

Origin and Evolution of Trojan Asteroids

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The regions around the L_4 and L_5 Lagrangian points of Jupiter are populated by two large swarms of asteroids called the Trojans. They may be as numerous as the main-belt asteroids and their dynamics is peculiar, involving a 1:1 resonance with Jupiter. Their origin probably dates back to the formation of Jupiter: the Trojan precursors were planetesimals orbiting close to the growing planet. Different mechanisms, including the mass growth of Jupiter, collisional diffusion, and gas drag friction, contributed to the capture of planetesimals in stable Trojan orbits before the final dispersal. The subsequent evolution of Trojan asteroids is the outcome of the joint action of different physical processes involving dynamical diffusion and excitation and collisional evolution. As a result, the present population is possibly different in both orbital and size distribution from the primordial one. No other significant population of Trojan asteroids have been found so far around other planets, apart from six Trojans of Mars, whose origin and evolution are probably very different from the Trojans of Jupiter.

1. INTRODUCTION

As of May 2001, about 1000 asteroids had been classified as Jupiter Trojans (<http://cfa-www.harvard.edu/cfa/ps/lists/JupiterTrojans.html>), some of which had only been observed for a few nights and some that had no measured absolute magnitude. The preceding cloud, L_4 , consists of 618 known members; there are 375 objects in the trailing cloud, L_5 . Out of these 1000, only 426 are numbered asteroids with reliable orbits; 284 are at L_4 and 142 at L_5 , according to Bowell's list (<ftp://lowell.edu>). From spectroscopic surveys it appears that most Trojans belong to the D taxonomic type, while only a few are classified as P and C type. All these objects have low albedos (average around 0.065) and share spectral similarities with short period comets, Centaurs, and transneptunian objects. In addition to Jupiter Trojans, there are five Trojans orbiting in the trailing cloud of Mars. Apart from Jupiter and Mars, observational searches have so far failed to detect Trojan asteroids of any other planet and it is still uncertain if this is due to an intrinsic instability of the tadpole orbits of those planets or to detection difficulties. Putative Trojans of Saturn, Uranus, and Neptune would in fact be very faint, while a possible population of Trojans of Earth or Venus would occupy a large projected area in the sky and would be unfavorably placed with respect to the Sun. A few interesting Trojan configurations have been found among the satellites of Saturn, and they may

originate from the collisional disruption and subsequent reaccumulation of larger primordial bodies.

A basic understanding of why asteroids can cluster in the orbit of Jupiter was developed more than a century before the first Trojan asteroid was discovered. In 1772, Joseph-Louis Lagrange demonstrated the existence of five equilibrium points in the restricted three-body problem where an object of negligible mass orbits under the gravitational effect of two larger masses (*Lagrange, 1772*). Three of these points, L_1 , L_2 , and L_3 lie on the line joining the two masses and are unstable to small perturbations. Each of the remaining two points, L_4 and L_5 , lies at the apex of an equilateral triangle with base equal to the separation of the two masses (see Fig. 1); stable motion is possible around them. Successive attempts to understand the dynamical properties of the three-body problem have provided a rich mathematical vein that continues to be mined to this day. Recent summaries of the literature are contained in *Szebehely (1967)* and *Marchal (1990)*. There are also particularly relevant chapters in *Brown and Shook (1993)* and *Murray and Dermott (1999)*. A series of papers have also been published by *Érdi* in *Celestial Mechanics* (1978, 1979, 1981, 1984, 1988) on the dynamics of Trojan asteroids. He has derived a second-order solution to the three-dimensional motion of Trojans within the framework of the elliptical restricted three-body problem. His perturbative approach leads to analytical expressions for the secular evolution of

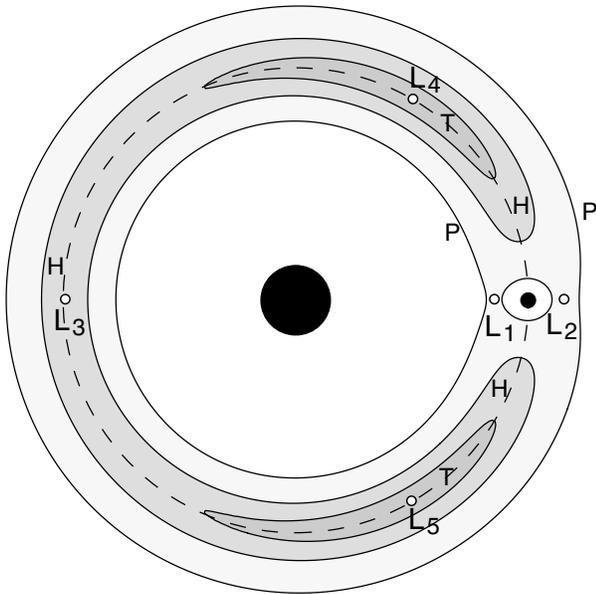


Fig. 1. The location of the five Lagrange equilibrium points in the circular-restricted three-body problem. The primary and secondary masses (the Sun and planet in our examples) are denoted by the large and small filled circles respectively. The selected zero-velocity curves (see text) are closely related to the types of orbits that can occur in the system. The letters P, H, and T denote passing, horseshoe, and tadpole orbits respectively. Note that the two masses and each of the L_4 and L_5 points form an equilateral triangle.

the perihelion and eccentricity and for the node as a function of the libration amplitude and inclination.

An important cosmogonical and dynamical question concerning Trojan orbits is their long-term stability. The first attempt to solve the problem analytically was made by *Rabe* (1967) who derived a region in the eccentricity-libration amplitude space for indefinite stability in the frame of the planar-circular-restricted three-body problem. More recently, *Georgilli and Skokos* (1997) used the same model but with a higher-order perturbative expansion and a different method to determine stability. They were able to mathematically prove the stability in only a small fraction of the region found by *Rabe* (1967). Unfortunately, the stability area found by *Georgilli and Skokos* (1997) does not compare with the Jupiter Trojan population; in fact, it comprises only a few of the observed Trojans. The same authors admit that their criterion is probably too restrictive.

When comparing the results obtained within the restricted three-body problem to the Jupiter Trojan populations, there are additional potential sources of instability that have to be taken into account. These are related to the gravitational influence of the other planets. *Levison et al.* (1997) resorted to a numerical approach to study the stability of Jupiter Trojans within a full N-body model that included all the outer planets. They found that there is a region for Jupiter Trojans where the asteroids may orbit in

a stable fashion over the age of the solar system, and that this region, as expected, is significantly wider than that defined by *Georgilli and Skokos* (1997). An immediate question arising from the *Levison et al.* (1997) results arises: What causes the instability outside this region? Is the destabilizing mechanism intrinsic in the three-body problem, or are the external perturbations of the other planets responsible for this instability? Secular resonances cross the region in the phase space where Trojans may orbit, and even the 5:2 near-resonance between Jupiter and Saturn (the “great inequality”) may cause Trojan orbits to become unstable. Eventually, a synergy among all these mechanisms may explain the presently observed structure of the Jupiter Trojan clouds.

The same reasoning concerning the stability should be applied to the study of the evolution of tadpole orbits for the other giant planets Saturn, Uranus, and Neptune. Of course, the orbital frequencies of the Trojan orbits and of the perturbing forces change from planet to planet, rendering the stability analysis an independent task for each planet. It is even possible that the strong mutual perturbations among the outer planets prevent Saturn, Uranus, and Neptune from having stable Trojans over the age of the solar system. The few observational surveys have failed so far to detect any Trojans. The reason might be purely dynamical, but we cannot exclude the possibility that Trojans exist but are too dark and distant to be readily detected. The possibility of terrestrial planet Trojans is a different story. While for the outer planets, the stability issue can be investigated over the age of the solar system, in the case of tadpole orbits for the Earth, Venus, and Mars, we can only perform numerical surveys limited in time, since to maintain the accuracy in the integration of the equation of motion a short timestep has to be used. The present studies on the dynamics of tadpole orbits for Earth, Venus, and Mars are in fact limited to a timespan of 100 m.y. Under this condition, we can only have a glimpse at their long-term dynamical behavior, and we cannot give a definitive answer to whether, for example, the six Mars Trojans are primordial, sharing a common origin with Mars, or were captured in more recent times. Even Earth and Venus may well possess stable Trojan-type orbits, but, for the same reason, we cannot assess their stability over the age of the solar system. Moreover, the possible detection of these Trojans would be very difficult because of unfavorable observational conditions.

