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INTERAGENCY REPORT: ASTROGEOLOGY 33

SIZE-FREQUENCY DISTRIBUTIONS OF FRAGMENTS OBTAINED FROM AREAL COUNTS OF SECTIONS

by

Diane McLoughlin

August 1971

Prepared under NASA Contract Nos. R-66, W-13, 130

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Size-frequency distributions of fragments obtained from areal counts of sections

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Abstract

Frequency distributions of fragments counted on surfaces cut through cement blocks containing known mass distributions are compared with theory.

As a result of the study it is found that procedures used previously to estimate frequency distributions from counts of fragments seen on spacecraft imagery may yield reasonable estimates.

Introduction

Size-frequency distributions of particulate materials shown on imagery taken by unmanned spacecraft may be obtained by direct counting of visible fragments and grains on the imagery. This procedure has been employed using Surveyor imagery (Shoemaker, et al, 1969, p. 68-96) where the results for Surveyor III, V, and VI yield results that are in substantial agreement with grain size estimates obtained from footpad imprints and engine exhaust analyses (Choate, et al, 1969). In contrast, the results from Surveyor I and VII require that a lower boundary be placed on the grain size in order to obtain satisfying results (Shoemaker, et al, 1969, p. 90). This problem was illustrated for Surveyor I using preliminary data (Meloy and O'Keefe, 1968).

This report explores the validity of using such procedures to obtain estimates of size-frequency distributions by experiment and theory. The experiment compares counts of grains on the faces of sections cut through cement blocks containing

known distributions with those distributions and theory. The theoretical approach differs from previous ones (Shoemaker, et al, 1969; Hapke, 1968) in that the effect of spherical particle shape (Krumbein, 1935) and limits on the distribution are considered.

These results may be helpful in assessing grain size distributions of grains and fragments imbedded in surfaces such as those on Mars where the distributions may not arise chiefly from the bombardment of the surface by meteoroids and other debris, but rather from other geologic processes such as wind erosion.

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Procedures

Four types of distributions were prepared, mixed with water and cement, and poured into containers roughly 150 mm on an edge: (1) uniform marbles, (2) a distribution similar to a crushed product of glass and quartz (Charles, 1957), (3) a distribution similar to the ejecta from the explosive crater pre-Schooner II (Frandsen, 1967), and (4) a distribution similar to those found on the Moon near Surveyor III (Shoemaker, et al, 1967, p. 31). For the distributions, rock particles were sieved using Tyler standard screens and then weighed to give the desired distributions. The weighed distributions were thoroughly mixed with water and cement and then poured in the boxes, partially filling them. After the cement dried, the blocks were cut into four equal volumes using a diamond saw so that six faces were available for counting (see figure 1). Counts on the block faces were made using a measuring

magnifier graduated in 0.1 mm intervals. Maximum horizontal dimensions of each grain were measured (Krumbein, 1935). The experimental conditions are summarized in tables 1 and 2. For the fourth distribution, grain counts revealed that the amount of grains in each $\log_2$ interval for sizes between 5.6 and 44.8 mm was about 2.7 times larger than planned. This was confirmed by dissolving one-fourth of the experiment and weighing the size fractions recovered.

Table 1. Experimental conditions and results for distribution 1.

Conditions:

Dimensions of fill (mm) 130 x 145 x 150

Value of ℓ * (mm) 145

Number of marbles 540

Average diameter of

marbles (mm) 13.5**

Number of faces counted 6

Results:

Diameter measured (mm)	Cumulative Particles Counted	Theoretical Particles Expected
1	303	301
2	296	298
3	296	294
4	294	288
5	286	280
6	280	270
7	266	258
8	246	243
9	229	225
10	208	203
11	175	175
12	140	138
13	66	81.4
14	1	0

* ℓ is the dimension of fill measured perpendicular to the faces the sections.

** Several marbles had diameters slightly larger than 14 mm.

Table 2. Experimental conditions and ingredients for distributions 2, 3, and 4.

