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Technical Note

An evaluation of interpolation methods for Mars Orbiter Laser Altimeter (MOLA) data

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Abstract. The Mars Orbiter Laser Altimeter (MOLA) instrument on the Mars Global Surveyor (MGS) spacecraft has returned a large amount of data on the topography of Mars. It is possible to generate high resolution digital elevation models (DEMs) from these data by employing data interpolation techniques. Four interpolation algorithms were selected for testing on MOLA data: Delaunay-based linear interpolation, splining, nearest neighbour, and natural neighbour. These methods were applied to the MOLA data of Korolev crater for qualitative analysis. In addition, a DEM of a part of Iceland was used for quantitative testing by simulating MOLA data acquisition, interpolating those data, and then calculating the mean absolute error (MAE) between the interpolated and original DEM. Execution speeds were measured for the four algorithms. The natural neighbour method proved superior both quantitatively and qualitatively to other methods tested, but is relatively slow computationally.

1. Introduction

The Mars Orbiter Laser Altimeter (MOLA) instrument on the Mars Global Surveyor (MGS) spacecraft acquired a large topography dataset from the orbit of Mars. MOLA fired 1064 nm laser pulses at the surface at a rate of about 10 per second (Zuber *et al*. 1992). When the laser strikes the surface, some fraction of the laser energy is backscattered in the direction of the spacecraft, and range is calculated from round-trip travel time. From September 1997 to June 2001, over 600 million data points have been collected. The MGS spacecraft is in a near-polar orbit with an inclination of 93° (Albee *et al*. 1998), so the density of coverage by MOLA increases with latitude.

Currently available digital elevation models (DEMs) of Mars, known as experiment gridded data records (EGDRs), are constructed by taking the median observed topography within a specified degree area. At the time of writing, the highest resolution EGDR is 1/32 degrees/pixel (Smith *et al*. 2001).

However, using data interpolation techniques instead of calculating median elevations can result in higher resolution DEMs. But, it should be noted that most

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common interpolation algorithms were formulated to work with randomly distributed data (Watson 1992), and give visible artefacts when applied to MOLA tracks. The data points within each MOLA track are regularly spaced, and the tracks themselves are either nearly parallel to each other or intersect each other at a latitude-dependent angle. The challenge is to find an algorithm that minimizes visual artefacts and is quantitatively as accurate as possible.

The goal of this project was to test several common interpolation techniques, namely Delaunay-based linear interpolation, splining, nearest neighbour (also known as inverse distance weighting), and natural neighbour. These techniques were applied to MOLA data for qualitative testing. In addition, quantitative testing of simulated MOLA data obtained from a complete DEM was performed.

2. Interpolation methods

2.1. *Linear interpolation*

Linear interpolation performs a Delaunay triangulation of a planar set of data points. After the irregularly gridded data points have been triangulated, the surface values are interpolated to a regular grid. Given the values of some observable (f_i, $i=1, 2, 3$) at the nodes of a Delaunay triangle (x_i, y_i, $i=1, 2, 3$), the interpolated value at any point (x, y) interior to the triangle is given by

$$f(x, y)=\sum_{i=1}^{3} \phi_i(x, y) f_i \tag{1}$$

where $\phi_i\ (x, y)$ is a two-dimensional basis function which varies linearly from a value of one at the node (x_i, y_i) to zero at nodes (x_j, y_j), $(j \neq i)$ (Sambridge *et al.* 1995). Linear interpolation of MOLA data was performed using the Interactive Data Language (IDL) software.

2.2. *Splining*

Splining is a curve fitting method, which fits a least-cost mathematical function through observed data points (Hutchinson and Gessler 1994). Physically, it is similar to fitting a thin elastic sheet through the given points; the values on its surface become the interpolated data. Mathematically, a spline function $z(x, y)$ satisfies the following constraints:

$$z(x_k, y_k)=z_k \qquad \text{for all data } (x_k, y_k, z_k),\ k=1, \ldots, n \tag{2}$$

$$(1-t)\nabla^2(\nabla^2 z)+t\nabla^2 z=0 \quad \text{elsewhere}$$

where t is the tension, $0 \leq t \leq 1$. At $t=0$, a minimum curvature solution is obtained, and $t=1$ yields a harmonic solution (Smith and Wessel 1990). To minimize visible artefacts, a value of $t=0$ was used.

