UNITED STATES
DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

INTERAGENCY REPORT: ASTROGEOLOGY 30

A TEST OF SELF-STATIONARITY

By

Robert D. Regan

July 1971

Prepared under NASA Contract H-82013A

Prepared by the Geological Survey for the National Aeronautics and Space Administration

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The work presented in this report was accomplished as part of a study investigating the analysis of traverse geophysics data. The study was directed toward developing methods of extracting the maximum amount of information from geophysical measurements obtained during proposed automated lunar vehicle traverses.

However, it became apparent that a computer program for testing stationarity would be of use in many fields of investigation. Thus it was decided to publish this report separately.

CONTENTS

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ABSTRACT

A method for testing the stationarity of a single time series has been devised. The method utilizes two pre-existing tests of stationarity. A computer program that permits routine testing of any time series has been developed, and the test has been successfully applied to several known series. In certain cases the self-stationarity determined can be equated to stationarity.

INTRODUCTION

In time-series analysis of any body of observational data, the fundamental assumption is that of stationarity. Very little work has been done with nonstationary time series, and the effects of nonstationarity upon time-series analysis are little known. In the practical application of time-series techniques the assumption of stationarity is routinely invoked without adequate tests for the validity of such an assumption. Yet the utilization of some of the more essential techniques of time-series analysis, such as the Wiener-Khintchine theorem in the indirect method of power spectra computation, is valid only for stationary series.

This situation stems from the fact that, to date, no rapid, accurate, routine method has been available to test the stationarity of a time series. Such a test is clearly required so that stationarity or nonstationarity of a series can be routinely determined and the validity of the application of time-series analysis be established. The method and computer program outlined in this report is designed to fulfill this requirement.

Basically, the method outlined and programmed is a combination of tests proposed by Bendat and Piersol (1966) and Bryan (1967). Their tests and assumptions have been combined with some additional logic to produce a computer test of the stationarity of a single time series. The method has proved successful in several test cases.

BRIEF THEORETICAL SUMMARY

Before the test method and rationale are explained it is essential to discuss some of the concepts and terms pertinent to stationarity.

Time Series-Stochastic Process

A stochastic process can be considered as being composed of a family of random variables, and viewed as a function of two variables t and w. The sample space associated with this stochastic process is doubly infinite and the set of time functions which can be defined on this space is called an ensemble.

A strict definition of a time series is that it is one realization (outcome) of a stochastic (random) process. A more popular definition is that it is a set of observations of a parameter arranged sequentially.

The label "time-series" is perhaps a misnomer, and is directly applicable only when the observations are made chronologically. However, for the independent variable, time, there may be substituted any other parameter -e.g., distance.

The basic idea of the statistical theory of time series analysis is to regard the time series as a set of observations made on a family of random variables, i.e., for each t in T, x (t) is an observed value of a random variable. The set of observations $\{ x(t);\ t \in T \}$ is called a time series.

Stochastic Process Functions

The basic process functions are

a) <u>mean of the process</u>

$$\mu_x(t) \stackrel{d}{=} E\{x(t)\} = L\int_{-\infty}^{\infty} x\, f(x,t)\, dx$$

<u>where</u>:

$L\int$ = Lebesgue Integral

$f(x,t)$ = Probability density function

E = Expectation operator

In the general case the means are different at different times and must be calculated for each t.

b) <u>autocorrelation function of the process</u>

$$R_x(t_1,t_2) \stackrel{d}{=} E\{x(t_1),\ x(t_2)^*\}$$

<u>where</u>:

$$x(t_2)^* = \text{complex conjugate of } x(t_2)$$

c) <u>autocovariance function of the process</u>

$$C_x(t_1,t_2) \stackrel{d}{=} E\{[x(t_1) - \mu_x(t_1)],[x(t_2) - \mu_x(t_2)]^*\}$$

In the general case these functions must be calculated for each t_1, t_2 combination.

Stationarity

In general the properties of a stochastic process will be time dependent. A simplifying assumption which is often made is that the series has reached some form of steady state in the sense that the statistical properties of the series are independent of absolute time. Stationarity can be pictured as the absence of any time-varying change in the ensemble of member functions as a whole.

A sufficient degree of stationarity for most time-series analysis is wide-sense stationarity. A process has wide-sense stationarity if its expected value is a constant and its autocorrelation depends only on the time difference, $t_1 - t_2 = \tau$, and not on the absolute value of the respective times.

i.e.

$$E\{x(t)\} = \mu_x(t) = \text{constant}$$

and

$$E\{x(t+\tau),\ x(t)^*\} = R_x(\tau)$$

Wide-sense stationarity is also termed stationarity of the second order, i.e., the series is stationary through its second order statistical moments. Stationarity of order n implies that all statistical moments less than n depend only on the time differences.

Ergodicity

Ergodicity relates to the problem of determining the statistics of the stochastic process from the statistics of one time series. A stochastic process for which the statistics are thus determinable is said to be ergodic, and the single time series is representative of the ensemble. The ensemble moments can then be equated to the time moments.

For example the time average equals the ensemble average of an ergodic process

$$\mu_x(t) = \frac{1}{T} \int_{-T}^{T} x(t)\, dt = E\{x(t)\}$$

If a time series is ergodic we need only to measure the time averages which are available rather than the postulated ensemble averages. Since ergodicity is a subclass of stationarity, a time series must be shown to be stationary before the question of ergodicity can be considered.

STATIONARITY OF A SINGLE TIME SERIES

In the transformation from the theoretical to the empirical, strict adherence to theory must be tempered with reasonable practical considerations. In most practical applications the single observed time series is the only information available on the parent process. Hence ergodicity must be assumed and time-domain statistics utilized. Also all analyses are performed on the sample series and thus the stationarity of this series is of prime interest. Bendat and Piersol (1966) have termed the concept of stationarity of this one time series as self-stationarity. Thus we speak of the series being stationary rather than the process being stationary.

