The Regime of Zonostrophic Macroturbulence and
Its Application for Characterization of Large-Scale Circulation on Jupiter and Other Giant Planets

**Boris Galperin**
University of South Florida

With: R.M.B. Young, S. Sukoriansky, N. Dikovskaya,
P.L. Read, A.J. Lancaster, D. Armstrong

Crossing the Boundaries in Planetary Atmospheres:
From Earth to Exoplanets
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Preamble

➤ The Reynolds numbers typical of the Jovian circulation are O(10^{16}) $\Rightarrow$ **well developed turbulence**
➤ Yet the striped structure of the Jovian disk has not changed in centuries!
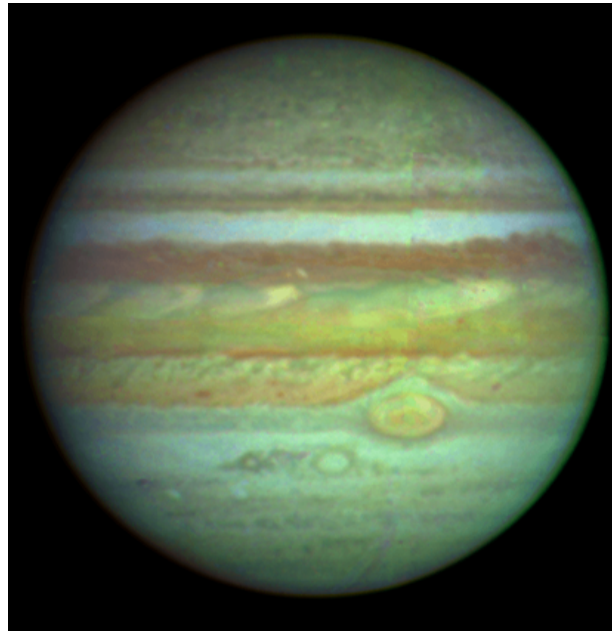
➤ How is this possible?
➤ **Clue**: the flow is not laminar; a scale matters!
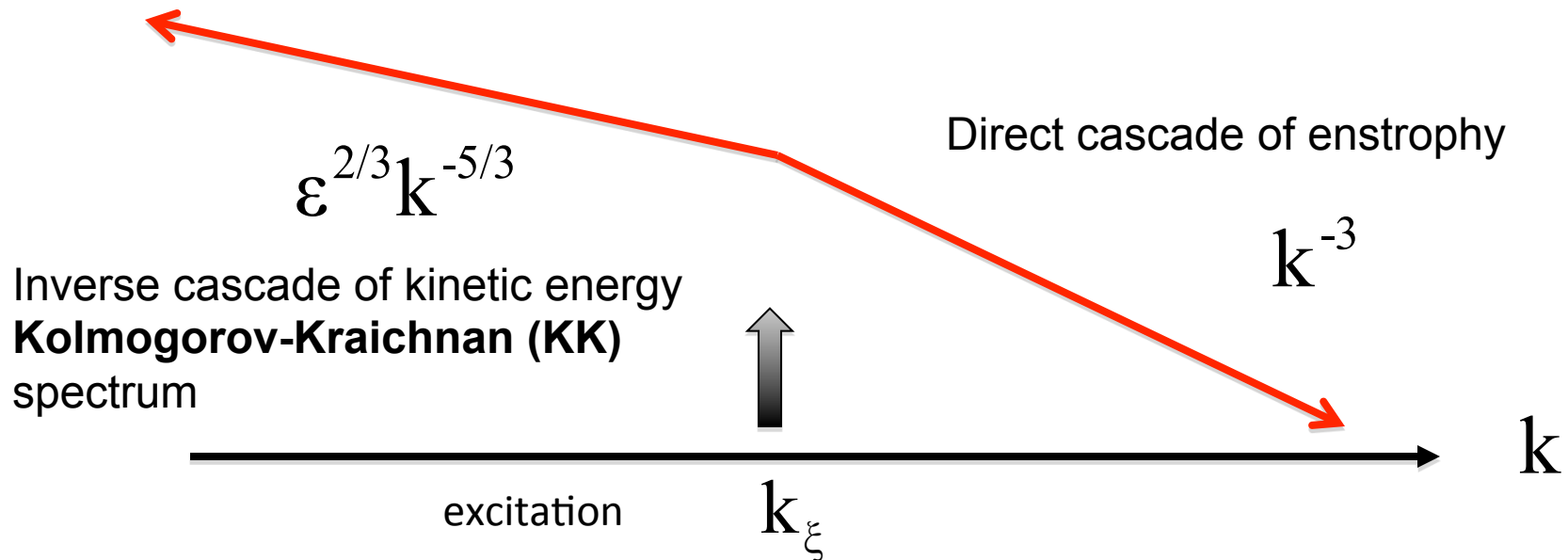➤ Different scale ranges harbor different physics