Another conundrum in the Trojan saga is represented by the high average inclination of Jupiter Trojans. The most accepted theory on their origin assumes that they were planetesimals orbiting near Jupiter that were then captured in the final phase of the planetary formation process. If this were the case, their present orbits should lie in the plane of the proto-nebula disk and close to the orbital plane of Jupiter. Which dynamical mechanism acted after their capture as Trojans to drive them into high-inclination orbits? Alternative scenarios include the raising of inclinations by secular resonances (*Yoder, 1979; Marzari and Scholl, 2000*) or by close encounters with lunar-sized bodies similar to a mecha-

nism proposed by *Petit et al.* (1999) for main-belt asteroids. It is even possible that they were stirred up prior to capture by proto-Jupiter (*Kokubo and Ida*, 2000).

In this chapter we summarize the present state of the research on Trojans. After an overview of the known orbital and size distributions of Jupiter Trojans, we analyze the general dynamical properties of tadpole orbits in the context of the three-body model. We then explore models of the origin of Trojans and investigate how they may have subsequently evolved to their present configuration through interactions with other planets of the solar system.

2. PRESENT POPULATION OF JUPITER TROJANS

Dedicated surveys for Trojans have been made by *van Houten et al.* (1970a) with the Palomar Schmidt telescope during several apparitions: March 1971 (L_5), September 1973 (L_4), and October 1977 (L_5). The Trojans discovered during these runs were designated T-1, T-2, and T-3. Also, the Palomar Leiden Survey of Asteroids (*van Houten et al.*, 1970b) was close to the preceding Lagrangian point. *Lagerkvist et al.* (2000) used the ESO Schmidt Telescope for a dedicated survey of L_4 with observations during the oppositions in 1996, 1997, and 1998. *Jewitt et al.* (2000) made a campaign to estimate the number of small Trojans in L_4 (radii <20 km) with the University of Hawai'i Mauna Kea Observatory 2.2-m telescope.

2.1. Asymmetry Between L_4 and L_5 ?

The first question Trojan surveys could help to answer is whether there is any difference between the leading and trailing Trojan populations. From a theoretical point of view, there is no known difference between the dynamics of the L_4 and L_5 points for Jupiter. However, it has been shown (*Peale*, 1993; *Murray*, 1994; *Marzari and Scholl*, 1998a) that in the presence of gas drag, the orbits of small Trojan asteroids around L_5 are more stable than those around L_4 . On the contrary, planetary migration seems to destabilize more easily the orbits of Jupiter Trojans around L_5 (*Gomes*, 1998). A possible difference between the two populations in size distribution or in orbital distribution would confirm either of the hypotheses: that the capture of Trojans occurred in presence of gas drag or that a significant planetary migration took place in the early phases of the solar system. *Lagerkvist et al.* (in preparation, 2002) applied a simple statistical test, the Kruskal-Wallis test (*Siegel and Castellan*, 1988), to compare the orbital elements of the two Trojan populations. They compared the distributions of the mean distance (a), eccentricity (e), and inclination (i) between the L_4 and L_5 populations, discriminating between numbered and unnumbered (more recently discovered and hence presumably smaller) Trojans. The elements a and e showed no such differences between L_4 and L_5 , or between numbered and unnumbered objects. However, they found the inclination to be different between the two clouds in the sense that

L_5 contained more high-inclination orbits. The reason for this is not clearly understood, but may be due to observational bias. An investigation of the discovery circumstances might reveal the true nature of this difference. There is, however, no difference between the inclinations of the numbered and unnumbered Trojans in the L_5 cloud. For absolute magnitudes, they found no difference between the two Trojan clouds. Even the Spacewatch data, once bias-corrected, show no significant discrepancies between the magnitude distributions in the two swarms (see *Jedicke et al.*, 2002). The two populations have therefore been treated together in the following discussion of the size distribution even if this is focused on L_4 , the most-studied Lagrangian point. In recent years, only a few new Trojans have been found to be brighter than absolute magnitude $H = 9.5$, and the population seems to be more or less complete down to this limit ($H = 9.5$ corresponds to a radius of 43 km for an assumed geometric albedo of 0.04; $H = 10$ corresponds to 34 km). Presently there are 60 Trojans with $H \leq 10.0$, of which as many as 18 have been found during the last five years.

Of interest for the stability of the Trojan orbits are the proper elements. These have been calculated numerically for the L_4 members known to date by *Karlsson* (personal communication, 2001), following the method given in *Schubart and Bien* (1987). There is a large spread in the libration amplitudes ranging from 0.6° to 88.7° , with a mean around 32.7° for the L_4 cloud. The Kruskal-Wallis test gave no significant differences (*Lagerkvist et al.*, in preparation, 2002) of the libration amplitudes of the numbered and unnumbered Trojans in L_4 . Even in the proper elements computed by *Milani* (1993) with a slightly different method and with a smaller sample of Trojan orbits (174), there was no significant difference between the distribution of the libration amplitude in L_4 compared to that in L_5 . Recently, *Beaugè and Roig* (2001) have developed a semianalytical method to estimate proper elements and they have applied it to a sample of 533 Trojans, observing only minor differences between the L_4 and L_5 populations. From the proper elements they proceeded to identify asteroid families as in *Milani* (1993) and found a possible asymmetry between the two swarms: The L_4 region shows more families than L_5 and they are even more robust. Is this an indication of a larger number of potential projectiles in L_4 for catastrophic disruption events, or is it the orbital distribution to be really asymmetric in the two swarms leading to a different collisional rate? We need additional data to answer to this question.

2.2. Size Distribution

The total number of Trojans down to an absolute magnitude of $H = 13.0$ was estimated by *van Houten et al.* (1970a) to be about 700. From the same observational data, *Shoemaker et al.* (1989) deduced that there were 995 L_4 Trojans, a factor of 1.4 larger than that claimed by *van Houten et al.* (1970a). *Lagerkvist et al.* (2000; in preparation, 2002), from a large observational sample, after correcting for incompleteness down to the limiting magnitude, suggested that

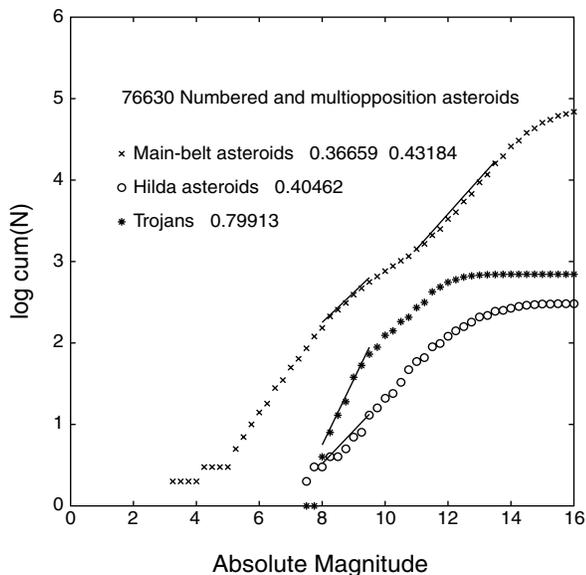


Fig. 2. Cumulative numbers of main-belt asteroids, Hildas, and Trojans plotted on a logarithmic scale vs. absolute magnitude. For each population, the slope of the magnitude distribution is given for selected magnitude ranges. At the high-magnitude end, Trojan asteroids have the steepest slope.

there were 1100 L_4 Trojans in the same cloud. *Jewitt et al.* (2000) got as many as 3300 L_4 Trojans down to same sizes. The large discrepancies between some of these estimates might be due to some extent to the statistical limitations in the detection capabilities of the surveys. Bias correction for the selection effects in the observed populations is a difficult task, in particular for Trojans. Their librational motion can generate nonuniform density patterns once projected onto the sky. Figure 1 shows, for example, that the distribution of Trojans around L_4 is by no means the same in the direction of Jupiter as in the opposite direction and also that latitude plays an important role.