For splining interpolation of MOLA data, the 'surface' program included in the Generic Mapping Tools (GMT) package was used, with a maximum of 10^6 iterations per computational cycle.

2.3. *Nearest neighbour*

In general, for every point of the output grid, a weighted average of n closest data points is performed. The weighting factor used most frequently is $1/r_i^2$, where r_i is the distance from the point being interpolated to the data point i (Eckstein 1989).

There is a nearest neighbour program available in the GMT package; however, it performs a sector-based neighbour search, which does not work well for MOLA data if a large n is used. Other widely available programs do not allow the user to rigorously define an output grid, or were found to have other problems. As a result, a simple nearest neighbour procedure was implemented in the C programming language. A value of $n=50$ and a weighting factor of $1/r_i^2$ was used for all tests.

2.4. *Natural neighbour*

The natural neighbour interpolation method has some features in common with linear and nearest neighbour techniques. In particular, it involves Delaunay triangulation and a weighted average, but it successfully avoids some of the problems of the aforementioned techniques. It differs primarily by the method of neighbour selection and the fact that weights are based on proportionate areas, rather than distances (Sibson 1981).

The natural neighbours of any point are defined as those to which the point is connected by the sides of Delaunay triangles, or equivalently, those in the neighbouring Voronoi cells. In the natural neighbour interpolation, a Voronoi diagram of existing data points is constructed. Subsequently, a new Voronoi cell is created about the interpolation point. If there are n natural neighbours of the interpolation point, the overlap of the new Voronoi cell with the original cells creates n new cells. The normalized area of the new cells is used as a weighting factor for the natural neighbours. A concise summary of this algorithm is available in Sambridge *et al.* (1995).

'Natgrid' is a natural neighbour interpolation package which is part of the 'ngmath' library distributed with NCAR Graphics. It contains both linear and nonlinear implementations of the natural neighbour algorithm. For this study, the linear version was used, as it is roughly an order of magnitude faster, and no visual differences were discerned between the two approaches.

3. Qualitative analysis

Linear, splining, nearest neighbour, and natural neighbour techniques were used to interpolate MOLA data of the Korolev crater region (161–167° E, 72–74° N). The interpolated DEMs generated using the above methods were visually compared for visible artefacts. Figure 1 shows the visualization results.

4. Quantitative analysis

For quantitative analysis, a DEM of a part of Iceland (15°45′–17°15′ W, 64°35′–66°10′ N), produced by the Icelandic Geodetic Survey, was used. The general concept is to sample data from it simulating MOLA data acquisition, interpolate those data, and then numerically compare the interpolated DEM to the original DEM (figure 2).

For step 1 in figure 2, actual MOLA points were taken from the Korolev crater dataset. In step 2, elevations from the Iceland DEM were assigned to these MOLA points. These points were then used as input for the interpolation techniques.

The output grids were created in low, medium and high resolutions, corresponding to the resolutions of 1000, 250 and 82 pixels/degree on Mars. Figure 3 presents the interpolations to the medium (250 pixels/degree) grid. The resulting interpolated DEMs were then compared with the original Iceland DEM on a point-by-point basis, and the mean topography difference was calculated as