Rotation

Jovian circulation is subject to strong rotation ⇒ Taylor-Proudman theorem, two-dimensionalization; QG dynamics; inverse energy cascade

Classical 2D turbulence ??

???

> But what happens on large scales where β-effect becomes important?

$$\varepsilon^{2/3} k^{-5/3}$$

Direct cascade of enstrophy

$$k^{-3}$$

Inverse cascade of kinetic energy
Kolmogorov-Kraichnan (KK)
spectrum

excitation k_ξ k

Barotropic Vorticity Equation on a sphere

Small-scale forced BVE in spherical coordinates

$$\frac{\partial \zeta}{\partial t} + J(\psi, \zeta + f) = \nu \nabla^{2p} \zeta - \lambda \zeta + \xi$$

$\zeta = \Delta \psi$ - vorticity; ψ - stream function; $f = 2\Omega \sin\theta$ - Coriolis parameter; Ω - angular velocity; θ - latitude; ϕ - longitude; ν - hyperviscosity coefficient; λ - linear friction coefficient; sets the large-scale friction wave number n_{fr}. Used R-truncation; R133 and R240 resolutions.

Random energy injection with constant rate ε at about $n_\xi = 100$.

Analyze anisotropic spectrum

$$E(n) = \frac{n(n+1)}{4R^2} \sum_{m=-n}^{n} \langle |\psi_n^m|^2 \rangle \Delta m = E_Z(n) + E_R(n)$$