Figure 2 gives the cumulative absolute magnitude distributions (logarithmic) of the main belt asteroids, the Hildas (asteroids captured in the 3:2 mean-motion resonance with Jupiter) and the Trojans (known objects). Only data from numbered and multiopposition objects have been used. For the Trojans, this means that somewhat fewer objects are available, but the data should still be complete down to $H = 9.5$ for all three categories. The slope of the Trojan distribution at small magnitudes (large size end) is about twice that of main-belt asteroids and Hildas of similar size. Assuming a mean geometric albedo of 0.04 from IRAS data (*Tedesco*, 1989) the differential size distribution is adequately fitted by a power law with index $q = 5.5 \pm 0.9$ for radii larger than 30–45 km (*Jewitt et al.*, 2000). For smaller Trojans, both *Shoemaker et al.* (1989) and *Jewitt et al.* (2000), with a more detailed survey, found a slope around 0.4 in magnitude that corresponds to an index of $q = 3.0 \pm$

0.3. *Jewitt et al.* (2000) predicts, however, a cumulative number of small Trojans larger than that of *Shoemaker et al.* (1989), due to a significantly different scaling of the power laws. What is the interpretation of the different slopes at large and small sizes for Trojans? A possible answer is that large objects represent a primordial planetesimal population captured during the formation of Jupiter, while smaller Trojans represent fragments produced during the subsequent collisional evolution of the population (*Shoemaker et al.*, 1989; *Marzari et al.*, 1997). This interpretation is reinforced by the fact that the index q of the differential size distribution is close to the Dohnanyi's value of $q = 3.5$ (*Dohnanyi*, 1969), typical of a collisionally relaxed population. Moreover, from the larger mean light-curve amplitude of large-sized Trojans compared to their low albedo main-belt counterparts, *Binzel and Sauter* (1992) argued that large Trojans might have retained their primordial spherical forms.

If future, more complete surveys of the Trojan swarms confirm the slopes and size distribution derived by *Jewitt et al.* (2000), then Trojan asteroids would outnumber main-belt asteroids. From the debiased observational data of the SDSS survey (*Ivezić et al.*, 2001), there would be about 5.3×10^5 main-belt asteroids with a diameter larger than 1 km, while Trojans, thanks to their steeper slope at the small size end, would number around 1.28×10^6 according to *Jewitt et al.* (2000).

3. DYNAMICS OF THE THREE-BODY PROBLEM

One of the great triumphs of celestial mechanics was the prediction by Lagrange in the eighteenth century that points of stable equilibrium could exist in the orbit of Jupiter and that objects might one day be found there. However, it was only in the twentieth century that the first Trojan asteroid was discovered, thereby validating Lagrange's prediction. Many of the world's greatest mathematicians have studied the "three-body problem," i.e., the motion of a small object moving under the gravitational effect of a planet and the Sun. In the simplest version of the three-body problem, Jupiter and the Sun move in circular orbits around their common center of mass while the asteroid is treated as a test particle perturbed by the two masses but without, in turn, affecting their motion. This is a reasonable approximation because Jupiter's eccentricity is small ($e_J \approx 0.048$) and even the largest asteroid, (1) Ceres, has a mass that is $<10^{-6}$ that of Jupiter. Although the angular momentum and energy in this idealized system are not conserved quantities, the circular restricted problem has a constant of the motion, the Jacobi constant C , that is a function of the position and velocity of the asteroid (e.g., see *Murray and Dermott*, 1999). Although it is not possible to find a practical, analytical solution for the resulting motion of the asteroid, the existence of the Jacobi constant implies that it is sometimes possible to identify regions from which the asteroid is excluded. This is achieved by plotting the so-called "zero-velocity curves" for the asteroid. Figure 1

shows three sets of such curves for a mass ratio $\mu = m_j / (m_s + m_j)$ of 10^{-2} , or approximately $10\times$ larger than the actual value for the Sun-Jupiter system. The three shaded areas denote excluded regions for the zero-velocity curve associated with the corresponding value of C .

It is clear from Fig. 1 that the locations of the Lagrangian equilibrium points (small unfilled circles) are related to the critical points of zero-velocity curves. In appropriate units for small values of μ , the value of the Jacobi constant at L_4 and L_5 is a minimum ($C_{L_{4,5}} = 3 - \mu$) while it is a maximum at L_1 ($C_{L_1} = 3 + 3^{4/3} \mu^{2/3} \approx C_{L_2}$) with an intermediate value at L_3 ($C_{L_3} = 3 + \mu$). Therefore, an asteroid with $C < C_{L_{4,5}}$ will have no zero-velocity curve and hence, on this basis, there are no regions from which it is excluded. That does not imply that its motion is unbounded or that it will escape from the system; it just means that the Jacobi constant cannot be used to place bounds on its motion.

The fact that Jupiter and the Sun maintain a fixed separation in the circular problem means that it is customary to consider the motion of the asteroid in a frame rotating with the (constant) angular velocity or mean motion, n , of Jupiter. One consequence of choosing such a reference frame is that the motion of the asteroid can appear to be quite complicated. However, it is important to remember that the asteroid's motion is dominated by the Sun and that in the nonrotating frame, it is following a near-constant Keplerian elliptical path around the Sun. The effect of Jupiter is to perturb that ellipse and introduce variations in the orbital elements of the asteroid. We can see this and gain a useful insight into the dynamics of Trojan asteroids by investigating the asteroid's motion in the vicinity of the L_4 point.

In addition to identifying the triangular equilibrium points, Lagrange also investigated their linear stability. This involved carrying out an analysis of how an object (the asteroid) would respond if it underwent a small displacement from the equilibrium point. It can be shown (e.g., see

Murray and Dermott, 1999) that provided the displacement is small and the condition

$$\mu < \mu_{\text{crit}} = (27 - \sqrt{621}) / 54 \approx 0.0385$$

is satisfied, the asteroid moves in a stable path with two distinct components. The first is a long-period, elongated librational motion around L_4 with a period

$$T_1 = T_J / \sqrt{(27/4)\mu}$$

where T_J is the orbital period of the secondary mass (Jupiter) around the central mass (the Sun). Taking $\mu \approx 10^{-3}$ and $T_J \approx 12$ yr gives a librational period of ~ 146 yr for Jupiter Trojans. Superimposed on this is a short-period motion of period

$$T_2 = T_J / \sqrt{1 - (27/8)\mu}$$

This is usually referred to as the epicyclic motion of the asteroid and is simply its Keplerian motion viewed in the rotating reference frame. Note that $T_2 \approx T_J$ for small μ . Furthermore, as the eccentricity of the test particle (asteroid) tends to zero, so the amplitude of the epicyclic motion is suppressed.

It can be shown that for small amplitude librations around L_4 or L_5 , the ratio of the axes for the elongated librational ellipse is $\sqrt{3}\mu$ while that for the epicyclic motion ellipse is $1/2$. Figure 3a shows the two components of the motion for a sample case, while Fig. 3b shows the equivalent resulting path in the rotating frame. In this case, there is a zero-velocity curve, albeit a very narrow one, that the particle just touches at the cusps of its motion (see the interior part of the trajectory in Fig. 3b). Again, although the path is complicated, this is simply a consequence of viewing the trajectory in the rotating frame. Note that, except for Pluto