Diameter interval (mm)	Distribution 2 Mass in interval (g)	Distribution 2 Cumulative Mass (g)	Distribution 3 Mass in interval (g)	Distribution 3 Cumulative Mass (g)	Distribution 4 Intended Mass in interval (g)	Distribution 4 Intended Cumulative Mass (g)	Distribution 4 Mass added in error (g)	Distribution 4 Actual Cumulative Mass (g)
0.351-0.701	46.8	46.8	570.9	570.9	1997.0	3969.1		4340.1
0.701-1.4	93.7	140.5	571.3	1142.2	1000.0	1972.1		2343.1
1.4 - 2.8	187.5	328.0	573.2	1715.4	501.4	972.1		1343.1
2.8 - 5.6	375.0	703.0	570.8	2286.2	252.1	470.7		841.7
5.6 -11.2	750.0	1453.0	570.4	2856.6	124.9	218.6	212.0	589.6
11.2 -22.4	1500.0	2953.0	572.0	3428.6	62.5	93.7	106.0	252.7
22.4 -44.8	3004.0	5957.0	569.5	3998.1	31.2	31.2	53.0	84.2
Volume(mm^3) assuming density of $2.5 \times 10^{-3} g/mm^3$	2.383×10^6		1.599×10^6		1.588×10^6		1.736×10^6	
Dimensions of fill (mm)	150x150x128		157x157.5x140		160x160x145			
Value of ℓ *	150		157.5		160			
Volume (mm^3)	2.88×10^6		3.47×10^6		3.71×10^6			
Dimensions of faces counted	150x128		157.5x140		160x145			
Number of faces counted	6		6		4			
Total area counted (mm^2)	1.15×10^5 ($0.115 m^2$)		1.32×10^5 ($0.132 m^2$)		0.928×10^5 ($.0928 m^2$)			

* ℓ is the dimension of fill measured perpendicular to the faces of the sections.

Results

Counts for the marbles were found to be in reasonable agreement with previous work (Krumbein, 1935). Expectations were predicted using Krumbein's result, for a section through a volume containing a large number of spherical grains with one radius, which yields the cumulative fraction of grains (f) with one radius (r) as a function of the radii measured in the section (x). His equation is:

$$f = \sqrt{\frac{r^2 - x^2}{r}} \qquad (1)$$

The probability of encountering a grain (P) by passing a plane parallel to the edges of a rectangular volume is equal to twice the radius of the particle (2r) divided by the distance (ℓ) between the faces of the volume measured perpendicular to the plane:

$$P = \frac{2r}{\ell} \qquad (2)$$

If there are N grains in the volume, then the probability of encountering grains (P x N) becomes:

$$PxN = \frac{2r}{\ell} N. \qquad (3)$$

From equations 1 and 3, the expectations for measurements of these grains for each surface counted are:

$$PxNxf = \frac{2r}{\ell} N \sqrt{\frac{r^2 - x^2}{r}} \qquad (4)$$

Substitution of the appropriate numbers in equation 4 yield the expected number of particles shown in table 1. The distance between the block faces normal to the planes of the sections (ℓ) was 145 mm; the average radii of the marbles (r) were

13.5/2; the number of marbles in the volume was 540; and six faces were counted.

Inspection of table 1 and figure 2 shows excellent agreement for the cumulative number obtained for the smaller measured diameters between 1 and 4 mm and for the larger ones between 8 and 13 mm, and reasonable agreement was found for the other sizes.

Expectations for distributions 2, 3, and 4 require that the incremental number of particles (dn) for the distribution be substituted for N in equation (4) which then must be integrated to yield the cumulative frequency distribution of particles as a function of their measured diameters from some small value of r to the largest size in the distribution.

The input for distribution 2, which is similar to a crushed product of quartz or glass, can be described by:

$$\frac{V_g}{V} = \frac{r}{k} \qquad (5)$$

where: V_g is the cumulative volume of grains with radii equal to or larger than r,

V is the total volume of the grains (2.38×10^6 mm^3), and

k is the maximum radius of the grains (22.4 mm).