zonal (m=0)   **residual**

$$\Delta m = 1$$

Zonostrophic turbulence
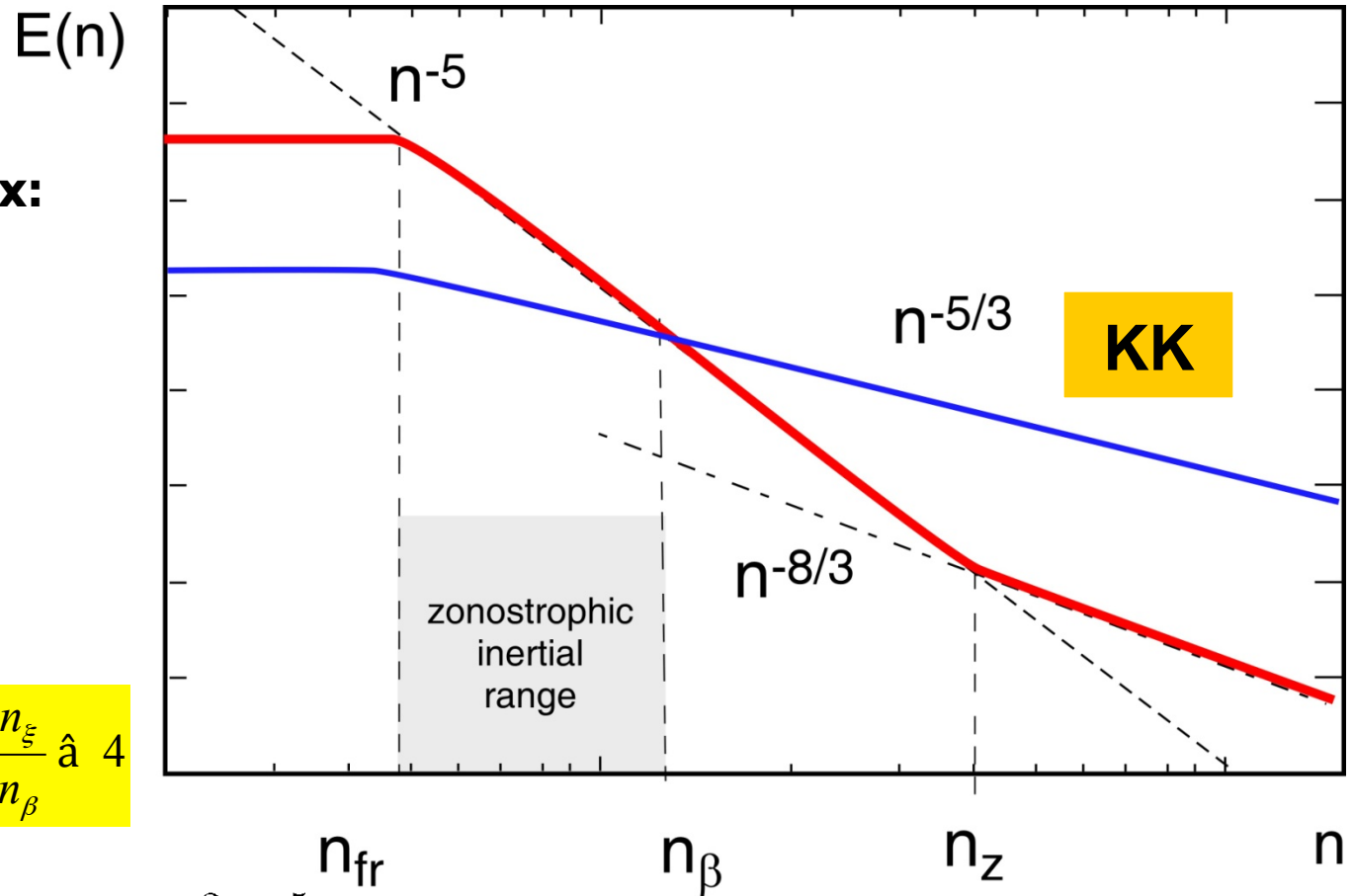
(from Greek ζωνη - band, belt, and στροφη - turning)

Zonostrophy index:

$$R_\beta = \frac{n_\beta}{n_R} \hat{a} \ 2.5$$

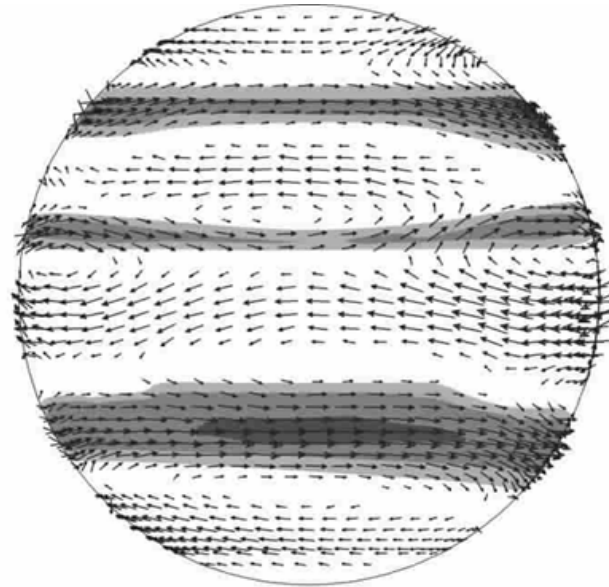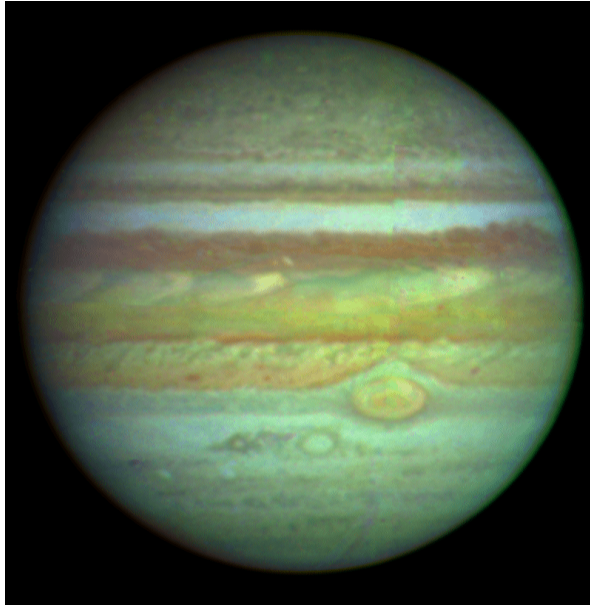$$n_\beta \ ; \ 0.5(\beta^3 / \acute{U})^{1/5}$$

