Fractal analysis of drainage basins on Mars

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[1] We used statistical properties of drainage networks on Mars as a measure of martian landscape morphology and an indicator of landscape evolution processes. We utilize the Mars Orbiter Laser Altimeter (MOLA) data to construct digital elevation maps (DEMs) of several, mostly ancient, martian terrains. Drainage basins and channel networks are computationally extracted from DEMs and their structures are analyzed and compared to drainage networks extracted from terrestrial and lunar DEMs. We show that martian networks are self-affine statistical fractals with planar properties similar to terrestrial networks, but vertical properties similar to lunar networks. The uniformity of martian drainage density is between those for terrestrial and lunar landscapes. Our results are consistent with the roughening of ancient martian terrains by combination of rainfall-fed erosion and impacts, although roughening by other fluvial processes cannot be excluded. The notion of sustained rainfall in recent Mars history is inconsistent with our findings. *INDEX TERMS:* 1824 Hydrology: Geomorphology (1625); 1848 Hydrology: Networks; 3250 Mathematical Geophysics: Fractals and multifractals; 5415 Planetology: Solid Surface Planets: Erosion and weathering; 6225 Planetology: Solar System Objects: Mars

1. Introduction

[2] Extensive phenomenological studies [*Horton*, 1945; *Hack*, 1957; *Tarboton et al.*, 1988] of terrestrial drainage (river) networks showed that their geometrical and topological structure is characterized by absence of well-defined length scale. This fractal character of drainage networks is corroborated [*Rodriguez-Iturbe and Rinaldo*, 1997; *Rinaldo et al.*, 1998] by appearance of power laws in probability distribution functions (PDFs) of several network quantities, such as contributing area, $P(a) \propto a^{-(1+\tau)}$, stream length, $P(l) \propto l^{-(1+\gamma)}$, and dissipated potential energy, $P(e) \propto e^{-(1+\beta)}$. These quantities are defined at any point S on a network: a is the area of a sub-basin terminating at S, l is the length of the longest upstream path starting from S, and e is a product of water flow and the elevation rise along a link terminating at S. Drainage density, D, is defined for a sub-basin terminating at S and measures the average area drained per unit length of stream. D is an important network attribute, however it does not bear upon basin fractal structure, and typically has a gaussian-like PDF. We define ρ, the ratio of the mean to the dispersion of the random variable D, to measure the uniformity of D. A morphology of a given network can be encapsulated in a four-dimensional point $A = (\tau, \gamma, \beta, \rho)$, which we call the network descriptor.

[3] The morphology of a drainage network reflects a roughness of an underlying landscape [*Dodds and Rothman*, 2000]. Although all natural and many artificially contrived roughening processes lead to surfaces that yield fractal drainage networks, the networks can be sorted into different classes (sets) on the basis of their A values. Thus the network descriptor may serve as a diagnostic tool, linking, in a statistical sense, the nature of a surface to the process by which it formed. Drainage basins can be extracted from landscapes regardless of whether any liquid actually flowed over these terrains. The roughening of the terrestrial landscape is predominantly due to rainfall-fed fluvial erosion, thus the water is both the agent responsible for the topology of the network and the actual subject of the drainage. Phenomenology of those terrestrial basins reveal that $A \in \mathbf{A}_{\mathrm{Ter}}$ [*Rodriguez-Iturbe and Rinaldo*, 1997; *Rinaldo et al.*, 1998], regardless of geology, climate, vegetation etc. The location of $\mathbf{A}_{\mathrm{Ter}}$ in a four-dimensional space (τ, γ, β, ρ) encapsulates common characteristics of terrestrial landscapes and links them to the rainfall-fed erosion process, while its range is caused by specificity of local conditions. (Antarctic Dry Valley basins do not experience rainfall but do have streams fed by glaciar runoff. Preliminary analysis indicates that their network descriptors are outside of $\mathbf{A}_{\mathrm{Ter}}$ suggesting that Antarctic basins belong to a different class.) On the other hand, the roughening of the lunar landscape is due to meteorite bombardment. A 'drainage' network can be computationally extracted; it encapsulates connectivity of the bombarded terrain and is characterized by $A \in \mathbf{A}_{\mathrm{Lun}}$, where $\mathbf{A}_{\mathrm{Lun}} \neq \mathbf{A}_{\mathrm{Ter}}$ links the lunar landscape to erosion by bombardment.