However, the concept of self-stationarity is not restrictive. A necessary condition to extend this self-stationarity to stationarity is that the process be ergodic, i.e. that the series is representative of the process. Ergodicity is impossible to prove (except in special instances) when the entire process is not known.

Bendat and Piersol (1966, p. 12) state that "in actual practice, random data representing stationary physical phenomena are generally ergodic".

If the assumption of ergodicity is justified then self stationarity becomes equivalent to stationarity, since the single time series is representative of the ensemble.

TEST FOR SELF-STATIONARITY

The test for self-stationarity is based on the methods proposed by Bendat and Piersol (1966) and Bryan (1967). The basis of these methods is that in a stationary series certain statistical properties of the time series are considered invariant with time. The tests are for second order self-stationarity.

Bendat and Piersol Test

Bendat and Piersol (1966) suggest that the series be divided into n equal time intervals (either contiguous or non-contiguous) and that the mean and variance of these intervals be calculated. The two series thus formed, composed of means and variances, are then tested for underlying trends or variations by the Run test and the Trend test, Bendat and Piersol (1966, p. 156). If no trends or variations are suggested by the application of these tests, the original series is assumed to be stationary.

Bryan Test

Basically, Bryan's test (1967) is quite similar in that he tests the invariance of the means and variances obtained from independent, equal time segments. Rather, than using the sample mean and sample variance he constructs two combinations of the data to serve as estimates of the population mean and population variance. Those two estimates are independent, and independently distributed.

Using these two variates, m, a linear function of the data and an unbiased estimate of the population mean, and Q, a quadratic function of the data, he develops a test for the hypothesis that the time series is stationary, and two test variables, L_1 for

the Neyman-Pearson L test, and F for the F-distribution test.

Test modifications

The tables for the Run and Trend tests (Bendat and Piersol 1966, p. 170) were extrapolated to include the range n = 1 to n = 200. With values of n less than 12 the results proved unreliable. Hence a low limit cut off at n = 12 is utilized. The Acceptance region was extended to include the lower bound.

The Bryan test was extended to include the 97.5 percent confidence interval.

Self-Stationarity Test

In the proposed test both methods are combined and used with some restrictions and modifications.

First the sampling interval for independent samples is determined. If independent samples cannot be determined, i.e., the autocorrelation function does not damp to zero, the test is aborted and the series can be considered non-stationary if a reasonable number of points has been used.

Once the sampling interval has been determined, the series is segmented into independent samples of length N. Initially the series is tested with N = 5, then N is increased to 10, and the final test, if there are enough data points, is for N = 15.

The minimum test is for KK = N samples of length N. If there are not enough data points for this number of samples, the requirement for independent samples is relaxed slightly (i.e. the sample separation interval is steadily decreased to a limiting value equal to the number of lags necessary for the autocorrelation function to damp to .100). If at this sample interval there are not N samples, the test is aborted. If this happens at N = 5, it may be an indication of nonstationarity.

If we have KK independent samples of length N the series is tested for stationarity at three confidence intervals (95%, 97.5%, 99%) in the following manner

a) if KK is greater than or equal to 12, the Bendat

and Piersol test statistics and the Bryan test statistics are both utilized.

b) if KK is less than 12 only the Bryan test statistics are utilized.

The series is considered stationary if both the Run and Trend tests show no trends or variations for the mean and variance series and if the two test statistics in the Bryan test indicate stationarity.

In the case where only one method is utilized (i.e. KK < 12) stationarity is tested on the merits of the Bryan test statistics alone.

It should be noted that the 95 percent confidence interval is the most restrictive (i.e. the smallest acceptance region) and the other confidence intervals progressively less restrictive.

Also results at the largest N used are preferable since more data points are used in each sample to determine the test statistic and the assumption of normality in the Bryan test statistics is more closely approximated.

COMPUTER PROGRAM STEST

The stationarity test has been programmed on an IBM 360/30 as computer program STEST. A copy of the computer program is contained in the Appendix.

The program requires approximately 53,000 bytes of storage and to test a series of 400 data points for all values of N requires approximately 2 minutes of computer time.

Program Input

The input to the program is simply the number of data points in the series and the values of the data points. An example is contained in the Appendix.

Program Output

The program output has several forms. Initially the lag required for the autocorrelation function to damp to zero is indicated along with the value of the autocorrelation function at

that lag.

If the autocorrelation function does not damp to zero, a statement is printed stating that this occurred and that it may be indicative of non-stationarity.

The stationarity test is then conducted for values of N = 5, 10, 15. If at any value there are not at least N samples available for testing, the sampling interval is progressively decreased to a limiting value to obtain N values. If N values are obtained in this manner a statement indicating that correlated samples are being used along with the autocorrelation lag and value at this sample interval is printed out. If N samples cannot be obtained the test is aborted.

If there are KK samples ($KK \geq N$) of length N, the test results are printed out for the 95%, 97.5%, and 99% confidence intervals.

A sample output is contained in the Appendix.

TEST CASES

Seven series that have been tested are series A-F as given in Box and Jenkins (1970) and a second order auto regressive process as given in Jenkins (1968). The results are shown in Table 1. In all cases except series A the test accurately indicated the stationarity or non-stationarity of the series. The discrepancy in series A may be attributable to the fact that correlated samples were used.

It is interesting to note that the series generated from the second order AR process is stationary and the process itself is stationary. Thus, in this case self-stationarity is indicative of stationarity.