$$\beta = \Omega / R$$

$$R_b = 0.7 \frac{\hat{E}\acute{U}b^2\hat{}^{1/20}}{\hat{A}l^5 \approx}$$

$$R_\xi = \frac{n_\xi}{n_\beta} \hat{a} \ 4$$

$E(n)$

n^{-5}

$n^{-5/3}$

KK

$n^{-8/3}$

zonostrophic inertial range

n_{fr} n_β n_z n

$$E_Z(n) = C_Z(\Omega/R)^2 n^{-5}, \quad C_Z \simeq 0.5$$

$$E_R(n) = C_K \epsilon^{2/3} n^{-5/3}, \quad C_K \simeq 4 \text{ to } 6$$

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When zonostrophic regime is attained, a system of powerful zonal jets emerges in physical space ⊠


zonostrophic macroturbulence

Zonal jets – manifestation of a slow manifold with a steep 1D spectrum and a very long characteristic time scale ⊠ low variability of the jets

Do giant planets' circulations conform to the zonostrophic regime?

Using various observations, we computed zonal spectra on Jupiter and Saturn - they agreed with those in simulations

Galperin et al. (2001)
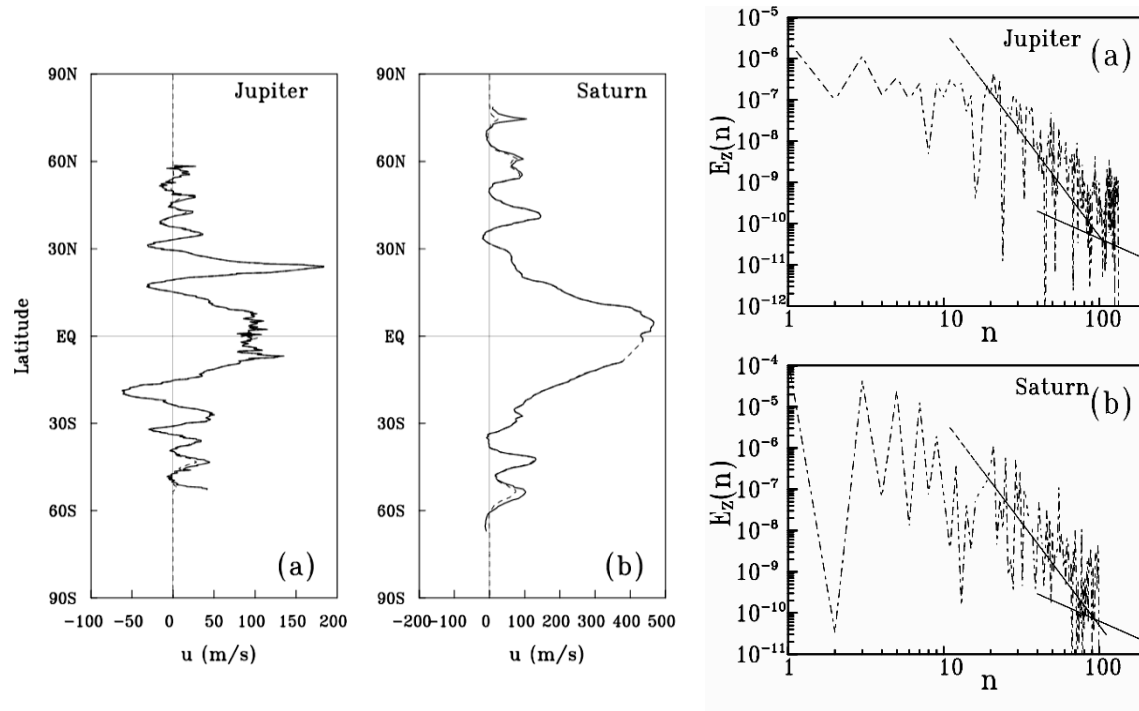
These spectra were also confirmed later by Barrado-Izagirre et al. (2009)

But do they really conform?