[4] While meteorite bombardment is also a factor in the roughening of the martian surface, features reminiscent of terrestrial river networks, visible from orbital images, point to the possibility of a fluvial factor. The descriptive morphology of visible martian valley networks has been used [*Pieri*, 1980; *Squyres and Kasting*, 1994] to argue for or against particular processes responsible for the observed structures. We submit that drainage basins, extracted from topography, are a more objective means to address this question, because they pertain to the overall roughness of the landscape rather than to its individual features. In particular, the landscape erosion caused by a sustained rainfall would imprint a characteristic drainage pattern on the martian surface, that should be recognizable by means of a network descriptor, even if prominent features such as sapping-induced valley networks and impact craters are interwoven into the original landscape. This is because it is thought [*Rinaldo et al.*, 1993; *Rigon et al.*, 1994] that a landscape developing due to the rainfall-fed erosion eventually self-organizes itself to form a drainage basin characterized by $A \in \mathbf{A}_{\mathrm{Ter}}$ regardless of factors such as initial form, gravity, geology, climate, etc. We propose using statistical properties of martian drainage basins, encapsulated in their network descriptors, as an objective measure of the martian landscape morphology. A collection of network descriptors calculated for a number of martian drainage basins defines a class set $\mathbf{A}_{\mathrm{Mars}}$; $\mathbf{A}_{\mathrm{Mars}} \approx \mathbf{A}_{\mathrm{Ter}}$ would point to rainfall-fed erosion and provide support for a warm period in the Mars past, whereas $\mathbf{A}_{\mathrm{Mars}}$ different from $\mathbf{A}_{\mathrm{Ter}}$ would indicate an absence of persistent and widespread rainfall.

2. Methods

[5] To study martian drainage networks we have constructed DEMs of 12 areas on the surface of Mars (Sinus Sabaeus (13S, 24N), Tyrrhena (22S, 101E), Hecates Tholus (33N, 150E), Ma'adim Valles (13S, 175E), Tader Valles (49S, 209E), Alba Patera (46N, 248E), Warrengo Valles (43S, 267E), Bosphorus Planum (34S, 288E), Melas Fossae (25S, 289E), Nanedi Vallis (5N, 312E),

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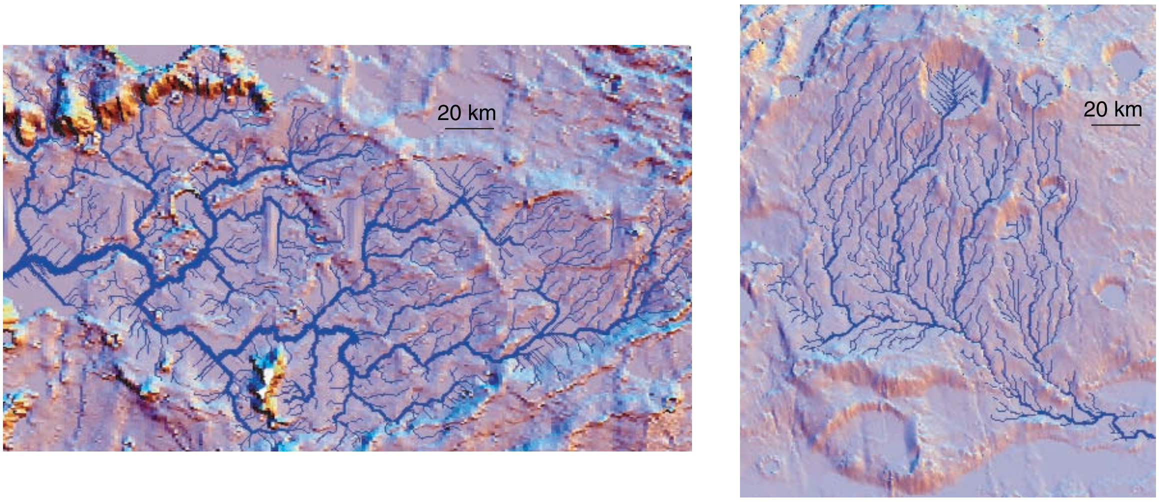

Figure 1. Two examples of martian drainage networks. The thickness of blue lines representing streams is proportional to the Horton-Strahler order, Ω, of the stream. The network is drawn on the top of a perspective view of terrain topography with exaggerated vertical dimension. The terrain on the left is centered on about 22S and 101E. A drainage network of $\Omega = 6$ was extracted with threshold of a_{ch} = 50 cells. The terrain on the right is centered on about 45.5S, 267.5E in the area where Viking images yield a classic example of a valley network strongly resembling terrestrial river networks. A drainage network of $\Omega = 5$ was extracted with threshold of a_{ch} = 50 cells.