Table 1

Series A - Non stationary

STEST Results

- Correlated samples used -

A) N = 5

95% confidence interval	:	non stationary
97.5% confidence interval	:	stationary
99% confidence interval	:	stationary

B) N = 10

Not enough data points for independent or correlated samples.

Series B - Non stationary

STEST Results

A) N = 5

There are not enough data points for independent or correlated samples. Since this occurred for a sample of length 5 and the length of the input series is 369, this may be indicative of non-stationarity.

Series C - Non stationary

STEST Results

A) N = 5

95% confidence interval	:	non stationary
97.5% confidence interval	:	non stationary
99% confidence interval	:	non stationary

B) N = 10

Not enough data points for independent or correlated samples.

Series D - Non stationary

STEST Results

A) N = 5

For N = 5 there are not enough data points for independent or correlated samples. Since this occurred for a sample of length 5 and the length of the input series is 310 this may be indicative of non-stationarity.

Series E - Stationary

STEST Results

A) N = 5

95% confidence interval	:	non stationary
97.5% confidence interval	:	stationary
99% confidence interval	:	stationary

B) N = 10

Not enough data points for independent or correlated samples.

Series F - Stationary

STEST Results

A) N = 5

95% confidence interval	:	stationary
97.5% confidence interval	:	stationary
99% confidence interval	:	stationary

B) N = 10

Not enough data points for independent or correlated samples.

Second Order A. R. Process - Stationary

STEST Results

A) N = 5

95% confidence interval	:	stationary
97.5% confidence interval	:	stationary
99% confidence interval	:	stationary

B) N = 10

95% confidence interval	:	stationary
97.5% confidence interval	:	stationary
99% confidence interval	:	stationary

C) N = 15

95% confidence interval	:	stationary
97.5% confidence interval	:	stationary
99% confidence interval	:	stationary