➢ Only zonal velocity profiles and zonal spectra were used. This is not enough. Non-zonal spectra are required. They are predicted to be Kolmogorov-Kraichnan (KK)

➢ From these spectra, compute ε and R_β $\Rightarrow$ positive regime identification

➢ For a long time, data was insufficient to perform this analysis

➢ Data was provided by Cassini flyby in December 2000

➢ Three days of this data were processed to compute 1D and 2D spectra

Next steps

➤ The data was independently processed and spectra were evaluated by David Choi and Adam Showman (CS) and our international team (Roland Young's poster)

➤ The results have confirmed the presence of ZMT regime

➤ Other issues addressed:

(i)   energetics; energy transfer to zonal flows and to Great Red Spot
(ii)   large apparent energy conversion rate to zonal flows
(iii) increasing strength of circulation with decreasing forcing
(iv) meridional diffusion
(v)   diffusion of gases and debris produced by asteroidal impacts
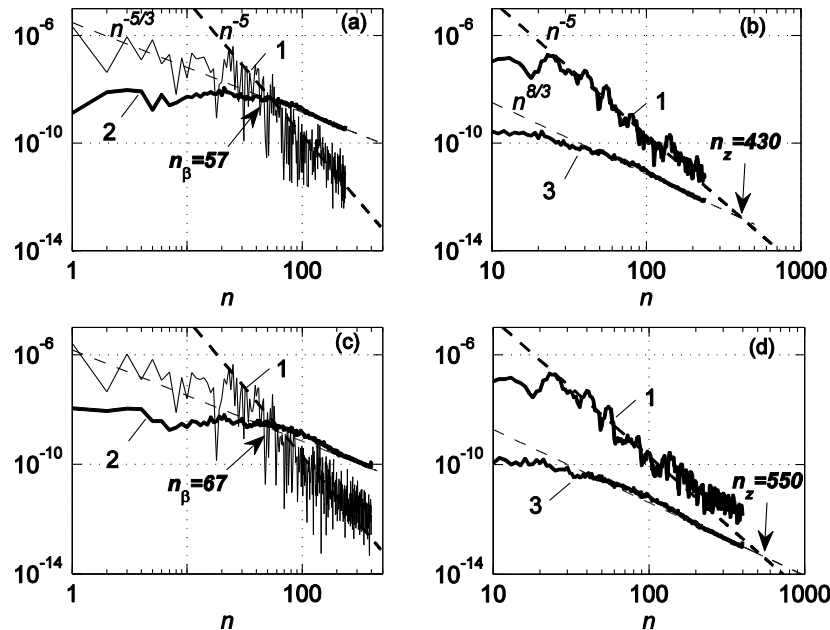
Zonal and residual spectra
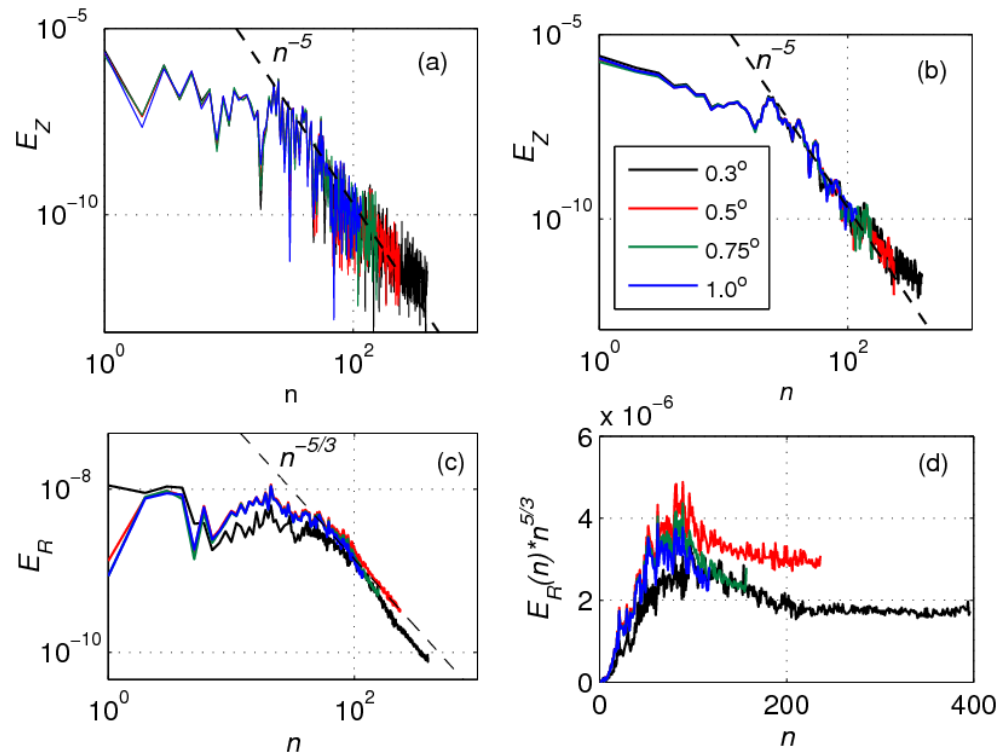
(a): Three-day averaged zonal spectra; $C_Z=2$
(b): Smoothed spectra using a moving box (5 point stencil)
(c): Residual spectra $E_R(n)$; the dashed line corresponds to KK spectrum
(d): Compensated residual spectra $C_R=E_R(n)n^{5/3}$ derived from (c).
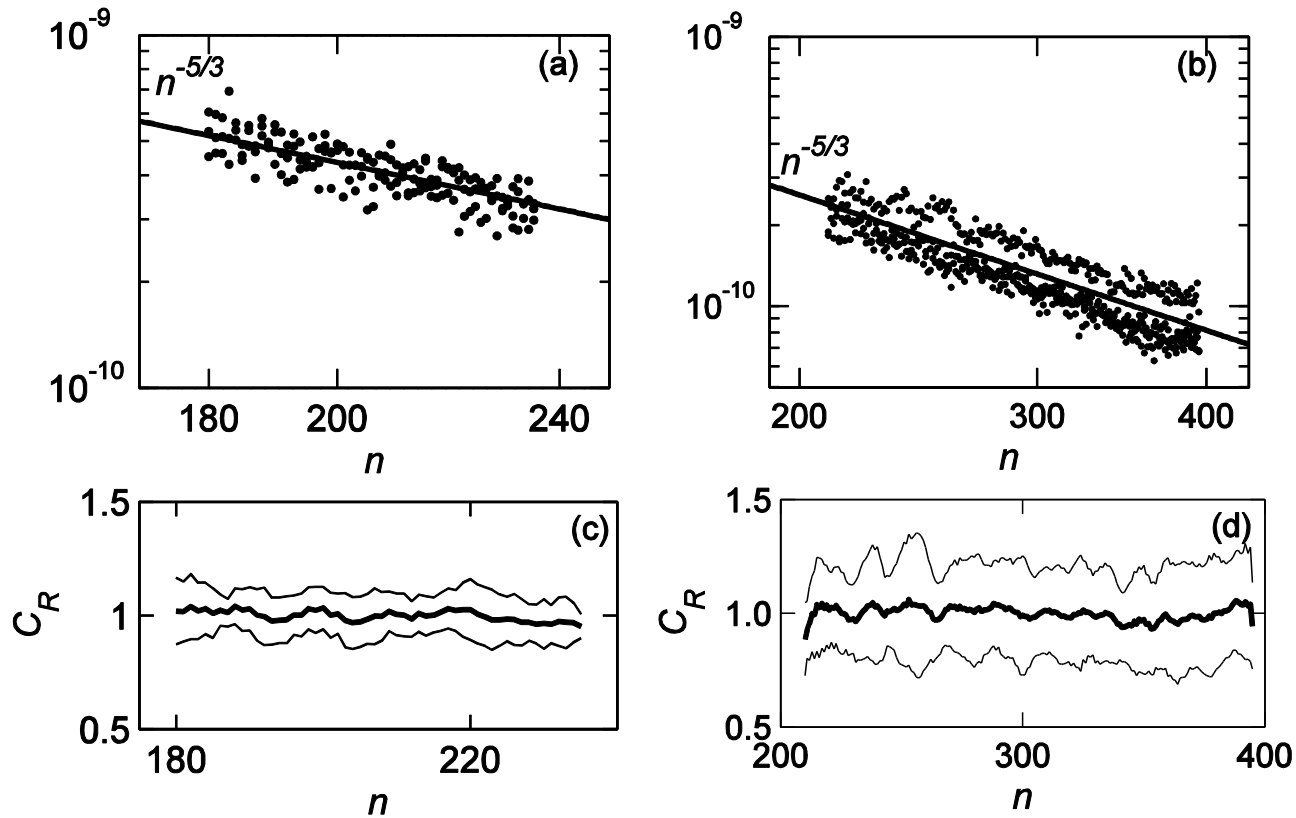
Blow-up of the three-day averaged residual spectra
(a,b): our and CS data points used for least-square determination of spectral slopes
(c,d) the corresponding compensated spectra, $C_R(n) \sim E_R(n)\, n^{5/3}$, scaled around unity, with $\pm\, \sigma$ error bars.