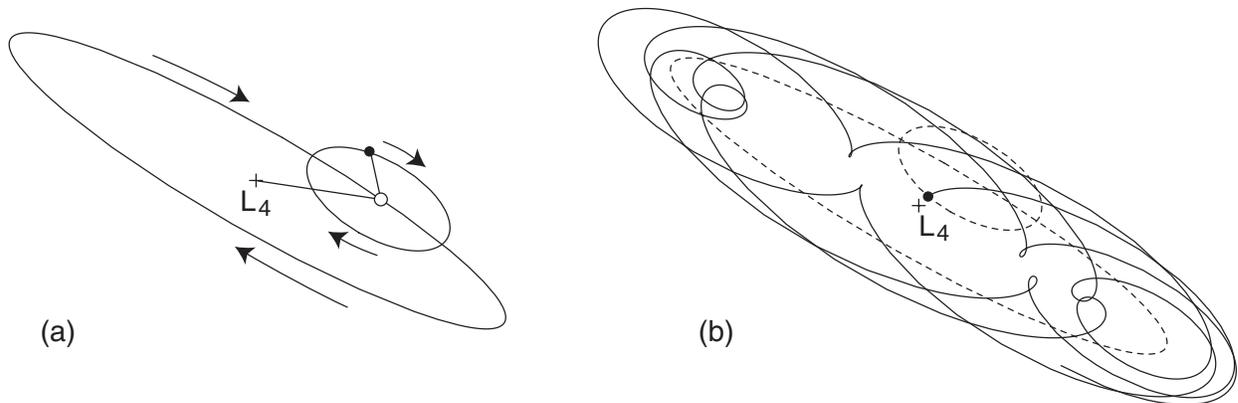


Fig. 3. The typical motion of an asteroid in a small amplitude tadpole orbit around L_4 . (a) The motion has two separate contributions: a long-period motion (elongated ellipse) about the equilibrium point combined with a short-period motion (small ellipse) arising from the Keplerian motion of the asteroid. (b) The resulting path appears complicated but the dynamical mechanism is straightforward. Adapted from Murray and Dermott (1999).

and its moon Charon, the condition for linear stability, $\mu < \mu_{\text{crit}}$ is satisfied by every planet-Sun and satellite-planet pair in the solar system, even though the vast majority show no signs of maintaining Trojan objects.

It is tempting to think of the curves shown in Fig. 1 as defining an asteroid's path in the rotating frame, despite the fact that these are zero-velocity curves rather than trajectories. In fact, as suggested above, for Trojan asteroids on near-circular orbits, there is a close correspondence between actual paths in the rotating frame and their associated zero-velocity curves [see *Murray and Dermott* (1999) for a more complete description; note that for larger eccentricities the correspondence breaks down and the paths can be significantly different as shown by *Namouni* (1999)]. More importantly, the relationship allows us to see how the orbits in the vicinity of L_4 and L_5 change as the initial displacement from the equilibrium points increases. The symmetry of the librational ellipse around the equilibrium point is lost and it develops an elongated "tail." These are referred to as tadpole orbits and the two examples in Fig. 1 are labeled with a T. As the amplitude of libration increases, the tail of the tadpole extends toward the L_3 point. For an object with $C_{L_3} < C < C_{L_1}$, a type of motion that encompasses L_3 , L_4 , and L_5 is possible. This is the so-called horseshoe orbit; an example is labeled H in Fig. 1. Although no Jupiter Trojans are currently known to exist in horseshoes, the saturnian satellites Janus and Epimetheus are known to be involved in a variation of this configuration. By increasing the Jacobi constant and hence the libration amplitude further, the horseshoe configuration can be lost and the object can circulate either interior or exterior to the orbit of the secondary mass. These can be referred to as passing orbits and are labeled P in Fig. 1.

Although many of the above results have been derived in the context of the circular-restricted problem, they can be extended to the case where the planet moves in an elliptical rather than a circular path. For example, *Danby* (1964) investigated the linear stability of L_4 and L_5 in the elliptical problem. Analytical and numerical studies show that the phenomena of tadpole and horseshoe orbits carry over into the elliptical regime and form part of a more general class of orbits that demonstrate what is called coorbital motion with important applications to planetary rings and studies of the formation of planets and satellites. In all these cases, the key quantity is the semimajor axis of the asteroid relative to that of the planet. If we define the coorbital width of a planet as the region within which tadpole or horseshoe motion is possible, this region has a half-width $\Delta = [m_p/3(m_p + m_s)]^{1/3} a_p$, where a_p is the semimajor axis of the planet and m_p is its mass. In fact, this is nothing more than the approximate distance of the L_1 or L_2 points from the planet (e.g., see *Murray and Dermott*, 1999).

The early analytical work on the dynamics of coorbital motion concentrated on the case where the eccentricity, e , and inclination, I , were small. This was because it was relatively easy to tackle analytically using expansions in terms of small quantities. However, *Namouni* (1999) showed ana-

lytically that previously unknown types of coorbital motion were possible in the case of relatively large values of e and I . He predicted that in addition to the T, H, and P orbits, one could also expect to find, among others, RS-T (retrograde satellite-tadpole), RS-H (retrograde satellite-horseshoe), and T-RS-T (tadpole-retrograde satellite-tadpole) orbits. Indeed, additional work by *Namouni et al.* (1999) identified asteroids in such orbits associated with Earth and Venus. If variable a denotes the semimajor axis of the asteroid and a_p is that of the planet, we can then define the relative semimajor axis $a_r = (a - a_p)/a_p$. Similarly, we can define the relative longitude $\lambda_r = \lambda - \lambda_p$ (where λ denotes mean longitude) to help identify librational motion. Figure 4 is taken from *Namouni et al.* (1999) and shows plots of the time evolution of a_r and λ_r for three asteroids. Numerical integrations in a planetary system including all planets indicate the possibility that (3753) Cruithne and (3362) Khufu are or have been involved in coorbital interactions with Earth, while 1989 VA will be a coorbital of Venus. Figure 4 clarifies and explains the orbit of (3753) Cruithne ($e = 0.515$, $I = 19.8^\circ$) and shows that it will become a retrograde satellite of Earth within the next 10^4 yr. Although one might have expected such orbits not to survive the effect of perturbations from other planets, what was observed in the numerical integrations was a switching from one type of orbit to another. For example, 2×10^4 yr ago (3753) Cruithne was in a large eccentricity T orbit around Earth's L_4 point, before evolving into a T-RS-T orbit, a P orbit, and a H-RS orbit (its current state). In contrast, (3362) Khufu is currently in a P orbit but prior to that had been a retrograde satellite of Earth. *Christou* (2000a) confirmed and extended these results. There is no inherent reason why such orbits cannot exist among asteroid themselves. Indeed, using numerical integrations over 2×10^6 yr, *Christou* (2000b) found four asteroids temporarily trapped in coorbital configurations with (1) Ceres and (4) Vesta. As with all chaotic orbits, it is important to note that many numerical integrations of "clone" asteroids have had to be carried out before definite statements can be made regarding the exact nature and timings of the orbital evolution.

4. ORIGIN AND EVOLUTION

4.1. Jupiter Trojans

A natural question about Jupiter Trojans arises: Did they form where they are now? If this were the case, they would be the remnant of the planetesimal population that populated the feeding zone of the proto-Jupiter embryo, and their composition would give important clues on the interior of Jupiter. If the Trojan precursors were indeed planetesimals orbiting near Jupiter, they must have been trapped before the planet reached its final mass, since a fully formed Jupiter would have cleared up the region around its orbit on a very short timescale. There are various ideas on how planetesimals could have been captured as Trojans in the early phases of Jupiter growth, based on different physical pro-