Appendix

Computer Program STEST

```
              C  PROGRAM STEST
              C
              C THIS PROGRAM TESTS A TIME SERIES FOR WIDE SENSE STATIONARITY
              C
              C
              C  THE  PROGRAM  USES TWO TESTS
              C
              C              1) J.G. BRYAN--STATISTICAL TEST OF THE HYPOTHESIS THAT A
              C                               TIME SERIES IS STATIONARY---GEOPHYSICS-V.32-NO.3
              C                               TIME SERIES IS STATIONARY---GEOPHYSICS-
              C                               VOL.32---NO.3--P. 499
              C
              C              2) BENDATT AND PIERSOL---MEASUREMENT AND ANALYSIS OF
              C                               RANDOM DATA---P. 219
              C
              C
              C      THE SERIES IS READ INTO THE PROGRAM. THE ACF IS CALCULATED AND
              C      THE  NUMBER OF LAGS NECESSARY FOR THE ACF TO DAMP TO ZERO IS
              C      DETERMINED. THE SERIES IS THEN SAMPLED AT INTERVALS SEPARATED
              C      BY THIS LAG. THIS ASSURES INDEPENDENCE. THE SAMPLED SERIES IS
              C      THEN TESTED FOR STATIONARITY. IF THE NUMBER OF SAMPLES IS G.E.
              C      12 BOTH TESTS ARE USED, IF LT 12 ONLY BRYAN'S TEST IS USED.
              C
              C
              C                          INPUT DATA
              C
              C                    NN--LENGTH OF THE SERIES
              C                    X---TIME SERIES
              C
              C
              C
0001                DIMENSION X(1000),R(300),W(300),WX(300),WWW(15,15),RW(300),NTEST(4
                   1),JTEST(4),NNTEST(4)
0002                READ(1,100)NN
0003              100 FORMAT(I4)
0004                READ(1,101)(X(I),I=1,NN)
0005              101 FORMAT(F12.4)
              C
              C NOW CALCULATE THE ACF
              C
0006                CALL XVAR(X,1,NN,XBAR,VARX)
0007                ITAU=0
0008                AN=NN
0009                M=0.3*AN
0010                DO 13 IP=1,M
0011                NIP=NN-IP
0012                ASUM=0.
0013                DO 14 IN=1,NIP
0014               14 ASUM=ASUM+((X(IN)-XBAR)*(X(IN+IP)-XBAR))
0015                R(IP)=(ASUM/NIP)/VARX
0016                IF(ITAU.GT.0) GO TO 12
0017                IF(R(IP).LE.0.) ITAU=IP
0018               12 RW(IP+1)=R(IP)
0019               13 CONTINUE
```

```
      IF(ITAU.GT.0)WRITE(3,701)
  701 FORMAT('1',40X,'***** LAG INFORMATION *****',//)
      IF(ITAU.GT.0)WRITE(3,700)ITAU,ITAU,RW(ITAU+1)
  700 FORMAT(30X,'INDEPENDENT TAU =',I3,10X,'R(',I3,') =',F7.4,/)
      RW(1)=1.
      IF(ITAU.GT.0) GO TO 444
      WRITE(3,300)M
  300 FORMAT('1',5X,' THE ACF DID NOT DAMP TO ZERO IN',I4,'LAGS---THIS
     1MAY BE AN INDICATION OF A SIGNIFICANT TREND IN DATA',/,'IMPLYING
     2NON STATIONARITY--------TEST ABORTED')
      GO TO 999
  444 CONTINUE
      DO 20 N=5,15,5
      WRITE(3,702)N
  702 FORMAT(//,40X,'***** STATIONARITY TEST FOR N =',I3,1X,'*****',/)
      ICNT=0
      ISEG=N+ITAU
      KK=NN/ISEG
      IF(KK.LT.N) CALL PFIX(RW,ITAU,N,ISEG,KK,NN)
      IF(ITAU.LE.0)WRITE(3,304)N
  304 FORMAT(///,10X,'FOR N =',I3,2X,'THERE ARE NOT ENOUGH DATA POINTS F
     1OR INDEPENDENT OR CORRELATED SAMPLES-----TEST ABORTED.')
      IF(ITAU.LE.0.AND.N.EQ.5)WRITE(3,305)NN
  305 FORMAT(//,2X,'SINCE THIS OCCURED FOR A SAMPLE OF LENGTH 5 AND THE
     1LENGTH OF THE INPUT SERIES IS',I5,1X,'THIS MAY BE INDICATIVE OF
     2NON-STATIONARITY')
      IF(ITAU.LE.0)GO TO 999
      NFI=KK*ISEG
      DO 30 J=1,NFI,ISEG
      NNN=J
      MMM=J+(N-1)
      DO 40 IJ=NNN,MMM
      ICNT=ICNT+1
      W(ICNT)=X(IJ)
      WX(ICNT)=X(IJ)
   40 CONTINUE
   30 CONTINUE
      IF(KK.LT.12) GO TO 445
      CALL BPTEST(KK,N,WX,NTEST)
  445 DO 60 J=1,N
      DO 60 I=1,N
      JI=J-1
   60 WWW(I,J)=W((JI*N)+I)
      CALL BTEST(N,WWW,RW,JTEST)
      DO 1000 II=1,3
      IF(KK.LT.12)NTEST(II)=4
      NNTEST(II)=NTEST(II)+JTEST(II)
      GO TO(130,131,132),II
  130 CONI=95.0
      GO TO 133
  131 CONI=97.5
      GO TO 133
  132 CONI=99.0
  133 IF(NNTEST(II).GE.5)WRITE(3,788)CONI
```

```
0069          IF(NNTEST(II).GE.5)GO TO 1000
0070          WRITE(3,789)CONI
0071      788 FORMAT(20X,F5.1,1X,'PERCENT CONFIDENCE INTERVAL.',10X,'THE DATA TE
             1STS STATIONARY',//)
0072      789 FORMAT(20X,F5.1,1X,'PERCENT CONFIDENCE INTERVAL.',10X,'THE DATA TE
             1STS NONSTATIONARY',//)
0073     1000 CONTINUE
0074       20 CONTINUE
0075      999 STOP
0076          END
```

```
      SUBROUTINE BPTEST(KK,N,WX,NTEST)
      DIMENSION WX(1),XME(200),XXAR(200)
      DIMENSION ITEST(4),MTEST(4),NTEST(1)
      ICNT=0
      NEI=KK*N
      DO 40 J=1,NEI,N
      ICNT=ICNT+1
      NNN=J
      MMM=J+(N-1)
      CALL XVAR(WX,NNN,MMM,XMN,VAR)
      XME(ICNT)=XMN
      XXAR(ICNT)=VAR
   40 CONTINUE
      NNN=1
      KJL=ICNT
      MMM=ICNT
      CALL XVAR(XME,NNN,MMM,XMN,VAR)
      CALL RTEST(XME,NNN,MMM,XMN,ITEST)
      DO 32 KK=1,3
   32 MTEST(KK)=ITEST(KK)
      NNN=1
      MMM=KJL
      CALL XVAR(XXAR,NNN,MMM,XMN,VAR)
      CALL RTEST(XXAR,NNN,MMM,XMN,ITEST)
      DO 90 IJK=1,3
      NTEST(IJK)=MTEST(IJK)+ITEST(IJK)
   90 CONTINUE
      RETURN
      END
```

```
0001          SUBROUTINE XVAR(X,NNN,MMM,XMN,VAR)
0002          DIMENSION X(1)
0003          XSUM=0.0
0004          DO 244 K=NNN,MMM
0005          XSUM=XSUM+X(K)
0006      244 CONTINUE
0007          AN=MMM-NNN+1.0
0008          XMN=XSUM/AN
0009          A=0.0
0010          DO 220 I=NNN,MMM
0011      220 A=A+(X(I)-XMN)**2
0012          VAR=A/AN
0013          RETURN
0014          END
```

```
0001          SUBROUTINE RTEST(X,NNN,MMM,XM,ITEST)
0002          DIMENSION ITEST(1)
0003          DIMENSION X(1)
0004          DIMENSION XX(100,6),XXX(100,6)
0005          DATA XX/1.,1.,1.,1.,3.,3.,4.,5.,6.,6.,7.,8.,9.,10.,11.,11.,12.,13.
             *,14.,15.,16.,17.,17.,18.,19.,20.,21.,22.,23.,24.,24.,25.,26.,27.,2
             *8.,29.,30.,31.,32.,33.,33.,34.,35.,36.,37.,38.,39.,40.,41.,42.,43.
             *,44.,45.,46.,46.,47.,48.,49.,50.,51.,52.,52.,53.,54.,56.,56.,57.,5
             *8.,59.,60.,61.,62.,63.,64.,65.,66.,67.,68.,69.,70.,71.,72.,73.,74.
             *,74.,75.,76.,77.,78.,79.,80.,81.,82.,83.,84.,85.,86.,87.,88.,88.,1
             *.,1.,1.,1.,2.,3.,3.,4.,5.,6.,7.,7.,8.,9.,10.,11.,12.,12.,13.,14.,1
             *5.,15.,16.,17.,18.,19.,20.,21.,22.,22.,23.,24.,25.,26.,27.,27.,28.
             *,29.,30.,31.,32.,33.,34.,35.,36.,37.,37.,38.,39.,40.,41.,42.,43.,4
             *4.,45.,46.,46.,47.,48.,49.,50.,51.,52.,53.,54.,55.,56.,57.,58.,58.
             *,59.,60.,61.,62.,63.,64.,65.,66.,67.,68.,69.,70.,71.,72.,72.,73.,7
             *4.,75.,76.,77.,78.,79.,80.,81.,82.,83.,84.,85.,86.,86.,1.,1.,1.,1.
             *,2.,2.,3.,4.,4.,5.,6.,7.,7.,8.,9.,10.,11.,11.,12.,13.,14.,15.,16.,
             *17.,17.,18.,19.,20.,21.,21.,22.,23.,24.,25.,25.,26.,27.,28.,29.,30
             *.,31.,32.,33.,34.,34.,35.,36.,37.,38.,38.,39.,40.,41.,42.,43.,44.,