The volume of grains in an interval of dr is:

$$dV_g = \frac{V}{k} dr \qquad (6)$$

and the frequency of grains is:

$$dn = \frac{dV_g}{\frac{4}{3}\pi r^3} dr \qquad (7)$$

Substitution of dn for N in equation (4) yields:

$$Pxfxdn = \frac{2V}{k\frac{4}{3}\pi\ell} \quad \frac{\sqrt{r^2-x^2}}{r^3}\, dr. \tag{8}$$

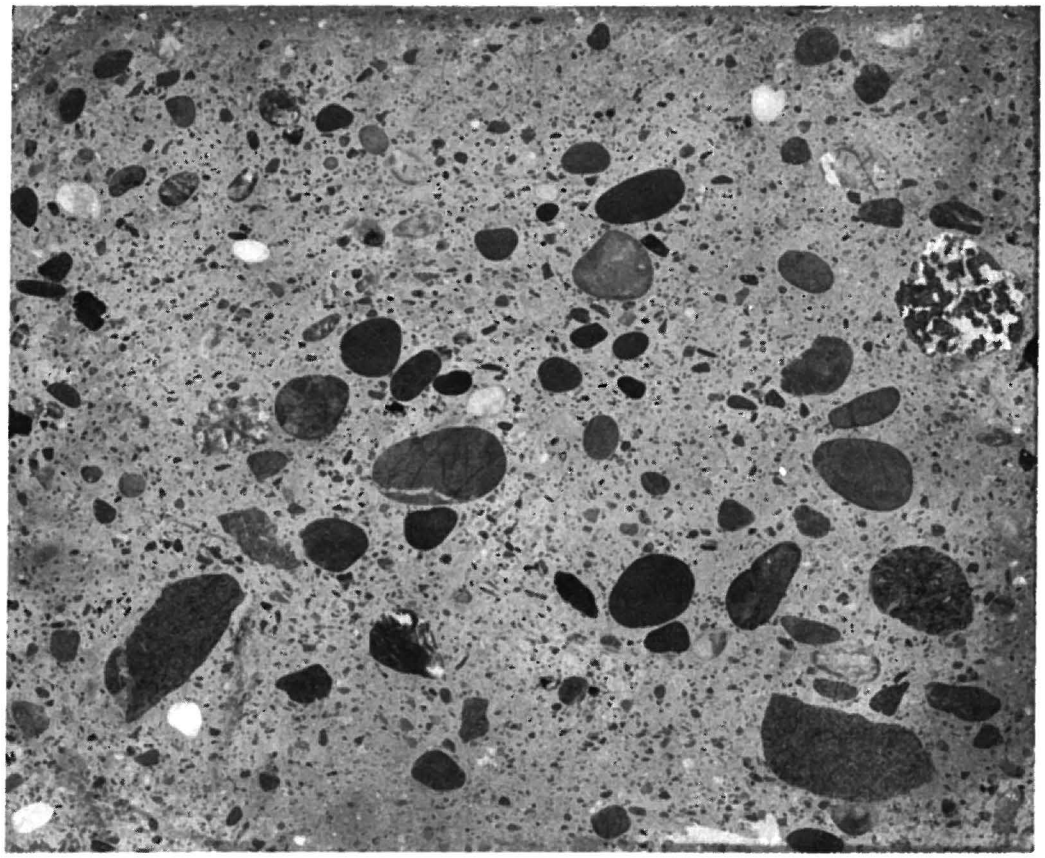

Figure 1. Photograph of section through block containing distribution no. 3.