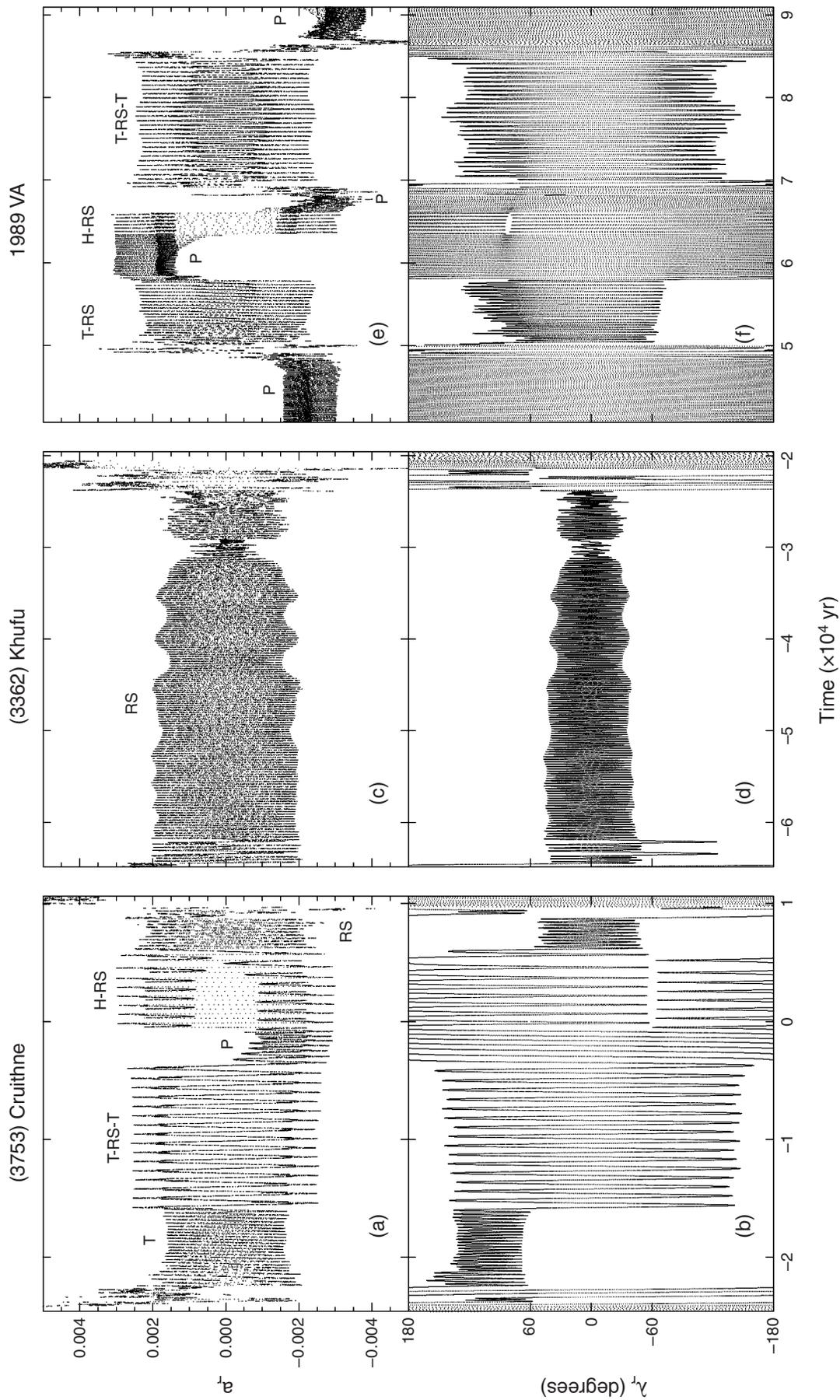


Fig. 4. Plots of relative semimajor axis a_r and relative longitude λ_r (see text) as a function of time for the asteroids (3753) Cruithne, (3362) Khufu, and 1989 VA. The letters in the plots indicate the type of coorbital motion that the asteroid is exhibiting. (3753) Cruithne and (3362) Khufu have coorbital interactions with Earth, while 1989 VA has interactions with Venus (courtesy of *Namouni et al.*, 1999).

cesses. *Shoemaker et al.* (1989) proposed that mutual collisions between planetesimals populating the region around Jupiter orbit might have injected collisional fragments into Trojan orbits. *Yoder* (1979), *Peale* (1993), and *Kary and Lissauer* (1995) showed that nebular gas drag could have caused the drift of small planetesimals into the resonance gap, where they could have grown by mutual collisions to their present size. The mass growth of Jupiter is also an efficient mechanism for trapping planetesimals into stable Trojan orbits, as shown by *Marzari and Scholl* (1998a,b). These mechanisms are by no means mutually exclusive, and each of them may have contributed in a synergistic manner to create the observed population.

There are also alternative ideas on the origin of Jupiter Trojans not related to early stages of the planet's formation. *Rabe* (1972) suggested that Trojans might be comets captured throughout the history of the solar system, while again *Rabe* (1954) and *Yoder* (1979) proposed that Trojans might be fragments of jovian satellites that leaked through the interior Lagrangian point. It seems implausible that these two mechanisms could have contributed significantly to the large populations of L_4 and L_5 Trojans. The origin of Trojan asteroids therefore appears to be more consistent with an early trapping of planetesimals during the formation of Jupiter.

Considerable quantitative work has been done to model the process by which planets like Jupiter may have formed first by accumulation of planetesimals and then by gas infall. *Pollack et al.* (1996) showed that Jupiter and Saturn could have formed on a timescale from 1 to 10 m.y., assuming an initial surface density of planetesimals $\sigma = 10 \text{ g cm}^{-2}$ in the region of the solar nebula where Jupiter formed. This value is about 4 \times that of the minimum-mass solar nebula proposed by *Hayashi et al.* (1985). According to their numerical simulations, in the final phase of its growth the planet increases its mass from a few tens of Earth mass to its present mass on a very short timescale, on the order of some 10^5 yr.

In this scenario, the capture of planetesimals into Trojan-type orbits by the mass growth of the planet described by *Marzari and Scholl* (1998a,b), and later revisited by *Fleming and Hamilton* (2000), is the most efficient mechanism for explaining the origin of Trojan asteroids. Unaccreted planetesimals within the feeding zone of the growing planet can be captured by the fast expansion of the libration regions that encompass the L_4 and L_5 Lagrangian points, where the expansion is caused by the increasing gravity field of the planet. Figure 5 illustrates the outcome of a numerical integration (*Marzari and Scholl*, 1998a) in which a planetesimal initially on a horseshoe orbit transits to a tadpole orbit when the mass of Jupiter grows at an exponential rate with a characteristic time of 10^5 yr from $10 M_{\oplus}$ to its present mass. A similar behavior is also observed for planetesimals orbiting near the planet and not necessarily in a horseshoe orbit. Hence, during the formation of Jupiter, a large number of planetesimals were presumably captured around the planet's Lagrangian points and the symmetrical nature of the tadpole solutions should have led to approxi-

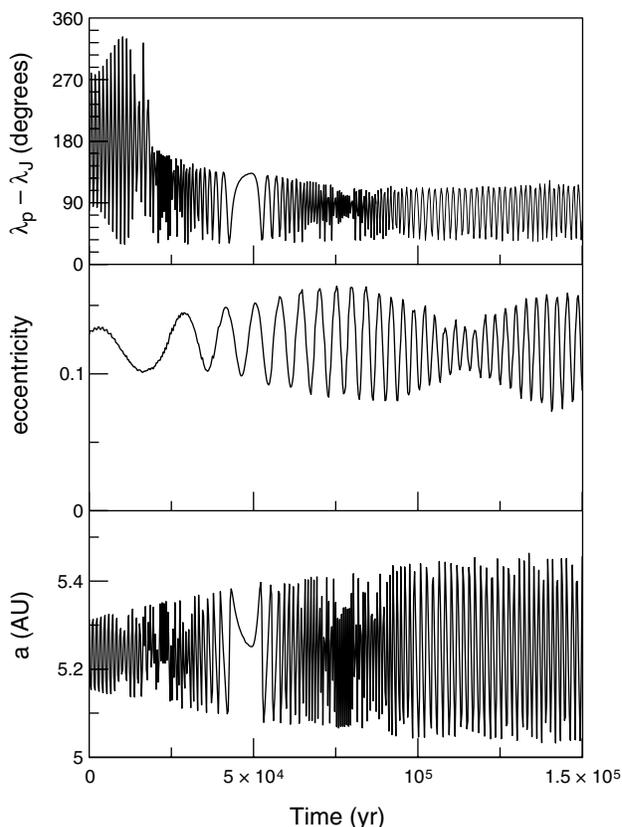


Fig. 5. Trapping of a planetesimal into a Trojan-type orbit by the mass growth of Jupiter. The planetesimal, originally in a horseshoe orbit with the protoplanet, starts to librate around L_4 as the mass of the planet grows. After the capture, the libration amplitude of the critical argument of the Trojan resonance $\lambda_p - \lambda_J$ (top plot), with λ_p and λ_J mean longitudes of the planetesimal and of Jupiter respectively, is further reduced by the increase in mass of the planet. The oscillation of the semimajor axis (bottom plot) widens as D decreases, while the average eccentricity is practically unchanged (middle plot).

mately equal size populations around L_4 and L_5 . The symmetry of the situation is, however, broken for small planetesimals if we consider the effects of the nebular gas drag. *Peale* (1993) showed by numerical simulation that the orbits around L_5 are more stable than those around L_4 for bodies smaller than ~ 100 m in diameter when Jupiter is on an eccentric orbit. *Marzari and Scholl* (1998a) found a similar effect on the trapping probability and explained this phenomenon by introducing in the equation for the Trojan motion derived by *Érdi* (1978, 1979, 1981) a term simulating the perturbation due to gas drag. This primordial asymmetry between the small size end of the two swarms might have been slowly erased by the subsequent collisional evolution of Trojan asteroids (*Marzari et al.*, 1997) so that at present it would be nearly undetectable.