             *45.,46.,47.,47.,48.,49.,50.,51.,52.,53.,54.,55.,56.,56.,57.,58.,59
             *.,60.,61.,62.,63.,64.,65.,65.,66.,67.,68.,69.,70.,71.,72.,73.,74.,
             *74.,75.,76.,77.,78.,79.,80.,81.,82.,83.,84.,3.,5.,6.,8.,9.,11.,12.
             *,13.,15.,16.,17.,18.,20.,21.,22.,23.,25.,26.,27.,28.,29.,31.,32.,3
             *3.,34.,35.,37.,38.,39.,40.,41.,42.,43.,44.,46.,47.,48.,49.,50.,51.
             *,52.,54.,55.,56.,57.,58.,59.,60.,62.,63.,64.,65.,66.,67.,68.,69.,7
             *0.,72.,73.,74.,75.,76.,77.,78.,79.,80.,81.,82.,84.,85.,86.,87.,88.
             *,89.,90.,91.,92.,93.,94.,96.,97.,98.,99.,100.,101.,102.,103.,104.,
             *105.,107.,108.,109.,110.,111.,112.,113.,114.,115.,116.,117.,3.,5.,
             *6.,8.,9.,10.,12.,13.,14.,15.,16.,18.,19.,20.,21.,22.,23.,25.,26.,2
             *7.,28.,29.,31.,32.,33.,34.,35.,37.,38.,39.,40.,41.,42.,43.,44.,45.
             *,46.,48.,49.,50.,51.,52.,53.,54.,55.,56.,58.,59.,60.,61.,62.,63.,6
             *4.,65.,66.,68.,69.,70.,71.,72.,73.,74.,75.,76.,77.,78.,79.,80.,82.
             *,83.,84.,85.,86.,87.,88.,89.,90.,91.,92.,93.,94.,95.,97.,98.,99.,1
             *00.,101.,102.,103.,104.,105.,106.,107.,108.,109.,110.,111.,113.,11
             *4.,115.,3.,4.,6.,7.,8.,10.,11.,12.,13.,15.,16.,17.,18.,19.,20.,22.
             *,23.,24.,25.,26.,27.,29.,30.,31.,32.,33.,34.,35.,36.,37.,38.,39.,4
             *0.,41.,43.,44.,45.,46.,47.,48.,49.,50.,51.,53.,54.,55.,56.,57.,58.
             *,59.,60.,61.,63.,64.,65.,66.,67.,68.,69.,70.,71.,72.,73.,74.,75.,7
             *6.,77.,79.,80.,81.,82.,83.,84.,85.,86.,87.,88.,89.,90.,91.,92.,93.
             *,95.,96.,97.,98.,99.,100.,101.,102.,103.,104.,105.,106.,107.,108.,
             *109.,110.,111.,113./
0006          DATA XXX/1.,1.,2.,3.,4.,5.,6.,8.,10.,13.,18.,21.,25.,30.,35.,41.,4
             *7.,54.,61.,69.,80.,90.,100.,110.,120.,130.,140.,150.,160.,171.,190
             *.,200.,215.,227.,240.,258.,270.,290.,305.,320.,345.,360.,380.,400.
             *,425.,445.,465.,485.,501.,514.,550.,570.,600.,610.,620.,665.,695.,
             *710.,722.,756.,800.,830.,850.,880.,910.,940.,970.,1000.,1015.,1045
             *.,1090.,1120.,1150.,1190.,1220.,1245.,1285.,1320.,1342.,1382.,1405
             *.,1440.,1495.,1515.,1550.,1600.,1640.,1690.,1710.,1766.,1800.,1830
             *.,1870.,1905.,1960.,2000.,2050.,2100.,2150.,2198.,1.,1.,1.,2.,3.,4
             *.,5.,7.,9.,11.,15.,18.,22.,27.,31.,38.,45.,50.,56.,64.,72.,82.,92.
             *,100.,110.,120.,130.,140.,149.,162.,172.,182.,195.,210.,225.,240.,
             *255.,270.,285.,305.,320.,340.,350.,370.,385.,405.,422.,442.,465.,4
             *95.,515.,535.,560.,580.,600.,625.,650.,670.,700.,731.,742.,770.,80
             *0.,820.,850.,880.,910.,935.,960.,1014.,1030.,1050.,1080.,1100.,114
```

```
                *0.,1180.,1205.,1240.,1285.,1344.,1360.,1390.,1410.,1450.,1500.,152
                *5.,1570.,1600.,1650.,1721.,1750.,1770.,1800.,1840.,1890.,1920.,199
                *0.,2005.,2060.,2145.,1.,1.,1.,1.,2.,3.,4.,6.,7.,9.,12.,16.,20.,24.
                *,29.,34.,39.,45.,52.,59.,65.,73.,80.,90.,100.,110.,120.,130.,140.,
                *152.,165.,180.,192.,205.,220.,232.,245.,261.,275.,290.,310.,325.,3
                *40.,360.,375.,395.,410.,430.,450.,473.,500.,518.,530.,560.,580.,60
                *5.,640.,650.,680.,702.,740.,760.,790.,810.,840.,870.,900.,930.,950
                *.,977.,1010.,1045.,1070.,1100.,1130.,1160.,1200.,1230.,1266.,1299.
                *,1330.,1365.,1400.,1430.,1460.,1500.,1540.,1590.,1625.,1668.,1700.
                *,1710.,1765.,1800.,1850.,1900.,1950.,2000.,2040.,2083.,3.,4.,6.,8.
                *,11.,15.,19.,23.,27.,35.,41.,49.,57.,66.,75.,85.,95.,107.,116.,130
                *.,140.,155.,170.,180.,195.,215.,230.,245.,265.,282.,305.,322.,345.
                *,368.,385.,405.,425.,450.,465.,489.,515.,540.,560.,580.,610.,640.,
                *660.,690.,720.,751.,780.,810.,840.,870.,905.,940.,970.,1000.,1030.
                *,1067.,1100.,1130.,1170.,1200.,1250.,1290.,1310.,1370.,1400.,1437.
                *,1460.,1500.,1560.,1600.,1650.,1680.,1705.,1750.,1800.,1860.,1880.
                *,1920.,1960.,2010.,2060.,2110.,2160.,2200.,2250.,2336.,2360.,2400.
                *,2450.,2500.,2560.,2620.,2680.,2730.,2800.,2866.,3.,5.,7.,8.,11.,1
                *5.,18.,22.,27.,33.,40.,47.,54.,63.,74.,81.,92.,102.,115.,125.,140.
                *,158.,170.,180.,192.,208.,220.,240.,255.,272.,290.,305.,325.,345.,
                *365.,385.,405.,430.,460.,474.,494.,518.,540.,560.,580.,610.,640.,6
                *60.,690.,729.,740.,770.,800.,830.,860.,890.,920.,950.,980.,1038.,1
                *060.,1100.,1140.,1170.,1200.,1230.,1270.,1305.,1350.,1400.,1420.,1
                *465.,1500.,1530.,1580.,1605.,1660.,1700.,1740.,1815.,1840.,1900.,1
                *930.,1960.,2010.,2050.,2105.,2160.,2200.,2283.,2310.,2350.,2400.,2
                *450.,2500.,2550.,2600.,2680.,2710.,2804.,3.,4.,6.,8.,10.,13.,17.,2
                *0.,25.,31.,36.,44.,52.,60.,68.,78.,86.,98.,110.,120.,135.,150.,165
                *.,180.,190.,200.,220.,235.,250.,263.,280.,300.,320.,330.,350.,375.
                *,390.,415.,440.,460.,490.,500.,530.,550.,570.,600.,630.,650.,670.,
                *710.,730.,760.,780.,810.,840.,870.,900.,930.,960.,1013.,1040.,1080
                *.,1105.,1140.,1180.,1205.,1250.,1295.,1310.,1369.,1400.,1440.,1485
                *.,1510.,1550.,1600.,1650.,1680.,1705.,1777.,1800.,1830.,1870.,1920
                *.,1980.,2005.,2055.,2100.,2150.,2238.,2260.,2300.,2350.,2400.,2450
                *.,2500.,2550.,2600.,2680.,2751./
0007             MNP=MMM
0008             IF(MMM.GT.100)MMM=100.
0009             N=MMM-NNN+1
0010             CNT=0.0
0011             L=MMM-1
0012             DO 430 I=NNN,L
0013             DO 431 J=I,L
0014             IF(X(I).GT.X(J+1))CNT=CNT+1.0
0015         431 CONTINUE
0016         430 CONTINUE
0017             RUN=1.0
0018             IF(X(NNN)-XM)403,402,402
0019         402 X(NNN)=4.0
0020             GO TO 404
0021         403 X(NNN)=3.0
0022         404 CONTINUE
0023             NJ=NNN+1
0024             IF(MNP.GT.200)MNP=200.
0025             NN=MNP-NNN+1
0026             DO 433 JJ=NJ,MNP
```

```
0027              X(JJ)=X(JJ)-XM
0028              IF(X(JJ).LT.0.0)GO TO 418
0029          419 IF(X(JJ-1)-3.0)900,421,420
0030          421 RUN=RUN+1.0
0031          420 X(JJ)=4.0
0032              GO TO 433
0033          418 IF(X(JJ-1)-3.0)900,424,423
0034          423 RUN=RUN+1.0
0035          424 X(JJ)=3.0
0036              GO TO 433
0037          433 CONTINUE
0038              MN=NN/2
0039              GO TO 444
0040          900 WRITE(3,503)
0041          503 FORMAT(7H  ERROR)
0042          444 ACNT=CNT
0043              IF(N.GT.100)RETURN
0044              DO 499 IJK=1,3
0045              ITEST(IJK)=0
0046              KJI=7-IJK
0047              TEST1=XX(MN,IJK)
0048              TEST2=XX(MN,KJI)
0049              TESTA=XXX(N,IJK)