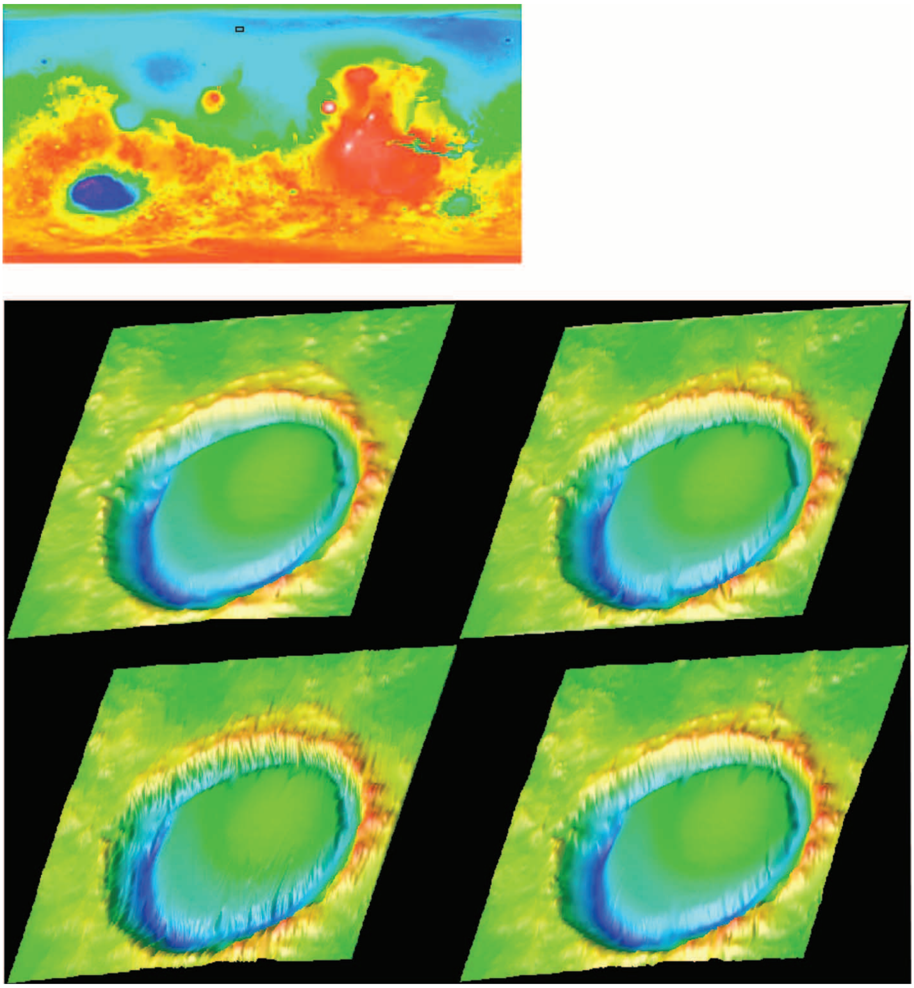

Figure 1. Interpolation techniques applied to the MOLA data of the 80 km Korolev crater (161–167° E, 72–74° N). (*a*) The location of Korolev crater is shown on a global MOLA topography map. (*b*) Clockwise from top left: natural neighbour, linear, nearest neighbour, splining. The resolution is 200 pixels/degree.

follows:

$$\text{Mean topography difference} = \sum_{i=1}^{N} \text{abs}(elev2_i - elev1_i)/N \qquad (3)$$

A summary of the mean topography differences is shown in table 1. In addition, computational times were measured for each algorithm, and are presented in table 2.

5. Discussion

The results indicate that the natural neighbour algorithm consistently outperformed other techniques both quantitatively and qualitatively. Comparing the DEMs of the Korolev crater produced by the four interpolation methods (figure 1), the DEM generated by the natural neighbour algorithm clearly contains the fewest