Once trapped as a Trojan, the libration amplitude of a planetesimal continues to shrink under the influence of the growing Jupiter, as can be seen in Fig. 5. *Marzari and Scholl* (1998b) have shown numerically how Jupiter's growth from

a mass of about $10 M_{\oplus}$ to its present mass would cause a reduction of the libration amplitude of a Trojan orbit to $\sim 40\%$ of its original size. This behavior, independent of the capture scenario, is relevant since it leads to a substantial stabilization of Trojan orbits when the planet gains mass. *Fleming and Hamilton* (2000) confirmed the results of *Marzari and Scholl* (1998b) and gave an analytical interpretation of this mechanism on the basis of a simplified Hamiltonian approach that describes the basic librational motion of a body in a 1:1 resonance. They derived a relation between the mass M_j , semimajor axis a_j of the planet, and the libration amplitude A of a Trojan orbit, defined as the difference between the maximum and minimum values of the critical argument, when the mass of the planet M_j changes sufficiently slowly (“adiabatically”)

$$\frac{A_f}{A_i} = \left(\frac{M_{jf}}{M_{ji}} \right)^{-1/4} \left(\frac{a_{jf}}{a_{ji}} \right)^{-1/4} \quad (1)$$

The subscripts i and f indicate the initial and final value of the variables. When i corresponds to the stage with a proto-Jupiter of $10 M_{\oplus}$ and j to the final stage with Jupiter fully formed, this equation predicts a reduction of $\sim 42\%$ between A_f and A_i in agreement with the numerical results. According to the above equation, radial migration of the planet toward the Sun caused by tidal interactions with the gas and planetesimal disk and, to a lesser extent, by the mass-accretion process, would not significantly affect the stability of Trojan orbits. A relative change of 20% in the semimajor axis of Jupiter (i.e., ~ 1 AU) would imply that the libration amplitude A is increased by only $\sim 4\%$. However, Jupiter and Saturn during their migration may have crossed temporary orbital configurations, where the combination of gravitational perturbations can destabilize Trojan-type orbits of either planet as argued by *Gomes* (1998). In particular, mean-motion resonances between the two planets are very effective and can cause a strong instability of tadpole orbits (*Michtchenko and Ferraz-Mello*, 2001).

The capture efficiency of the mass growth mechanism is high: Between 40% and 50% of the planetesimals populating a ring extending 0.4 AU around Jupiter’s orbit are trapped as new Trojans during the formation of Jupiter (*Marzari and Scholl*, 1998a). The total mass of Trojan precursors, assuming a surface density of planetesimals at Jupiter’s orbit of $\sigma = 10 \text{ g cm}^{-2}$ as in *Pollack et al.* (1996), should be about $5 M_{\oplus}$, much larger than the estimated present mass of $\sim 10^{-4} M_{\oplus}$ (*Jewitt et al.*, 2000). However, a simple estimate based on the value of σ does not take into account the density decrease in the planet feeding zone by accretion onto the core and by gravitational scattering of neighboring planetesimals by the protoplanet. This might have opened a gap around the protoplanet orbit in spite of the supply of new planetesimals due to (1) the expansion of the feeding zone caused by the planet growth, (2) gas drag orbital decay of small planetesimals into the feeding zone, and (3) collisional injection of planetesimals close to the borders of the feeding zone. A further significant re-

duction of the Trojan population occurred throughout the history of the solar system because of collisional evolution (*Marzari et al.*, 1997; *Davis et al.*, 2002) and dynamical outflow (*Levison et al.*, 1997). The latter loss mechanism might have been particularly effective if most of the Trojans captured by the mass growth mechanism had libration amplitudes extending beyond the stability zone.

This seems to be the case of the Trojan precursors, most of which are trapped by the mass growth mechanism in tadpole orbits with large libration amplitudes, in spite of postcapture shrinkage due to mass growth. A process that can reconcile the present population with a primordial population of large librators is, again, the collisional evolution. With a Monte Carlo method, *Marzari and Scholl* (1998b) showed that collisions, possibly with other local planetesimals before they were scattered by Jupiter, can alter significantly the distribution of the libration amplitudes by injecting initially large librators into more stable, small librating orbits. All the large librators would subsequently become unstable and would be ejected from the Trojan swarms, eventually becoming short-period comets (*Marzari et al.*, 1995, 1997; *Levison et al.*, 1997).

The effects of Jupiter’s mass growth on the eccentricities and inclinations of the Trojan precursors are essentially negligible (*Marzari and Scholl*, 1998a,b; *Fleming and Hamilton*, 2000). Hence, the present orbital distribution of the two swarms should reflect the original distribution of the planetesimals in the proto-Jupiter accretion zone. Large eccentricities are expected, since a massive protoplanet would stir up the random velocities of the neighboring planetesimals. However, it is difficult to explain the high orbital inclinations observed among Jupiter Trojans. Three mechanisms have been devised that might possibly have excited high inclinations. The first is related to the capture process: Planetesimals with large eccentricity and trapped in a Kozai resonance with the protoplanet can be captured by the mass growth mechanism while they are in the high-inclination stage (*Marzari and Scholl*, 1998b). This dynamical path can explain inclinations up to 10° assuming reasonable values of orbital eccentricities for the planetesimals. The second mechanism is described in *Marzari and Scholl* (2000) and is based on the synergy between the v_{16} secular resonance and collisions. The resonance would have excited Trojans with a libration amplitude of $\sim 60^\circ$ to high-inclination orbits, as suggested for the first time by *Yoder* (1979). Subsequently, collisions would have injected a fraction of these bodies into low-libration orbits. Recently, *Marzari and Scholl* (2000) have demonstrated that v_{16} not only covers a significant portion of the phase space populated by Jupiter Trojans, larger than previously expected, but that it also extends its effects to initially low-inclination orbits. Figure 6 shows the evolution of the inclination, the critical angle of the Trojan resonance $\lambda_T - \lambda_j$, and the critical argument of the v_{16} secular resonance of an L_4 Trojan orbit with a starting inclination of 4° . After frequent resonance crossings, the asteroid reaches a final inclination of $\sim 30^\circ$. The third hypothesis is suggested by the work of *Petit et al.* (1999) on the excitation of the asteroid belt by large