0050              TESTB=XXX(N,KJI)
0051              IF(RUN.LE.TEST2.AND.RUN.GE.TEST1)ITEST(IJK)=ITEST(IJK)+1
0052              IF(ACNT.LE.TESTB.AND.ACNT.GE.TESTA)ITEST(IJK)=ITEST(IJK)+1
0053          499 CONTINUE
0054              RETURN
0055              END
```

```
0001          SUBROUTINE BTEST(N,X,WW,JTEST)
0002          DIMENSION Q(15),QQ(15),AM(15),ZZ(225),ZII(225),ZI(15,15)
0003          DIMENSION X(15,15),WW(15),Z(15,15),G(15),W(15),B(15,15),Y(15,15)
0004          DIMENSION AAM(15),JTEST(1)
0005          DIMENSION TEST3(3,3),F3(3,3)
0006          DATA TEST3/.5951,.525,.4838,.8228,.795,.7791,.8887,.868,.8562/
0007          DATA F3/2.87,3.51,4.43,1.99,2.31,2.615,1.739,1.929,2.168/
        C                                                                 00000200
        C                                                                 00000230
        C  NOW CONSTRUCT THE AUTOCORRELATION MATRIX Z                     00000240
        C                                                                 00000250
0008          K=N                                                         00000260
0009          DO 10 I=1,N                                                 00000270
0010       10 Z(I,1)=WW(I)                                                00000280
0011          DO 20 J=2,K                                                 00000290
0012          J1=J-1                                                      00000300
0013          DO 30 I=1,J1                                                00000310
0014          I1=I-1                                                      00000320
0015       30 Z(I,J)=WW(J-I1)                                             00000330
0016          DO 40 I=J,N                                                 00000340
0017       40 Z(I,J)=WW(I-J1)                                             00000350
0018       20 CONTINUE                                                    00000360
        C                                                                 00000370
        C  CHANGE THE Z MATRIX TO MULTIPLEXED FORM ZZ FOR INVERSION       00000380
        C                                                                 00000390
0019          DO 50 J=1,K                                                 00000400
0020          DO 50 I=1,N                                                 00000410
0021          J1=J-1                                                      00000420
0022       50 ZZ((J1*N)+I)=Z(I,J)                                         00000430
        C                                                                 00000440
        C  NOW CALCULATE INVERSE OF AUTOCORRELATION MATRIX                00000540
        C                                                                 00000550
0023          CALL MAINE(N,ZZ,ZII)                                        00000560
        C                                                                 00000570
        C  TRANSFER INVERSE MULTIPLEXED ZII INTO REGULAR MATRIX ZI        00000580
        C                                                                 00000590
        C                                                                 00000600
0024          DO 80 J=1,K                                                 00000610
0025          DO 80 I=1,N                                                 00000620
0026          J1=J-1                                                      00000630
0027       80 ZI(I,J)=ZII((J1*N)+I)                                       00000640
        C                                                                 00000650
0028          DO 90 I=1,N                                                 00000740
0029          GSUM=0.0                                                    00000750
0030          DO 500 J=1,K                                                00000760
0031      500 GSUM=GSUM+ZI(I,J)                                           00000770
0032       90 G(I)=GSUM                                                   00000780
        C                                                                 00000790
0033          ASUM=0.0                                                    00000810
0034          DO 501 I=1,N                                                00000820
0035      501 ASUM=ASUM+G(I)                                              00000830
        C                                                                 00000840
0036          DO 502 I=1,N                                                00000870
0037      502 W(I)=G(I)/ASUM                                              00000880
```

```
                C                                                       00000890
                C                                                       00000930
0038                  DO 503 J=1,K                                      00000940
0039                  DO 503 I=1,N                                      00000950
0040                  B(I,J)=ZI(I,J)-G(I)*W(J)                          00000960
0041            503 CONTINUE                                            00000970
                C                                                       00000980
                C                                                       00001190
0042                  DO 504 J=1,K                                      00001200
0043                  DO 504 I=1,N                                      00001210
0044                  YSUM=0.0                                          00001220
0045                  DO 505 KK=1,N                                     00001230
0046            505 YSUM=YSUM+B(I,KK)*X(KK,J)                           00001240
0047                  Y(I,J)=YSUM                                       00001250
0048            504 CONTINUE                                            00001260
                C                                                       00001270
                C                                                       00001300
0049                  DO 506 J=1,K                                      00001310
0050                  QSUM=0.0                                          00001320
0051                  DO 507 I=1,N                                      00001330
0052            507 QSUM=QSUM+X(I,J)*Y(I,J)                             00001340
0053                  Q(J)=QSUM                                         00001350
0054            506 CONTINUE                                            00001360
                C                                                       00001370
                C                                                       00001380
0055            508 CONTINUE                                            00001480
                C                                                       00001490
                C                                                       00001530
0056                  DO 509 I=1,K                                      00001540