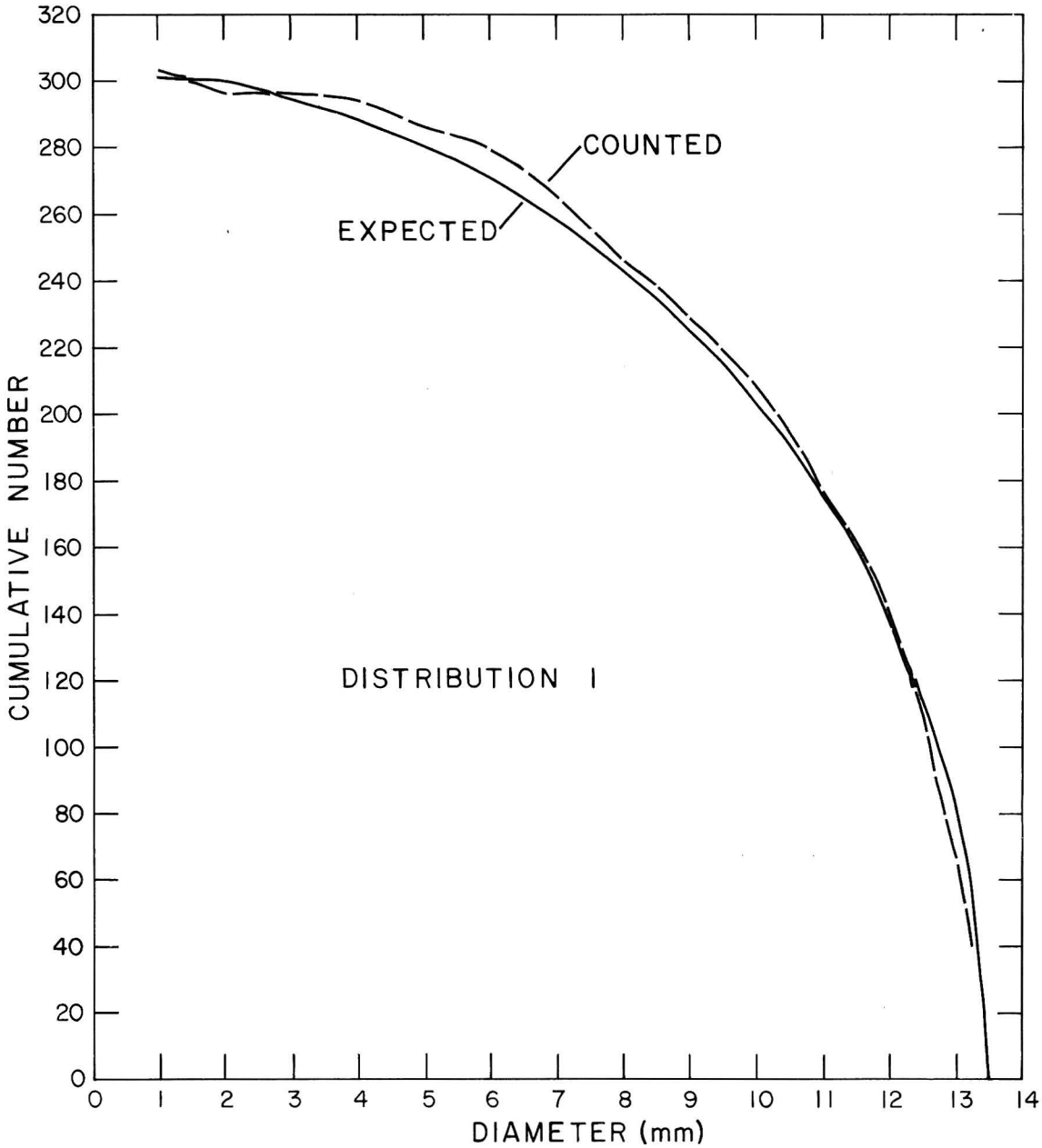

Figure 2. Expected and counted cumulative frequencies of particles for distribution 1, with measured diameters equal to or greater than that plotted.

Integration of equation 8 yields the distribution expected for the counts:

$$= \frac{2V}{k\,\frac{4}{3}\pi\ell}\left[\frac{1}{2x}\sec^{-1}\frac{r}{x} - \frac{\sqrt{r^2-x^2}}{2r^2}\right]_{r_e=x}^{r=k} \qquad (9)$$

Results from equation 9 are listed in table 3 and plotted in figure 3, for the intervals of r = 0.35 to k, r = 0.50 to k, r = 0.70 to k, and so forth, where k = 22.4 mm, V = 2.38×10^6 mm, and ℓ = 150 mm.

Fair agreement is found between equation 9 and the data for diameters larger than 4.2 mm. The discrepancy in number of grains counted and theory at the finer sizes is probably the result of resolution of grains in the cement. The strong bend in the curve for measured diameters between 16.8 mm and 44.8 mm is the result of the finite upper limit of the distribution and equation 9 which approaches zero as r approaches x. Additionally, the slope of the curve approaches -1 for large values of the ratio of $\frac{r}{x}$.

The input for distribution 3, which is similar to pre-Schooner II ejecta, may be approximately described by the equation:

$$\frac{V_g}{V} = 1 + B\,\log_e \frac{r}{k}, \qquad (10)$$

where B = 0.206 mm, k = 22.4 mm, and for the size interval of 0.175 mm to 22.4 mm. Using equation (10) and previous procedures it can be shown that the expected distribution for the counts is:

$$\frac{2BV}{\frac{4}{3}\pi\ell}\left[\frac{\sqrt{(r^2-x^2)^3}}{3x^2r^3}\right]_{r=x}^{r=22.4} \qquad (11)$$

Table 3. Comparison of actual number of particles counted in sections with expected number of spherical particles.