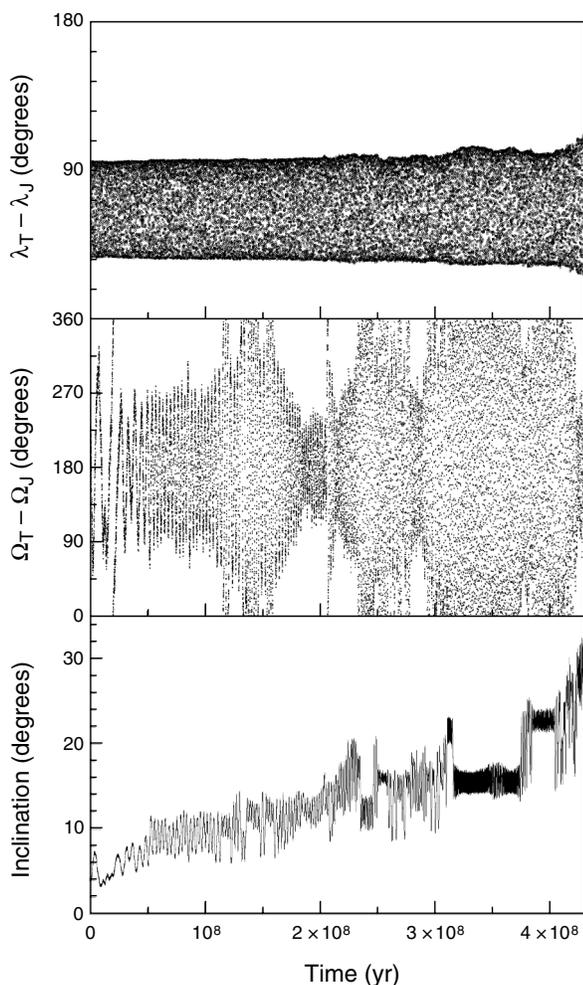


Fig. 6. Evolution of a Trojan orbit trapped in the ν_{16} secular resonance. The initial inclination of the body is $\sim 4^\circ$ and the libration amplitude D is $\sim 60^\circ$. The body has the inclination slowly pumped up by the resonant perturbations and it ends after 400 m.y. in a high-inclination orbit ($\sim 30^\circ$).

Uranus-Neptune scattered planetesimals. If large embryos, locally grown or coming from outside regions, were trapped in tadpole orbits in the final stage of the planet formation, they would have induced large inclinations by gravitationally scattering smaller Trojans, and, at the same time, would have possibly reduced the large populations trapped during the mass growth of the planet. The pitfall of this model is that it requires two almost-equal embryos to be trapped (one at L_4 and one at L_5), surviving on tadpole orbits for approximately the same time to produce equally stirred populations.

4.2. Saturn Trojans

The stages envisaged for the growth of Jupiter were presumably reproduced during the formation of Saturn, and, as a consequence, the planet should also have trapped local planetesimals as Trojans. However, even high-resolution surveys (Chen *et al.*, 1997) have so far failed to detect any

Saturn Trojan. A critical aspect of the Trojan-type orbits of Saturn is that they are mostly unstable. Early numerical experiments by Everhart (1973) suggested lifetimes for Saturn Trojans longer than 1 m.y. Subsequent numerical integrations by Holman and Wisdom (1993) over timescales of 20 m.y. found only small stability areas in the phase space characterized by large libration amplitudes. A more recent work by de la Barre *et al.* (1996) showed that on longer timespans (400 m.y.), only a few orbits remain stable, and that these are in a peculiar ω -librating state. The cause of instability was attributed by many authors to the close 5:2 resonance with Jupiter known as the “great inequality” (Innanen and Mikkola, 1989; Mikkola and Innanen, 1992; de la Barre *et al.*, 1996). By investigating the behavior of the critical argument of the most-relevant secular resonances, Marzari and Scholl (2000) found that the mixed secular resonance $2\tilde{\omega}_S - \tilde{\omega}_J - \tilde{\omega}_T$, where $\tilde{\omega}_S$, $\tilde{\omega}_J$, and $\tilde{\omega}_T$ are the longitudes of perihelion of Saturn, Jupiter, and

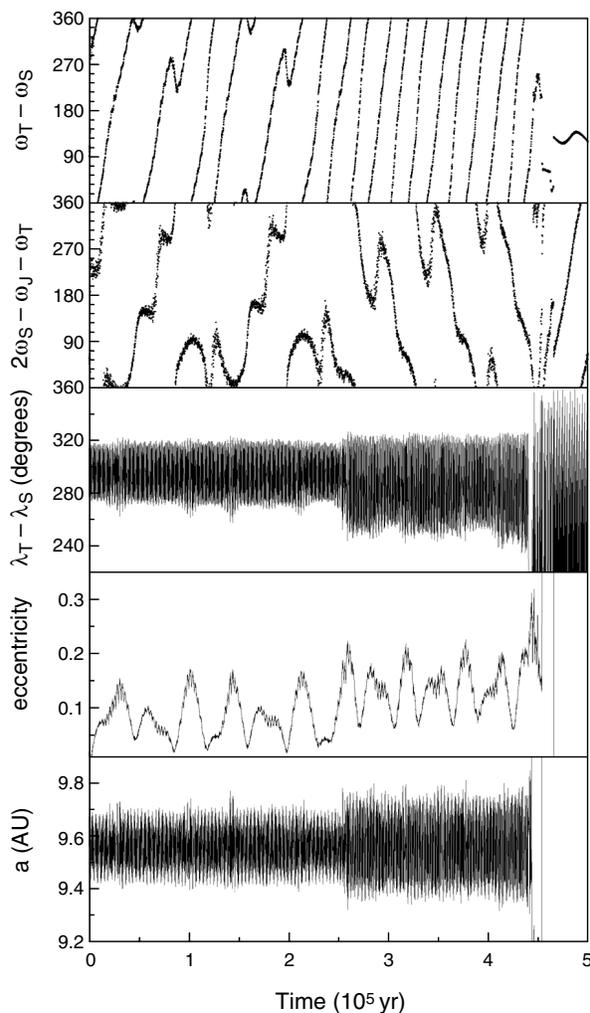


Fig. 7. Instability of a Saturn Trojan orbit caused by the crossing of the mixed secular resonance $2\tilde{\omega}_S - \tilde{\omega}_J - \tilde{\omega}_T$. The crossing occurs at about 2.5×10^5 yr and it is marked by the reversion of the circulation trend of the critical argument and by a sudden jump in both the semimajor axis oscillation amplitude and libration amplitude D .

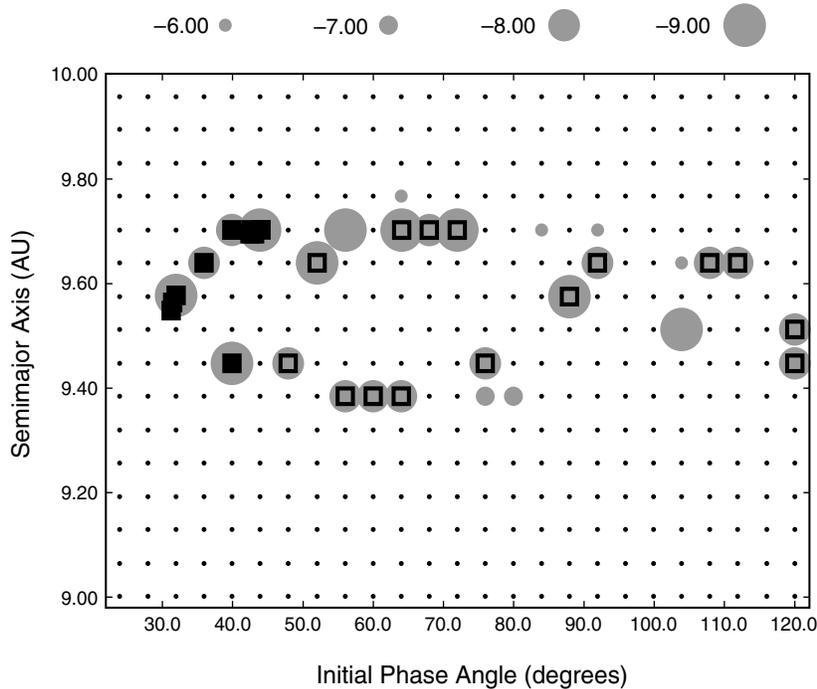


Fig. 8. Stability niches for Saturn Trojans adapted from *Melita and Brunini (2001)*. In the plane of the initial semimajor axis and phase angle $\lambda_T - \lambda_S$, the stable niches are shown as filled circles with radius proportional to the diffusion coefficient D that is an indicator of the decimal logarithm of the inverse of the dynamical lifetime of the orbit. Solid squares indicate bodies whose orbits have been integrated with starting conditions within the niche and that are stable over 1 b.y. Blank squares are orbits with stability timescales less than 1 b.y.