0057                  IF(Q(I).GT.0.)GO TO 900                           00001550
0058                  GO TO 999                                         00001580
0059            900 QQ(I)=ALOG10(Q(I))                                  00001590
0060            509 CONTINUE                                            00001600
                C                                                       00001610
                C                                                       00001620
0061                  AK=K                                              00001630
0062                  AN=N                                              00001640
0063                  QSUM=0.0                                          00001650
0064                  QQSUM=0.0                                         00001660
0065                  DO 510 I=1,K                                      00001670
0066                  QSUM=QSUM+Q(I)                                    00001680
0067                  QQSUM=QQSUM+QQ(I)                                 00001690
0068            510 CONTINUE                                            00001700
0069                  QMN=QSUM/AK                                       00001710
0070                  QQMN=QQSUM/AK                                     00001720
                C                                                       00001730
                C  FIND ANTILOG OF QQMN                                 00001740
                C                                                       00001750
0071                  ANOM=10**QQMN                                     00001760
                C                                                       00001770
0072                  TEST1=ANOM/QMN                                    00001780
                C                                                       00001790
                C                                                       00001930
                C  SECOND PART OF TEST                                  00001940
```

```
                C                                                                   00001970
0073                  DO 511 J=1,K                                                  00001980
0074                  SUMM=0.0                                                      00001990
0075                  DO 512 I=1,N                                                  00002000
0076              512 SUMM=SUMM+W(I)*X(I,J)                                         00002010
0077              511 AAM(J)=SUMM                                                   00002020
                C                                                                   00002030
                C  CALCULATION OF F                                                 00002040
0078                  SSSUM=0.0                                                     00002050
0079                  SMSUM=0.0                                                     00002060
0080                  DO 520 J=1,K                                                  00002070
0081                  SMSUM=SMSUM+AAM(J)                                            00002080
0082                  SSSUM=SSSUM+(AAM(J)*AAM(J))                                   00002090
0083              520 CONTINUE                                                      00002100
                C                                                                   00002110
                C                                                                   00002120
0084                  STAR1=(AK*(SSSUM)-(SMSUM)*(SMSUM))/AK                         00002130
0085                  STARM=ASUM*STAR1                                              00002140
                C                                                                   00002150
0086                  IK1=K-1                                                       00002160
0087                  IK2=K*(N-1)                                                   00002170
0088                  AK1=IK1                                                       00002180
0089                  AK2=IK2                                                       00002190
                C                                                                   00002200
0090                  F1=STARM/AK1                                                  00002210
0091                  F2=QSUM/AK2                                                   00002220
0092                  F=F1/F2                                                       00002230
                C                                                                   00002240
0093                  IX=N/5
0094                  DO 1000 IKI=1,3
0095                  JTEST(IKI)=0.0
0096                  TEST2=TEST3(IKI,IX)
0097                  F2=F3(IKI,IX)
0098                  IF(TEST1.GT.TEST2.AND.F.LT.F2)JTEST(IKI)=JTEST(IKI)+1.0
0099             1000 CONTINUE
0100              999 RETURN                                                        00002340
0101                  END                                                           00002350
```

```
0001          SUBROUTINE PFIX(RW,ITAU,N,ISEG,KK,NN)
0002          DIMENSION RW(1)
0003          WRITE(3,600)ITAU,N
0004      600 FORMAT(15X,'WITH TAU =',I3,3X,'AND N =',I4,3X,'THERE ARE NOT ENOUG
             1H DATA POINTS FOR INDEPENDENT SAMPLES',/)
0005        1 ITAU=ITAU-1
0006          IF(RW(ITAU+1).GT.0.1)ITAU=0
0007          ISEG=N+ITAU
0008          KK=NN/ISEG
0009          IF(ITAU.LE.0)RETURN
0010          IF(KK.LT.N) GO TO 1
0011          WRITE(3,601)ITAU,ITAU,RW(ITAU+1)
0012      601 FORMAT(25X,'CORRELATED SAMPLES USED WITH TAU =',I3,3X,'AND R(',I3,
             1') =',F7.4,/)
0013          RETURN
0014          END
```

```
0001              SUBROUTINE MAINE(N,A,B)
            C     VERSION 1 OF SUBROUTINE MAINE
            C     SYMMETRIC MATRIX INVERSION BY ESCALATOR METHOD
0002              DIMENSION A(400),B(400)
0003              B(1)=1.0/A(1)
0004              IF(N.EQ.1)RETURN
0005              NN=N*N
0006              DO 5 I=2,NN
0007            5 B(I)=0.0
0008              DO 50 M=2,N
0009              K=M-1
0010              MM=M+(M-1)*N
0011              EK=A(MM)
0012              DO 10 I=1,K
0013              DO 10 J=1,K
0014              MI=M+(I-1)*N
0015              IJ=I+(J-1)*N
0016              JM=J+(M-1)*N
0017           10 EK=EK-A(MI)*B(IJ)*A(JM)
0018              B(MM)=1.0/EK
0019              DO 30 I=1,K
0020              IM=I+(M-1)*N
0021              DO 20 J=1,K
0022              IJ=I+(J-1)*N
0023              JM=J+(M-1)*N
0024           20 B(IM)=B(IM)-B(IJ)*A(JM)/EK
0025              MI=M+(I-1)*N
0026           30 B(MI)=B(IM)
0027              DO 40 I=1,K
0028              IM=I+(M-1)*N
0029              DO 40 J=1,K
0030              MJ=M+(J-1)*N
0031              IJ=I+(J-1)*N
0032           40 B(IJ)=B(IJ)+B(IM)*B(MJ)*EK
0033           50 CONTINUE
0034              RETURN
0035              END
```

