002 in the lunar moment of inertia. (Sinclair, W. et al, 1976, Gapcynski, el al 1975). If a lunar density model having a core of 300 km radius is assumed, the large uncertainty in the lunar moment of inertia causes a density uncertainty of over 4 gm/cm^3. However, with the following proposed lunar mission and data processing scheme the uncertainty in the lunar moment of inertia can be reduced to near 0.0001 and thus the uncertainty in the core density example previously given can be brought to near 0.1 gm/cm^3. This error level provides a meaningful constraint which can be used with other precision data types to develop deep interior lunar density models.

A Lunar Polar Orbiter Mission using two satellites has been proposed to gather global geophysical and geochemical data. The proposed mission consists of a close orbiter (100 km altitude) and a relay satellite (2000 km altitude), both in near polar circular orbits. This study is directed at improving the accuracy of the low degree and order harmonics coefficients using only the relay satellite data since its orbit provides a natural attenuation of the high degree gravity terms, which are the major corrupting factors to the low degree terms.

A statistical analysis is performed to determine the sensitivity of the gravity parameters to the data noise, arc length, orbital inclination, geometric and bias parameters (station location, tropospheric model error, timing bias between the stations, etc.), higher degree and order harmonic coefficient errors and other force parameters such as solar radiation and re-radiation effects. The statistical analysis is performed using a maximum likelihood estimator assuming Gaussian errors. Doppler tracking data are generated assuming a count time of 60 seconds with a data noise of 1 mm/sec. The nominal mission assumes an orbit of 95^o inclination and a semi-major axis of about 4000 km and the eccentricity of 0.1 for the relay satellite. The study is performed considering 28 days of tracking data from three Deep Space Network stations.

Table 1 shows the sensitivity of harmonic coefficients (C_{20}, C_{22}) to orbital inclination. Studies show that optimum inclination for determining C_{20} is about 52^o and for C_{22} is about 75^o. For moment of inertia computations it is only necessary to determine either one of the two harmonics due to a constraint between C_{20}, C_{22} and the libration parameters (β,γ) (Williams, et al, 1975). If the inclination were kept at the nominal design value of 95^o, the degradation from its corresponding optimal inclination is less for C_{22} than for C_{20}. The error in the estimate of C_{22} is only twice that at the optimal 75^o inclination. Studies also show that other gravity parameters C_{31} and C_{33} are not very sensitive to orbital inclination. Improved determinations for C_{31} and C_{33} are needed to refine the current accuracy of libration parameters β and γ. However, currently the largest uncertainty in the lunar moment of inertia is due to uncertainty in errors in C_{20} or C_{22}.

Table 2 shows the results from the statistical analysis. These results are obtained for the nominal inclination of 95^o. It can be seen that the un-

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certainty in C_{22} is improved over a magnitude when semi-major axis (a) of 7000 km is used, however, the improvement in C_{20} is not significant. The study also shows that an orbit of a = 7000 km is necessary to achieve the desired accuracy. The harmonic coefficients C_{31} and C_{33} are not very sensitive to orbital altitude. Studies also show that errors up to 10 m in station locations, tropspheric model with less than 10% error and timing biases of less than a tenth of a millisecond between the stations do not affect gravity terms. However, gravity terms are very sensitive to lunar mass and solar radiation pressure errors. When both lunar mass and solar radiation coefficients are estimated along with the gravity terms their contribution to the error in gravity terms is negligible.

Table 1: Sensitivity to Orbital Inclination

Inclination degrees	$\sigma_{C_{20}}$ *	$\sigma_{C_{22}}$ *
40	0.7 E-5	
52	0.1 E-5	4.4 E-7
60	0.4 E-5	2.0 E-7
75	1.2 E-5	1.2 E-7
85 (95)	2.0 E-5	2.9 E-7

*where a full 4x4 harmonic field is estimated and the unmodelled effects of 5 and 6 degree and order harmonic coefficients and lunar mass error and solar radiation pressure error are considered.

Table 2: Results from Error Analysis

Harmonic Coefficient	Current Best Known Uncertainty† (Formal Statistics)	Uncertainty* When the Effects of Unmodelled Parameters are Included	
		a = 4000 km	a = 7000 km
C_{20}	1.5 E-6	8.6 E-7	2.3 E-7
C_{22}	1.2 E-7	8.2 E-8	9.6 E-9
C_{31}	1.5 E-6	2.9 E-7	9.4 E-8
C_{33}	9.9 E-7	4.0 E-8	4.3 E-8

*When a full 4x4 harmonic field and certain 5 and 6 degree and order harmonic coefficients, lunar mass, solar radiation effects are estimated while the unmodelled effects of certain other 5, 6, 7 and 8 degree and order harmonic coefficients are considered.

† W. Sinclair, et al (1976).