the Trojan respectively, also has a dominant role in destabilizing Saturn Trojans. Figure 7 shows the evolution of a test Saturn Trojan belonging to the L_5 swarm. After 2.5×10^5 yr, the body crosses the mixed secular resonance and becomes unstable until it is ejected by a close encounter with the planet. The resonance crossing is clearly marked by the change of the circulation direction and a corresponding step in the libration amplitude, semimajor axis oscillation amplitude, and eccentricity. The ν_6 secular resonance seems also to be involved in the escape of possible Saturn Trojans from the 1:1 resonance. The most appealing scenario of the Saturn Trojan evolution is that, after capture by the mass growth mechanism, the low-amplitude librators ($A < 50^\circ$), if any, escaped on a short timescale due to the perturbing effects of the 5:2 near-resonance. On longer timescales, the mixed secular resonance and, eventually, the ν_6 secular resonance depleted the regions at high-libration amplitudes, leaving only a few bodies on the small stability niches found in the phase space by *Melita and Brunini (2001)* from an analysis of diffusion maps of the proper elements (Fig. 8). Their results are limited to orbits initially close to the orbital plane of Jupiter and should be extended to higher inclinations. There might, however, be an upper limit to the possible inclination of Saturn Trojans, according to *Teger (1999)*: Beyond 30° no orbit would be stable. However, the work by *Teger* is based on a few short-term numerical integrations and needs to be confirmed by further simulations.

4.3. Other Trojans

More uncertain is the possibility that planetesimals were trapped as Trojans during the growth of both Uranus and Neptune. According to *Holman and Wisdom (1993)*, the two

planets have stable regions around both the equilateral points where Trojans could have been captured, but their study was limited in time (20 m.y.). *Weissman and Levison (1997)* performed a numerical exploration of the Trojan orbits for Neptune over 4 b.y., the age of the solar system, and found a small stability region at libration amplitudes lower than 60° and proper eccentricities smaller than 0.05. Their study is, however, limited to 70 test bodies and the timestep of the numerical integration was 1 yr, long compared to the orbital period of Jupiter. Additional tests are needed to assess the possible existence of long-living Trojan orbits at the Lagrangian points of Uranus and Neptune. Even the formation process of the two smaller ice giants is not well understood at present. It has been proposed that the proto-Uranus and proto-Neptune formed via the core-accretion model on a timescale of $\sim 10^7$ yr (*Bryden et al., 2000*). Close encounters between leftover planetesimals and the core would have led to orbital drift and migration of the two planets into outer orbits. In this scenario, the capture of local planetesimals as Trojans may have occurred in the final phase of growth of the two planets, but it is crucial to understand whether Trojan orbits would have been stable during the subsequent phase of orbital migration. *Gomes (1998)* and *Fleming and Hamilton (2000)* have shown that a low or moderate migration would not remove the Trojan populations of the two planets.

According to a recent theory by *Thommes et al. (1999)*, Uranus and Neptune would not have formed in their present location but in the region between Jupiter and Saturn at the same time as the two giant planets. When Jupiter and, maybe, Saturn accreted the nebular gas, proto-Uranus and proto-Neptune would have been gravitationally scattered outward. The subsequent evolution caused by gravitational interactions with the planetesimals of the outer disk of the

protosolar nebula would have pushed them to their present orbits. In this context, primordial Trojans could not have survived the phase of scattering by Jupiter.

Surprisingly enough, the only known Trojan asteroids in the solar systems not related to Jupiter are martian Trojans. (5261) Eureka, 1998 VF31, 2001 DH47, 2001 FG24, and 2001 FR127 orbit around the L_5 Lagrangian point of Mars, while 1997 U37 orbits around L_4 . The stability of the orbits of martian Trojans has been investigated by *Mikkola and Innanen* (1994) and *Tabachnik and Evans* (1999, 2000). Via numerical integrations, they showed that there are stable zones located at inclinations between approximately 15° and 40° with respect to Mars' orbit and at libration amplitudes between 0° and 45° . According to *Mikkola and Innanen* (1994) and *Tabachnik and Evans* (2000), the destabilization of low-inclination martian Trojans is mainly due to a secular resonance with the perihelion of Jupiter $3\Omega_T + 2\tilde{\omega}_T - \tilde{\omega}_J$, with the subscripts T and J referring to Trojan and Jupiter respectively, while additional instability for inclinations around 30° is caused by the secular resonance $\tilde{\omega}_T - \tilde{\omega}_J$. Unfortunately, the numerical investigations performed so far only cover timescales up to some 10^7 yr, since a short timestep must be used in the integration to follow accurately the orbits of the terrestrial planets. This timespan is too short to allow even speculations on the stability of martian Trojans over the age of the solar system. Certainly, if they are primordial, they were not trapped by the mass growth mechanism, since the mass-accretion rate of Mars was very slow. It is possible that they were injected into Trojan orbits by collisions in the highly violent environment that accompanied the final stage of planetary formation. A giant impact on Mars might have created a ring of fragments surrounding the orbit of the planet, from which a few were captured as Trojans. Alternatively, they may have been trapped in temporary tadpole orbits in relatively recent times, and it is possible they will become unstable within timescales of some million years. Even for Earth and Venus, *Tabachnik and Evans* (2000) found rich families of stable tadpole orbits. Trojan orbits of Venus with mean eccentricity ~ 0.03 and inclinations lower than 16° survive for about 25 m.y., while in the case of Earth there appear to be two extended bands of stability over 50 m.y.: for inclinations lower than 16° , as for Venus, and for inclinations between 24° and 34° . Mercury seems to be unable to retain any Trojan orbit for more than 100 m.y.

Many of the resonant structures we find in our solar system are shared in miniature by the Saturn system. Among the satellites, there are two examples of Trojan configurations discovered during the Pioneer 11 and Voyager 1 and 2 flybys to Saturn: Helene shares the orbit of Dione 60° ahead of its larger companion, while Telesto and Calypso are the leading and trailing Trojans of Tethys respectively. What is the origin of these Trojan satellites? It is believed (*Greenberg*, 1984; *Pollack and Consolmagno*, 1984) that during the early history of Saturn the inner satellites were involved in repeated collisional disruption and reaccretion. After the breakup of a satellite, a ring of debris is formed

around the orbit of the largest remnant. Most of the debris reaccumulates onto it on timescales of a few thousand years (*Burns et al.*, 1984; *Stevenson et al.*, 1986; *Marzari et al.*, 1998), but some large fragments may be trapped in Trojan orbits and protected from collisions with the larger coorbiting body.

5. OPEN PROBLEMS AND PERSPECTIVES

Future observational surveys should concentrate on the regions around the Lagrangian points of Saturn, Uranus, and Neptune, and of Earth and Venus. Dynamical models do not exclude the existence of Trojan swarms for these planets. Detection or evidence of absence of Trojans around Uranus and Neptune may contribute to the still-open problem of planet formation and the Trojan capture process. A collisional origin seems to be the most appealing interpretation of the martian Trojans, although a temporary capture event may be another possible explanation.

Detection of smaller objects in the Jupiter Trojan clouds is necessary to lower the limit of completeness, which is at present around 70–80 km in diameter. It would then be possible to confirm the existence of families in the Trojan clouds (*Milani*, 1993; *Beaugè and Roig*, 2001) and to estimate their size distribution, giving stringent constraints on the collisional evolution of the entire Trojan population. The morphology of the families may also shed new light on the physical properties of the Trojans. Comparing kilometer-sized Trojans at L_4 and L_5 might reveal the asymmetry predicted theoretically in presence of gas drag during capture or caused by planetary migration.

The origin of the comparatively high inclination of the Trojans is another open problem presumably related to the cosmogony of the planetary system. The small libration amplitudes of Trojans are an enigma in the context of the capture hypothesis by a growing Jupiter. Further investigation on the effects of collisions among Trojans during Jupiter growth might help to understand this problem.

The stability of Trojans around all planets and satellites over the age of the solar system is not yet fully explored. Jupiter Trojans are the most intensively studied population, with integrations covering at least 10^9 yr. Similar long-term simulations covering a wide range of orbital parameters are still missing for the other outer planets. Furthermore, additional progress on our understanding of the long-term stability for the tadpole orbits of the terrestrial planets is needed to complete our general view of the Trojan behavior.

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