Measured diameter (mm)	Distribution 2 Cumulative particles counted	Distribution 2 Theoretical particles expected	Distribution 3 Cumulative particles counted	Distribution 3 Theoretical particles expected	Distribution 4 Cumulative particles counted	Distribution 4 Theoretical* particles expected
0.7	2088	4,460	6663	16300	6559	15,800
1.0	1600	3,100	3871	7980	3128	5,250
1.4	1258	2,190	2386	4070	1526	1,910
2.1	900	1,430	1073	1810	530	567
2.8	707	1,050	637	1010	231	239
4.2	558	668	370	447	122	70.8
5.6	407	479	235	249	77	29.8
8.4	248	290	105	107	27	8.76
11.2	164	195	60	57.8	11	3.64
16.8	93	101	25	22.6	3	1.01
22.4	44	55.6	9	10.4	1	0.376
33.6	10	13.7	4	2.05	0	0.352
44.8	0	0	0	0	0	0

*For the intended distribution of table 2.

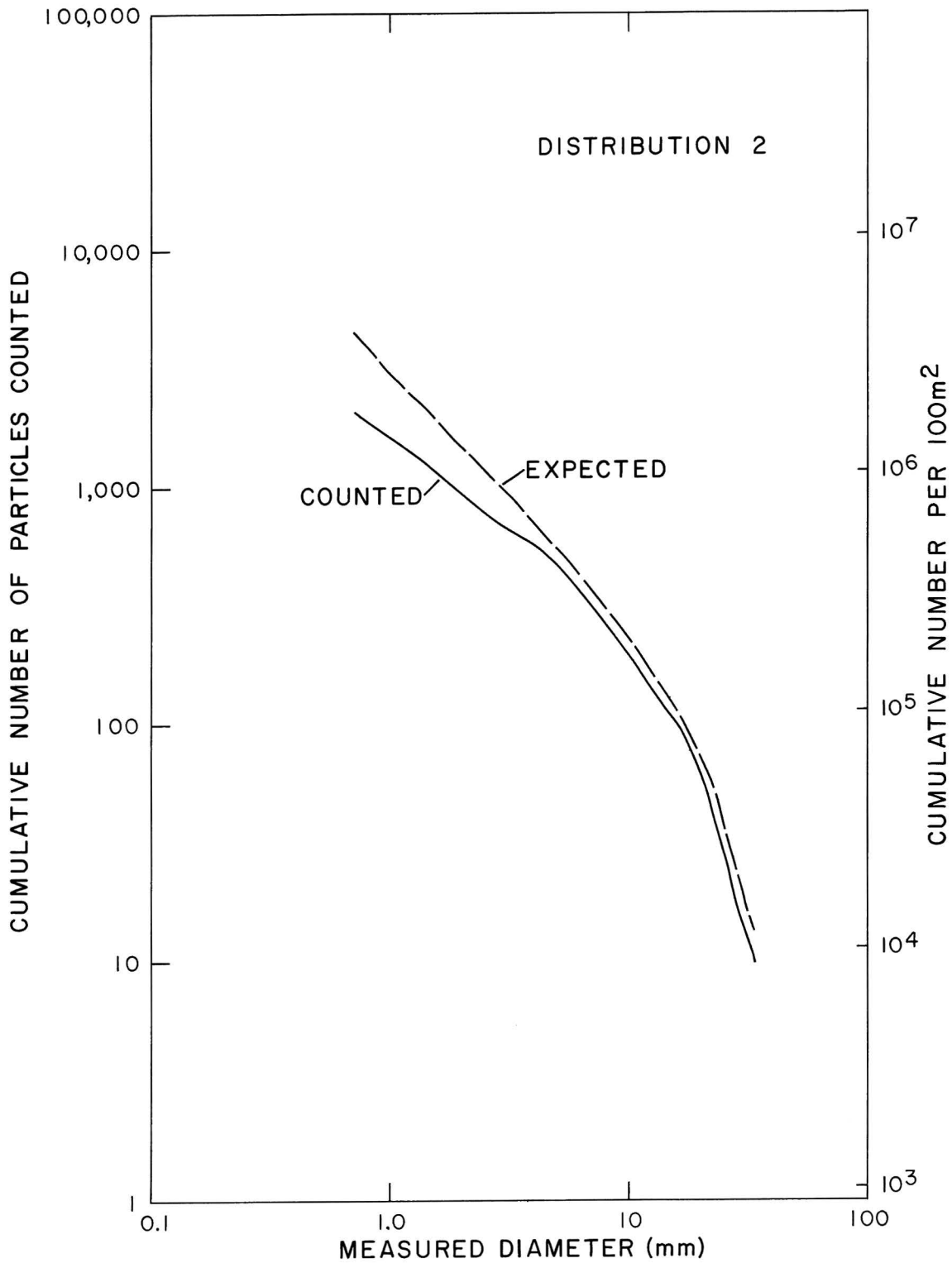

Figure 3. Expected and counted cumulative frequencies of particles for distribution 2 with measured diameters equal to or greater than that plotted.

Solutions to equation 11, using ℓ = 157.5 mm, agree very well with the data from the largest diameter of 44.8 mm down to 4.2 mm (fig. 4, table 3) where a significant departure occurs. This departure may be the result of resolution. The predicted slope of the curve, -2, is rapidly reached because the effects of a finite distribution and $\frac{r}{x}$ ratios of 1 are less important than in distribution 2.

The cumulative volume for the intended distribution 4 can be approximately described by:

$$\frac{V_g}{V} = 1.007874 \frac{C}{r} - 0.007874 \qquad (12)$$

where C is 0.175 mm. Solution for the cumulative number of grains counted on one surface yields:

$$Pnf = - \frac{1.007874\ 2VC}{\frac{4}{3}\pi\ell} \left[\frac{r^2-x^2}{8x^2r^4} \sqrt{r^2-x^2} + \frac{1}{8x^3} \sec^{-1} \frac{r}{x} \right]_{r=x}^{r=22.4} \qquad (13)$$

Numerical values for equation 13 have been calculated, listed in table 3, and plotted in figure 5. Good agreement is found for sizes between 1.4 and 5.6 mm, with a progressive departure at the finer sizes. As noted earlier, the amount of fragments put in the batch for fragments larger than 5.6 mm was too large by a factor of 2.7. When this is introduced into equation 13, good agreement is found for the range of diameters between 5.6 and 22.4 mm.

