Rubble Pile Asteroids: Stability of Equilibrium Shapes

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The shapes of solar system bodies are limited by their composition, shape, spin, gravitational and tidal forces. Most bodies in the solar system are solid; composed of rocks, dirt, ices, or metals. There is a classical literature on equilibrium shapes for fluid bodies, with the primary applications to stars. The definitive reference, "Ellipsoidal Figures of Equilibrium", is by Chandrasekha, 1969 [1]. He presents the important contributions of Newton, Maclaurin, Jacobi, Meyer, Louiville, Dirichlet, Dedekind, Riemann, Poincare, Cartan, Roche, Darwin and Jeans in a thorough and unified way. However those theories are of limited use for solid bodies.

Holsapple [2], derives limits on equilibrium configurations for a solid ellipsoidal body, modeled as an elastic-plastic material with a Mohr-Coloumb yield model with zero tensile strength. Thus, the material is characterized by its angle of friction. That model is appropriate for "rubble-pile" structures that consist of an assemblage of a continuum of particles held together at moderate densities by mutual gravitational forces. It is now thought that characterization includes most asteroids. Those bodies are in equilibrium with a given material, shape and rotation rate. Increased rotation or increased elongation can destroy that equilibrium. The task is then to derive criteria for those limit states, and to determine the fate of the body when that limit is exceeded.

Here a new approach and substantial additions to that previous theory are presented, using a new energy formulation for an elastic-plastic body. First, new closed-form algebraic expressions are derived for the limit equilibrium states, with explicit algebraic dependences on the ellipsoidal shape and the friction angle. The material model is for an elastic plastic material. The elastic component is arbitrary, including any nonlinearity. The plastic component is governed by the Mohr-Coloumb yield function with the cohesion equal to zero. Various possibilities for the plastic flow rule are considered. Since the limiting case of zero friction angle gives an incompressible fluid model, the classical fluid equilibrium states are recovered as a special case. Thus, the results are a major generalization of the fluid results to an elastic-plastic body.

For the fluid results, it is known that many of the equilibrium states are not also stable, so are not possible physical states. For example, the Maclaurin Spheroids with the ratio α of the shortest semi-axis c to the longest a less than 0.3033 are not stable [1]. Also, the cases with α between 0.3033 and 0.5827 are neutrally stable, but that neutrality is lost if there is any amount of viscosity [1]. Thus, over half of the equilibrium states for a Maclaurin Spheroid are not also stable. For the Jacobi ellipsoids, the equilibrium states are not stable if α is less than 0.3451 [1].

Since the solid equilibrium states include those fluid states and many others, the question of the stability of the more general solid materials arises. When are those shapes stable, and when are they not? What is the defining criteria for the loss of stability? The fluid models used have zero friction angle, are incompressible, and have zero shear rigidity. Which of those characteristics matters regarding stability? What aspects of the material model determine stability limits?

That question is also addressed here: a stability analysis is presented. Because of the differences between the allowable motions of an elastic-plastic solid compared to a fluid, the classical perturbation methods of the fluid analyses cannot be used, but an variational energy approach yields the desired results. Stability depends on features of the stress-strain behavior. The classical fluid stability results are again recovered as a special case. Thus, the results are an important generalization of the fluid results to an elastic-plastic body.

The stability analysis is not just of academic interest. It is found that the permissible range of rotation rates may be substantially restricted by the stability considerations, see figures 1 and 2 below. The governing mechanism is the strain softening that occurs in soils and other particulate materials during plastic flow.

For those stability limits, the associated spin rates are distinctly less that those possible considering equilibrium only, and the periods are greater. The "rotation rate barrier" for rubble pile asteroids, often stated as about 2.1 hr/rev (Harris, 1996) is, according to this new analysis, about 3.2 hr/rev for spherical bodies, and even greater (slower rotation) for elongated bodies. However, the new stability limits are still above the majority of data points for the asteroids. The new refined analysis still does not rule out most asteroids as candidates for the rubble pile structure.

References: [1] Chandrasekhar , S. 1969. *Ellipsoidal Figures of Equilibrium.* Dover Publications. [2] Holsapple, K., "Equilibrium Configurations of Solid Ellipsoidal Cohesionless Bodies". To Appear in *Icarus*, 2001.

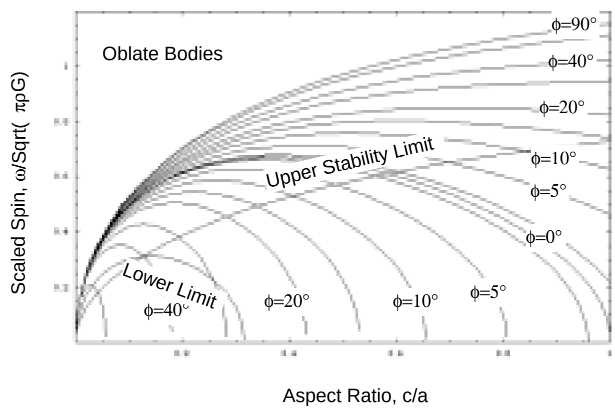

Figure 1. Equilibrium Curves and Stability Limits for Oblate Bodies with Zero Incremental Shear Stiffness.

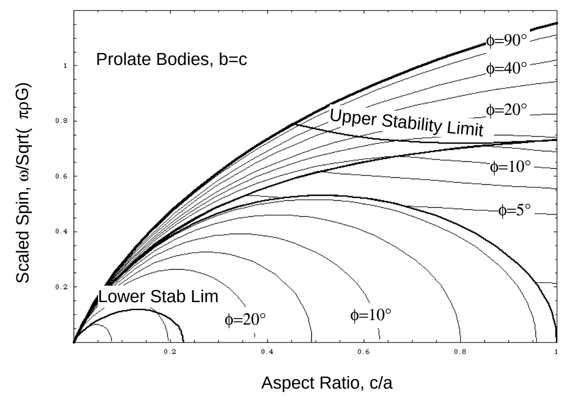

Figure 2. Equilibrium Curves and Stability Limits for Prolate Bodies with Zero Incremental Shear Stiffness.