Nirgal Vallis (27S, 316E), Margaritifer Valles (30S, 337E)) from which 20 basins were extracted. Several chosen areas contain visually identifiable fluvial features, while others do not. For each area, topography data were extracted from the Mars Orbiter Laser Altimeter (MOLA) [*Smith et al.*, 2001] CD-ROMs 2010 through 2044 (comprising the entire MGS Mapping Mission release at the time of this writing). To construct a DEM, data were assigned to 500 m bins and gridded using the 'surface' adjustable tension continuous curvature gridding algorithm of the Generic Mapping Tools (GMT) software package [*Wessel and Smith*, 1991]. With the 500 m resolution the filling factor is about 36%, unfilled bins were assigned values corresponding to a best-fit surface with tension factor 0.25. Typical sizes of the martian DEMs are about 1000 $\times$ 1000 bins.

[6] To facilitate comparison between different planetary surfaces we have also extracted 9 basins from two DEMs available for the lunar surface. The first is the Apollo 15 landing area (digitalized version of stereophotogrametric map 41B451(50) DMA) and the second the Lunar South Pole area (digital data provided by Anthony Cook from the Smithsonian Institution). We also have extracted 19 terrestrial river basins (Republican, KS; Marias, MT; Yellowstone, MT; Platte, NE; Blacks Fork, UT; Green, UT; Grands, SD; St. Miguel, CO; Big Blue, NE; Gila, NM; Clarks Fork, WY; Flathead, MT; Mimbres, NM; St. Joe, ID; Alamosa, NM; Racoon, PA, Brushy, AL, Big Creek, ID.) from US Geological Survey DEMs. Cell sizes for lunar DEMs are 30–100 m, 30–340 m for terrestrial DEMs (some terrestrial DEMs were degraded from their original 30 m resolution to be more directly comparable to martian DEMs), and the grids have on the order of 10^6 cells.

[7] Drainage basins are extracted from DEMs computationally using an algorithm developed for studies of terrestrial river basins [*Tarboton et al.*, 1989]. As the first step, the algorithm modifies the DEM by making it 'drainable' by 'flooding' all pits in the digital representation of the terrain. For terrestrial DEMs pits are usually data anomalies and typically only a few percent of cells are affected by flooding. However, martian and lunar surfaces contain natural pits, such as craters, and flooding affects typically about 20% of cells. Each cell in the modified DEM is assigned a drainage pointer to a neighbor cell in the direction of the steepest slope, that enables calculating values of a, l, and e for all cells in a grid. The drainage network is defined as cells with $a \leq a_{ch}$, where a_{ch} is a proxy for a minimum flow that causes channelization. A value of a_{ch} is determined objectively [*Tarboton et al.*, 1991] where possible, otherwise it is assumed. Empirical cumulative distribution functions (CDFs) for a, l, e, and D are constructed using their values at the nodes of the extracted network.

[8] Extracted martian basins range in size from 3.4×10^3 km^2 with the drainage network of 153 links having Horton-Strahler order of $\Omega = 5$, to 2.3×10^5 km^2 with the drainage network of 4965 links having Horton-Strahler order of $\Omega = 7$. Horton-Strahler order describes the branching hierarchy of a network, see [*Rodriguez-Iturbe and Rinaldo*, 1997; *Dodds and Rothman*, 2000] for details. Our selection of terrestrial basins has a similar range, but lunar basins are typically smaller, having $\Omega \leq 6$. Figure 1 shows examples of martian drainage networks extracted from our DEMs. The extracted networks closely follow the visual fluvial features where such features exist. For all considered basins the empirical CDFs of a, l, and e indicate power law distributions of these quantities, demonstrating the fractal character of drainage networks. Values of the exponents τ, γ, and β are estimated by a least-squares parametric fit to the respective CDFs. The value of ρ is calculated directly from the data; large values of ρ indicate high uniformity of D. Note that although values of D depend on the estimated (or assumed) value of extraction threshold a_{ch}, the value of ρ, which pertains to the shape of a distribution of D in a fractal network, does not depend on a_{ch}. Thus, the network descriptor A is calculated for each basin in our study. For graphing purposes, we split A into two subsets, (τ, γ) and (ρ, β). The first pair pertains to quantities describing the planar character of the network, whereas the second includes a quantity (β) pertaining to the vertical structure of the network.