Sample Input

SAMPLE INPUT

(Series C - Box and Jenkins, 1970, p. 528)

```
0226
26.6
27.0
27.1
27.1
27.1
27.1
26.9
26.8
26.7
26.4
.
.
.
.
20.2
19.7
19.3
19.1
19.0
18.8
```

Sample Output

SAMPLE OUTPUT

```
                    ***** LAG INFORMATION *****

INDEPENDENT TAU = 20          R( 20) = -0.0033

                    ***** STATIONARITY TEST FOR N = 5 *****

95.0 PERCENT CONFIDENCE INTERVAL.                THE DATA TESTS NONSTATIONARY
97.5 PERCENT CONFIDENCE INTERVAL.                THE DATA TESTS NONSTATIONARY
99.0 PERCENT CONFIDENCE INTERVAL.                THE DATA TESTS NONSTATIONARY

                    ***** STATIONARITY TEST FOR N = 10 *****

WITH TAU = 20  AND N = 10  THERE ARE NOT ENOUGH DATA POINTS FOR INDEPENDENT SAMPLES

FOR N = 10  THERE ARE NOT ENOUGH DATA POINTS FOR INDEPENDENT OR CORRELATED SAMPLES----TEST ABORTED.
```

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Jenkins, G. M., Watts, D. G., 1968, Spectral analysis and its applications: San Francisco, Holden-Day.