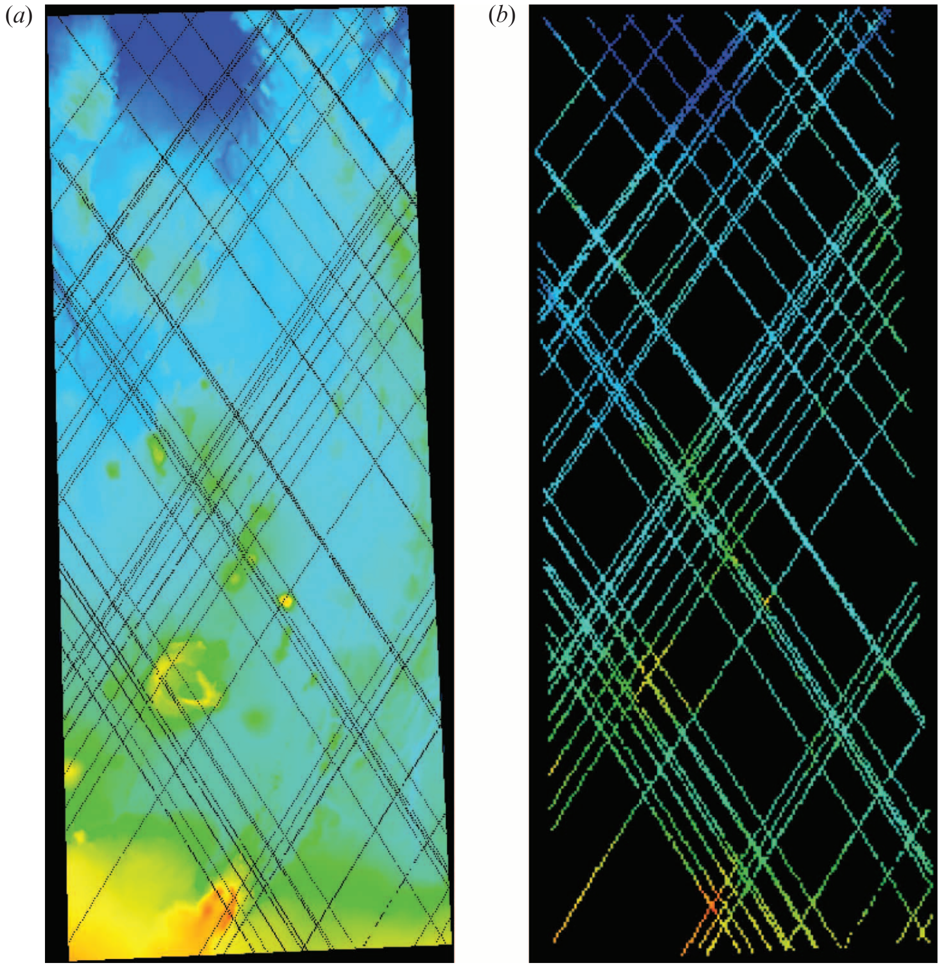

Figure 2. (*a*) Random MOLA tracks are superimposed over the DEM of the Jokulsa a Fjollum region of Iceland (15°45′–17°15′ W, 64°35′–66°10′ N), produced by the Icelandic Geodetic Survey. (*b*) Elevation values are obtained from the Iceland DEM at each MOLA point. An interpolated DEM is then created from these points and compared with the original DEM.

number of visible interpolation artefacts. These artefacts can be defined as systematic deviations from the original surface and usually take the form of ridges or grooves at the locations of MOLA tracks, terrain polygonization, and surface discontinuities at MOLA data points.

For the high resolution (1000 pixels/degree) interpolation of Icelandic data, the natural neighbour algorithm produced a realistic DEM with the fewest interpolation artefacts. It is also very promising that the overall lowest mean topography difference was achieved by the natural neighbour high resolution interpolation (table 1). The splining algorithm clearly did not work well for this resolution, and other methods had pronounced artefacts.

For the medium resolution (250 pixels/degree) interpolation (figure 3), the splining algorithm yielded a DEM that appears mostly artefact-free, but is highly