3. Results

[9] Figure 2 shows (τ, γ) and (ρ, β) diagrams for martian, terrestrial, and lunar drainage basins. The striking feature of the (τ, γ) diagram is that the martian and terrestrial data points are intermingled in the same, well bounded region of the diagram and disjoined from the much more scattered lunar data points. Moreover, there is the same significant correlation (r = 0.6)

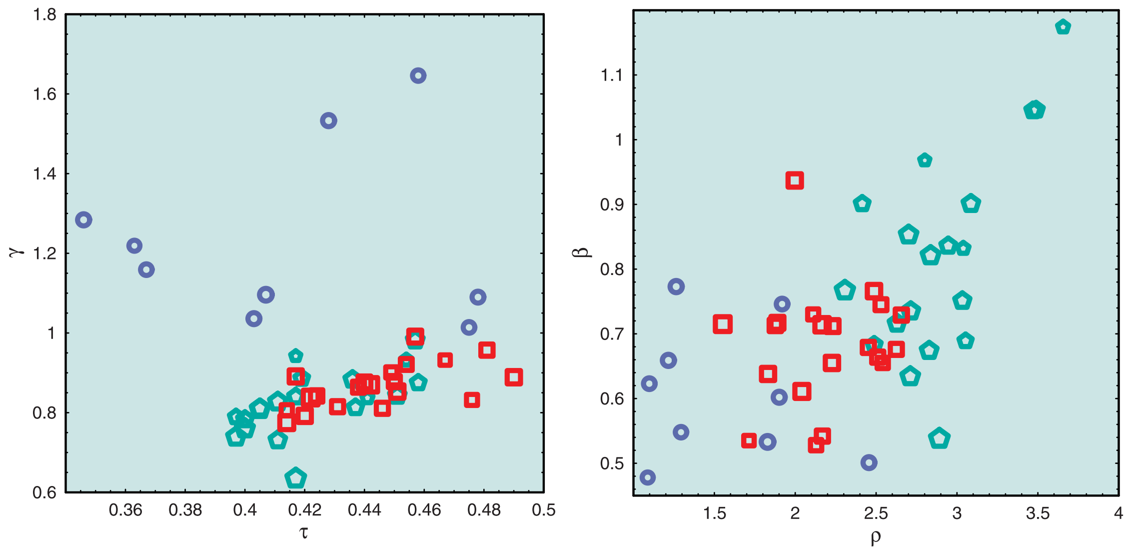

Figure 2. The network descriptor, *A*, for martian (red squares), terrestrial (green pentagons), and lunar (blue circles) drainage basins. The size of a symbol is proportional to the logarithm of basin total area. The left panel shows the planar characteristics of the networks as summarized by indices of power law distributions of sub-basin areas (γ) and stream lengths (τ). The right panel includes the vertical character of the networks in terms of the index of the power law distribution of potential energy dissipation (β) versus the dispersion of drainage density ρ.

between values of τ and γ in terrestrial and martian data, but the lunar data lacks any correlation. For terrestrial data the $\tau - \gamma$ correlation is consistent [*Dodds and Rothman*, 2000] with Hack's law, which thus seems to apply also to the martian basins but not to the lunar basins.

[10] The (ρ, β) diagram reveals that values of ρ for martian basins are systematically lower than those for terrestrial basins, but systematically higher than those for lunar basins. Thus, from the point of view of uniformity of *D*, martian basins can be classified somewhere between lunar and terrestrial basins. Martian and lunar basins are characterized by similar values of β, but terrestrial basins tend to have higher values of β. We have found that β correlates highly ($r \geq 0.9$) with, and can be used as a convenient proxy for, θ, an exponent often used [*Tarboton et al.*, 1989] in power law relations between sub-basin area *a* (itself a proxy for a flow rate under an assumption of an uniform rainfall) and the expected value of the random variable $|\nabla z|$ (local slope). For basins considered in this study, $\langle\theta\rangle_{Ter} = 0.38 \pm 0.11$, $\langle\theta\rangle_{Mars} = 0.22 \pm 0.08$, and $\langle\theta\rangle_{Lun} = 0.25 \pm 0.09$.

