

Cometary Orbit Determination and Nongravitational Forces

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The accuracies of the orbits and ephemerides for active comets are most often limited by imperfectly modeled rocket-like accelerations experienced by active comets as a result of the outgassing cometary nucleus near perihelion. The standard nongravitational acceleration model proposed by *Marsden et al. (1973)* has been updated by allowing the nucleus outgassing to act asymmetrically with respect to perihelion, providing for time-dependent effects through the precession of the cometary nucleus and accounting for the outgassing from discrete surface areas on a rotating nucleus. While the most accurate nongravitational models will likely require a detailed *a priori* knowledge of a comet's surface activity and rotation characteristics, it is becoming possible to use only astrometric data to actually solve for some of the parameters that describe the comet's outgassing and rotational characteristics.

1. ASTROMETRY AND THE ORBIT DETERMINATION PROCESS FOR COMETS

The accuracy of the orbit-determination process for comets depends on a number of factors, including the accuracy of the astrometric data, the interval over which these data are available, the extent to which the position of the photometric image represents a comet's true center-of-mass, the incorporation of planetary and asteroidal perturbations, the accuracy of the numerical integration and differential correction processes, and especially the correct modeling of the accelerations due to the comet's outgassing (i.e., nongravitational effects). These latter nongravitational effects, which are due largely to the recoil effect from the vaporization of water ices from the cometary nucleus, will be discussed in later sections.

1.1. Astrometric Data

Optical astrometric positions for celestial objects most often consist of pairs of right ascension and declination values for a given time; these positions are determined using measured offsets from neighboring stars whose positions have been accurately determined. Thus an object's astrometric accuracy depends upon the measurement technique and the accuracy of the reference star catalog employed. For modern astrometric positions, the highest accuracy is achieved when the star catalog has been reduced with respect to Hipparcos reference star positions. One example of this type of catalog is the star catalog (UCAC2) from the United States Naval Observatory (USNO), which includes more than 60,000,000 stars, thus providing the high star density necessary for most astrometric reductions. This cata-

log goes as deep as magnitude 16 and has an astrometric accuracy better than 0.1 arcsec (<http://ad.usno.navy.mil>).

Since cometary observations are influenced by random errors, astrometric observations should be selected and weighted before they are used for orbit improvement. Bielicki has elaborated objective criteria for the selection of cometary observations on the assumption that the distribution of observational errors is normal (*Bielicki and Sitarski, 1991*). Bielicki applied the process of selecting and weighting the observations to those observations belonging to one apparition of the comet and thus he could obtain a value of the mean residual for observations of this apparition. Using the values of the mean residuals found for each of several apparitions, one can calculate in advance the mean residual *a priori*, μ_{apr} , which can then be compared with the mean residual *a posteriori* μ_{apo} that results from the model of the comet's motion when linking a number of apparitions. A mathematical model of the comet's motion used for the linkage of several apparitions may be regarded as satisfactory if $\mu_{\text{apo}} \approx \mu_{\text{apr}}$.

The most powerful astrometric optical positional data for improving an object's orbit are taken when the object is closest to Earth. Unfortunately, active comets have substantial coma, or atmospheres, in the inner solar system and these atmospheres have optical depths that prevent a direct observation of the comet's nucleus. As a result, the observer normally must assume that the object's photometric center (center-of-light) is also the comet's center-of-mass, and this is not often the case. While linking a long data interval for Comet 26P/Grigg-Skjellerup, *Sitarski (1991)* found it necessary to adjust the observations using a radial offset that varied as the inverse cube of the heliocentric distance. *Yeomans (1994)* outlined a procedure whereby this center-of-mass/center-of-light offset at 1 AU from the Sun was