Estimation of the main parameters

Zonal (1), residual (2), and modal residual (3) spectra estimated from our (a and b) and CS data (c and d). (b) and (d) - zonal spectra smoothed by a five-point running average.

Assuming $C_K = 6$ we find:

Our velocity fields: $n_\beta = 57$, $n_z = 430$, $\varepsilon = 1.03 \times 10^{-5}$ m^2s^{-3}, $R_\beta = 5.2$
CS velocity fields: $n_\beta = 67$, $n_z = 550$, $\varepsilon = 0.46 \times 10^{-5}$ m^2s^{-3}, $R_\beta = 6.1$

Implications

A single zonal mode contains most of the kinetic energy

Total kinetic energy is $\quad \mathrm{E}_{tot} = \dfrac{5}{4} C_Z \beta^2 n_{fr}^{-4} = \dfrac{5}{4} C_Z (\Omega R)^2 \hat{n}_{fr}^{-4}$

Total kinetic energy ~~does not depend~~ on the forcing!

n_{fr} is not known *apriori* but can be easily determined 🎦 number of jets

Jupiter vs. Saturn: $\beta_{⚄} \simeq \beta_{♄}; \quad n_{fr⚄} \simeq 2 n_{fr♄} \Rightarrow E_{tot♄}/E_{tot⚄} \simeq (n_{fr⚄}/n_{fr♄})^4 = 2^4 = O(10)$

Equatorial jets: $U_{♄}/U_{⚄} \simeq (E_{tot♄}/E_{tot⚄})^{1/2} = 2^2$

Indeed, $400 \text{ m/s } ♄$ vs. $100 \text{ m/s } ⚄$

Meridional diffusivity K_Φ is scale-independent and given by $\quad K_\phi \propto \hat{U}^{3/5} \beta^{-4/5}$

For Jupiter's troposphere, $\beta \sim 2.5 \times 10^{-12} \text{ m}^{-1}\text{s}^{-1}$ and earlier estimate $\varepsilon \sim 10^{-5} \text{ m}^2\text{s}^{-3}$ yield $K_\phi \sim 2 \times 10^{10} \text{ cm}^2\text{s}^{-1}$

Observations of stratospheric diffusion and dispersion of debris after cometary impacts give similar estimates of K_Φ

Summary of diagnostic results

(1) the flow field is strongly anisotropic in both spectral and physical space
(2) the spectra conform to the spectral laws of ZMT
(3) $R_\beta > 5$, is significantly larger than the threshold value of ~ 2.5
(4) the energy flux to zonal flows is ε, about 10 times smaller than thought
(5) the diffusivity K_Φ is not only scale-independent but also in good agreement with multiple observations

Conclusions

ZMT regime imposes a hard constraint on the large-scale energetics
Provides convenient tools to characterize Jupiter-like planets
To simulate this regime, horizontal resolution of $n=O(10^3)$ is required!
Large-scale circulation is turbulent; mean velocity profile is only the first-order moment
Higher-order moments need to be considered $\rightarrow$ spectra and energy and heat transfers