[11] We have not found any correlations between values of components of *A* and the large-scale morphology and geology of martian sites. In particular, there are no statistical differences between basins incorporating visible fluvial features and those that are identified exclusively from topography. This suggests that the extent of $\mathbf{A}_{Mars}$, much like the extent of $\mathbf{A}_{Ter}$, is not caused by spatial variation of erosional processes, but rather by a non-causal scatter intrinsic to a single erosional process and uncertainties in deriving values of *A*.

[12] The network descriptors calculated in our study form three well-defined clusters, $\mathbf{A}_{Ter}$, $\mathbf{A}_{Mars}$, and $\mathbf{A}_{Lun}$ in the (τ, γ, β, ρ) space. The average values of network descriptors are $\langle A\rangle_{Ter} = (0.42 \pm 0.02, 0.83 \pm 0.08, 0.82 \pm 0.16, 2.90 \pm 0.36)$, $\langle A\rangle_{Mars} = (0.44 \pm 0.02, 0.87 \pm 0.06, 0.68 \pm 0.09, 2.19 \pm 0.32)$, $\langle A\rangle_{Lun} = (0.41 \pm 0.05, 1.23 \pm 0.22, 0.61 \pm 0.10, 1.56 \pm 0.48)$. The cluster $\mathbf{A}_{Mars}$ separates from $\mathbf{A}_{Lun}$ (which we associate with the process of meteorite bombardment) in all dimensions except for β, and it separates from $\mathbf{A}_{Ter}$ (which we associate with the process of rainfall-fed fluvial erosion) in ρ and β dimensions. Thus, although the planar characteristics of martian and terrestrial drainage networks have a high degree of similarity, indicating the possibility of rainfall-fed erosion, a relatively non-uniform drainage density and low values of β_{Mars} (and θ_{Mars}) are not consistent with an erosion by sustained and uniformly distributed rainfall. In particular, the value of θ is frequently used [*Tarboton et al.*, 1989] to infer an erosion mechanism; $0.35 \leq \theta \leq 0.6$ is usually attributed [*Whipple et al.*, 1999] to river sediment transport processes. We have found θ for the martian basins to be mostly below that range and in an agreement with θ for the lunar basins. This may suggest that martian landscapes were roughened by a different process, though for landscapes with $\theta < 0.3$ values of θ may not be indicative of any specific erosion mechanism [*Schorghofer and Rothman*, 2001].

[13] A plausible interpretation of our finding is that the roughness of the ancient martian landscape is not due to a single, predominant process, but rather due to a superposition of roughening by meteorite bombardment and the rainfall. Such an interpretation accounts for the Earth-like planar characteristics of the martian networks, they were imprinted during the time when Mars possessed an early climate that was conducive to rainfall, but which disappeared before the end of heavy bombardment (∼4 Ga). The pattern of the rainfall-eroded landscape was subsequently modified by impacts, causing the observed relative non-uniformity of *D* and the Moon-like character of the vertical structure of drainage networks. This interpretation implies that networks identified on younger terrains, such as Alba Patera, should have values of *A* different (in a statistical sense) from that for the networks extracted from older terrains. No such distinction is found in our results, although the number of younger surfaces in our sample is too small for any statistical inference to be significant. An alternative interpretation is that $\mathbf{A}_{Mars}$ does not reflect superposition of rainfall and impacts, but instead is associated with an another roughening process, such as erosion associated with local hydrothermal discharges or from fluvial processes at the base of glaciers and snow packs. Regardless of this ambiguity, our results are not consistent with the idea that Mars has experienced sustained rainfall after the end of heavy bombardment, or the notion that Mars underwent recurring episodes of rainfall of sufficient duration and intensity to redistribute the volume of water associated with a northern ocean (10^7 km^3) into the southern highlands multiple times during the planet's history [*Baker et al.*, 2000]. In both cases, we would expect the roughness of the martian surface to have an unambiguous signature of the rainfall-fed erosion.

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