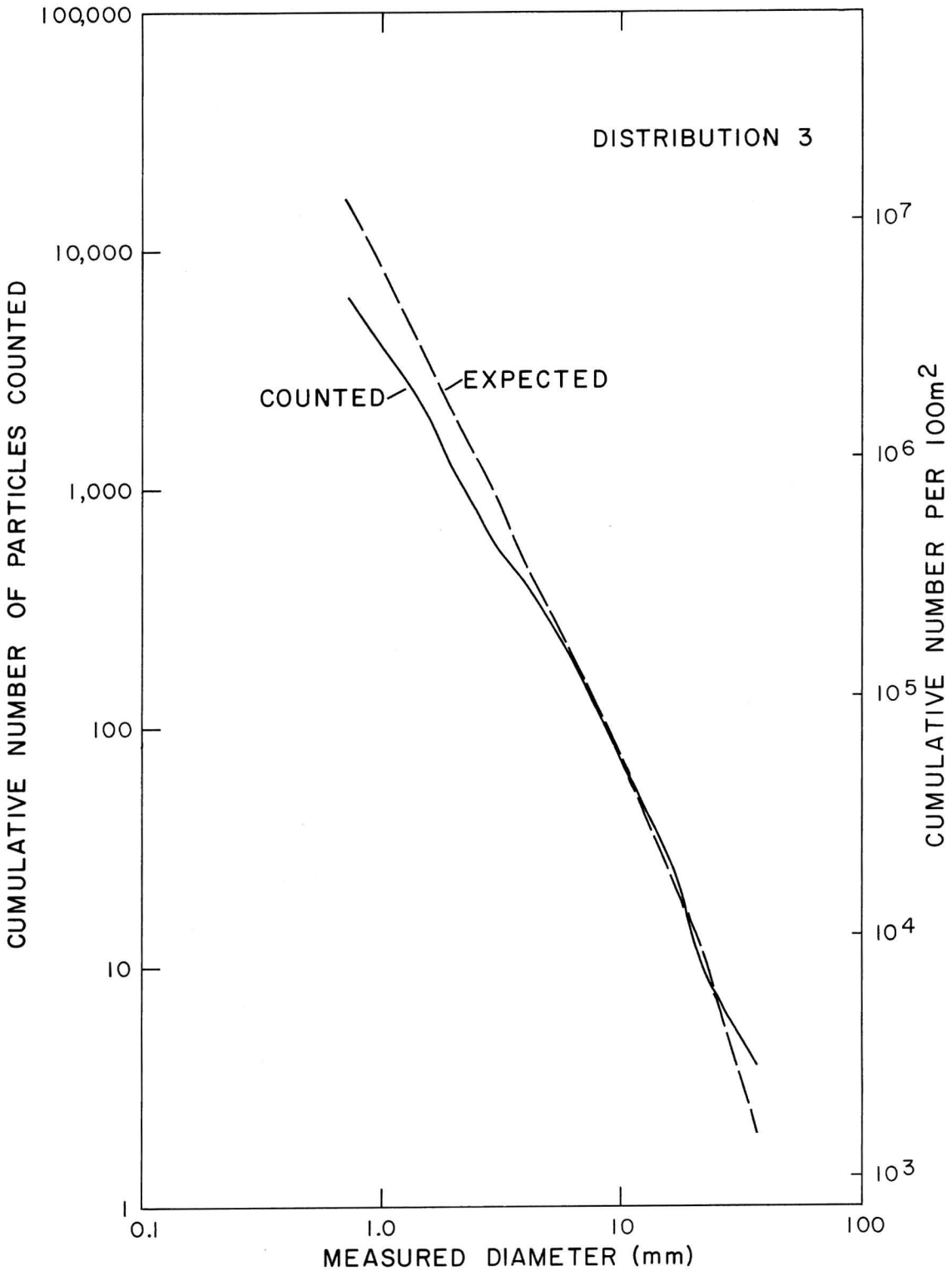

Figure 4. Expected and counted cumulative frequencies of particles for distribution 3 with measured diameters equal to or greater than that plotted.