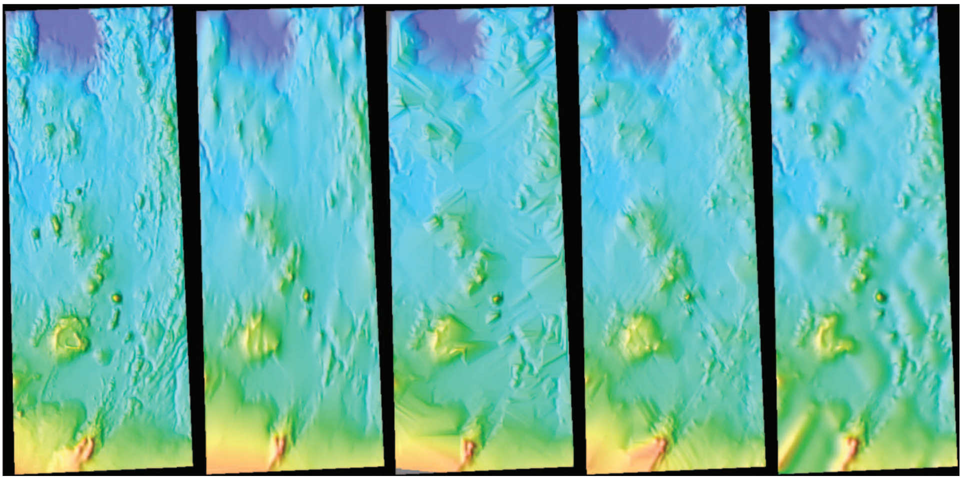

Figure 3. Medium resolution (equivalent to 250 pixels/degree on Mars) interpolations of the simulated MOLA data acquired from the DEM of the Jokulsa a Fjollum region of Iceland. The geographical location of the study area is 15°45′–17°15′ W, 64°35′–66°10′ N. From left to right: original DEM, natural neighbour interpolation, linear interpolation, nearest neighbour interpolation (with $n=50$), splining (minimum curvature, maximum iterations per cycle $=10^6$).

inaccurate when analysed numerically. Both linear and nearest neighbour methods result in a large number of visible artefacts and are quantitatively inferior.

The only exception is the low resolution (82 pixels/degree) interpolation results, which show a similar mean topography difference for all four algorithms and, with the exception of linear interpolation, have a roughly similar visual appearance. However, we anticipated that different interpolation methods would be best discriminated at high resolutions, and converge at low resolutions, so these results are not unexpected.

The time it takes for these interpolation algorithms to execute varies greatly (table 2). While natural neighbour produces the best-looking interpolations that are also the closest approximation of the original surface, it is relatively slow to execute. Thus, this algorithm is best suited for examining localized areas on Mars at high resolution. The natural neighbour algorithm has been successfully applied to both high latitudes, which have a high density of MOLA data points (Korolev crater, this paper) and equatorial regions (Athabasca Vallis, Burr *et al.* 2002), with sparser spatial coverage.

One disadvantage of the natural neighbour interpolation is that bad data points may not be recognized in the interpolated DEM, because they are made to look like natural features. Thus, it is important to eliminate bad data points prior to interpolation.

6. Conclusion

While further quantitative testing is desirable, it is clear that the natural neighbour algorithm yields excellent results when applied to MOLA data. Additional investigation in this area should include testing of the natural neighbour algorithm on other known DEMs and possibly combining it with the median observed topography technique. The current results indicate that natural neighbour should be the algorithm of choice when accuracy and realistic appearance are

Table 1. Summary of the quantitative analysis of interpolation techniques. All values are in metres. For the nearest neighbour technique, the number of nearest neighbours (*n*) was 50. For splining, 10^6 maximum iterations per cycle were used.

	High resolution interpolation (1000 pixels/degree)		Medium resolution interpolation (250 pixels/degree)		Low resolution interpolation (82 pixels/degree)	
	Mean topography difference	Standard Deviation	Mean topography difference	Standard Deviation	Mean topography difference	Standard Deviation
Natural neighbour	**25.51**	39.26	**27.23**	39.46	**36.22**	43.21
Linear	**34.03**	46.30	**32.43**	45.61	**34.64**	47.36
Nearest neighbour	**30.51**	43.81	**31.50**	44.30	**37.25**	48.29
Splining	**88.98**	116.49	**48.41**	92.05	**35.39**	48.64

Table 2. Execution times for the interpolation of 80 732 data points to produce a DEM with a resolution of 200 pixels/degree.

Interpolation algorithm	Execution time
Natural neighbour	02 h 08 min 47.53 s
Linear	00 h 00 min 05.70 s
Nearest neighbour	11 h 09 min 11.58 s
Splining	00 h 00 min 30.55 s

required, and splining can be used as a quick first-order interpolation technique. Also, interpolations with higher resolutions and better quality than those presented here are now possible with the recently released MOLA datasets MGSL2034–2054.

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