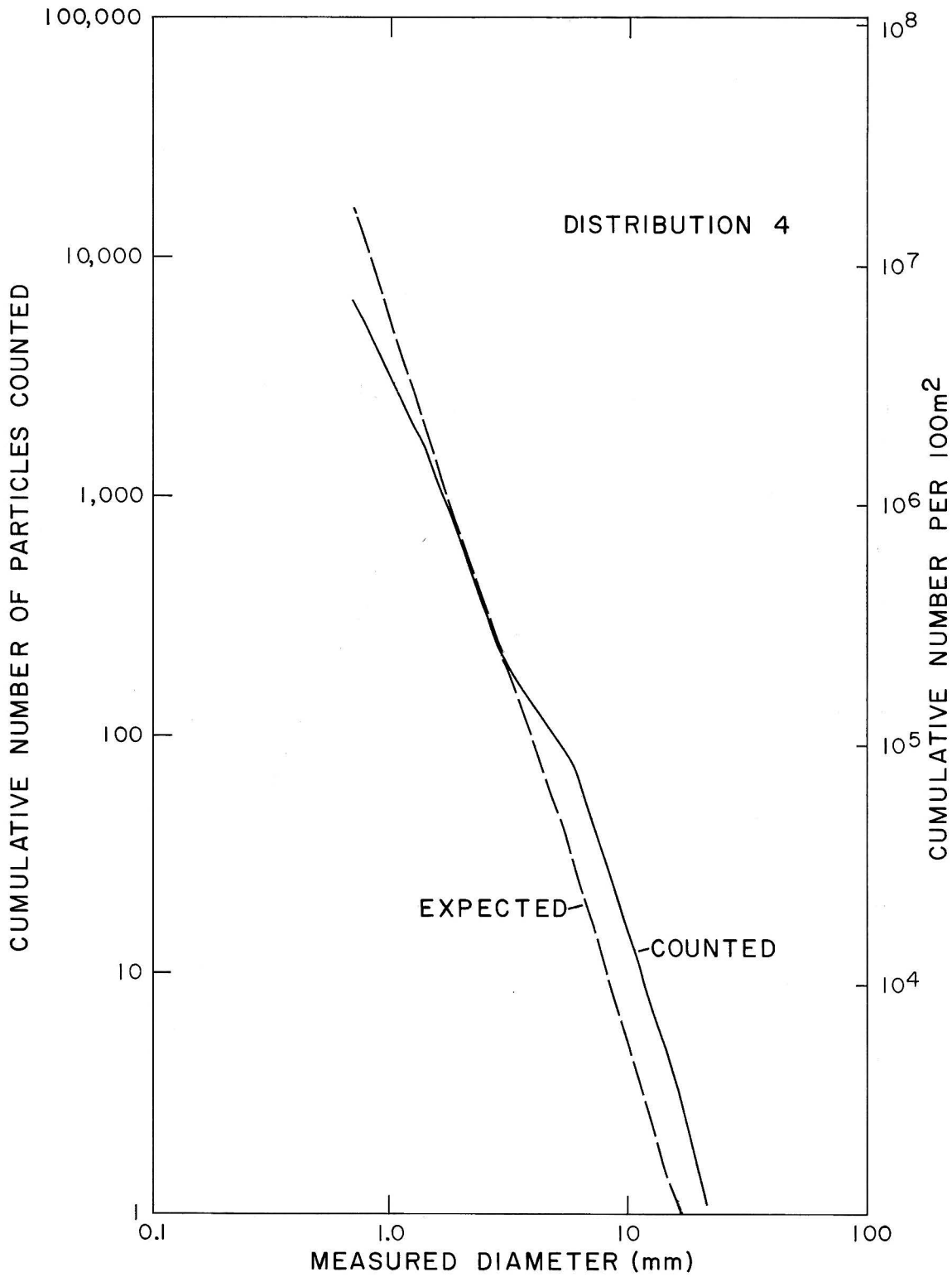

Figure 5. Expected and counted cumulative frequencies of particles for distribution 4 with measured diameters equal to or greater than that plotted. The expected curve is for the intended distribution. The offset of the curve about 5.6 mm is accounted for by the weighing error revealed when grains from one quarter of the block were reweighed.

Volumetric size distributions

As a result of studies of spacecraft imagery, estimates of frequency distributions of particles in the lunar regolith were made starting with the Rosiwal principle (Shoemaker, 1969) or, rather, the Delesse relation (Chayes, 1956, p. 12). In this analysis grains of radius r are considered to be analogous to a mineral species so that the area of the grain is a measure of the volume of grains of that size. Such an analysis neglects the fact that the measured radius (x) is less than and rarely corresponds to the actual radius (r). The volume of spherical grains can be related simply to the volume of the sample (V_b) by

$$V_b = \ell m q \tag{14}$$

where ℓ, m, and q are the dimensions of the fill. Substitution for ℓ from equation 14, and accounting for the number of faces counted, into equations 9, 10, 11, and 13 will yield the relationship between the volume of grains, the volume of fill, and the constants.

Discussion

This study indicates that the procedures suggested for estimating size distributions using counts shown on images (Hapke, 1968; Shoemaker et al, 1969) are reasonable provided there is evidence indicating the surface and its fragments can be treated like a sectioned volume. This is not always the case. For example, Shoemaker et al (1967, p. 31-35) noted imagery may yield frequencies of fragments directly. This is particularly true for lunar craters when blocks in the ejecta are either

lying on the surface or are larger than the thickness of the fine fraction of the ejecta. This may be the case for the 30 meter crater east of Surveyor I where the slope of cumulative frequencies curve for blocks between 20 and 65 cm is -3.17 and from 6 to 20 cm it is near -2.13. Here, the largest block observed was 65 cm across. Comparison of the integral of equation (7) and equation (9) show that a change in slope from about -3 to about -2 should be expected if the larger blocks are counted directly and then the smaller ones are counted in a manner analogous to a count of a sectioned volume. A similar effect could be obtained by integration of a finite distribution for which the blocks are counted directly, or for a finite distribution counted on a surface of a sectioned volume.

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