included within the orbit-determination process. This offset (S_o) is assumed to vary along the comet-Sun line with an inverse square dependence on heliocentric distance. For Comet Halley, the solution for the offset at 1 AU was about 850 km. In most cases, this offset is not easily determined from the available astrometric data. Of course, rather than trying to account for inaccurate observations of a comet's center-of-mass, it would be preferable if groundbased optical observations actually observed the comet's true position in the first place. In this regard, *Chesley et al.* (2001) compared the groundbased optical astrometric positions for Comet 19P/Borrelly in September 2001 with spacebased observations of the comet's true nucleus taken by the *Deep Space 1*'s optical imaging cameras prior to its close flyby on September 22, 2001. They concluded that the true nucleus position of the comet was more accurately defined by groundbased observations if the brightest pixel were used rather than positions based upon a best-fitting two-dimensional Gaussian fit to the photometric image. That is, the comet's center-of-light should not be assumed to be the comet's center-of-mass. We recommend that observers use the "brightest pixel" technique for reporting cometary astrometric observations.

While the extraordinary power of radar observations to refine an asteroid's orbit has been documented several times (for example, see *Yeomans and Chodas*, 1987; *Yeomans et al.*, 1992), radar observations are available for only five comets (http://ssd.jpl.nasa.gov/radar_data.html). This paucity of radar observations for comets is primarily due to the infrequency with which comets pass close enough to Earth.

1.2. Cometary Equations of Motion and the Orbit Determination Process

The orbit-determination process is a linearized, weighted least-squares estimation algorithm, in which astrometric observations are used to improve an existing orbit (*Lawson and Hanson*, 1974). At each time step of the numerical integration process, the dynamic model should include gravitational perturbations due to the planets and the larger minor planets, relativistic effects, and the accelerations due to the nongravitational effects. The partial derivatives necessary for adjusting the initial conditions should be integrated along with the object's equations of motion. The cometary equations of motion can be written

$$\begin{aligned} \frac{d^2\mathbf{r}}{dt^2} = & -\frac{k^2\mathbf{r}}{r^3} + k^2 \sum_j m_j \left[\frac{(\mathbf{r}_j - \mathbf{r})}{|\mathbf{r}_j - \mathbf{r}|^3} - \frac{\mathbf{r}_j}{r_j^3} \right] \\ & + \frac{k^2}{c^2 r^3} \left[\frac{4k^2\mathbf{r}}{r} - (\dot{\mathbf{r}} \cdot \dot{\mathbf{r}})\mathbf{r} + 4(\mathbf{r} \cdot \dot{\mathbf{r}})\dot{\mathbf{r}} \right] \quad (1) \\ & + A_1 g(\mathbf{r})\mathbf{r}/r + A_2 g(\mathbf{r})\mathbf{t} + A_3 g(\mathbf{r})\mathbf{n} \end{aligned}$$

The first term on the righthand side of the equation is the solar acceleration where the Sun's mass has been taken as

unity (*Marsden et al.*, 1973). The second term represents both the direct effects of the perturbing bodies on the comet and the indirect effects of the perturbing bodies upon the Sun. The perturbing bodies are often taken to be the planets and the three most massive minor planets. The second line of the equation represents one form of the relativistic effects (*Anderson et al.*, 1975) that should be included because for many objects with small semimajor axes and large eccentricities, these effects introduce a nonnegligible radial acceleration toward the Sun (*Sitariski*, 1983, 1992c). In addition, these effects are required to maintain consistency with the planetary ephemeris (*Shahid-Saless and Yeomans*, 1994). The accelerations are given in astronomical units/(ephemeris day)²; k is the Gaussian constant; m_j = the masses of the planets and Ceres, Pallas, and Vesta; \mathbf{r} and $r = |\mathbf{r}|$ are the heliocentric position vector and distance of the comet respectively; $r_j = |\mathbf{r}_j|$ are the planetary distances from the center of the Sun; and c is the speed of light in AU per day. The third line of this equation gives the standard-model expressions for the outgassing accelerations acting on the comet in the radial (Sun-comet), transverse, and normal directions. These so-called nongravitational effects are discussed in the following sections.

2. HISTORICAL INTRODUCTION TO NONGRAVITATIONAL EFFECTS

Comet 2P/Encke has played a central role in the historical evolution of ideas concerning the rocket-like thrusting of an outgassing cometary nucleus and the attempts to model these so-called nongravitational accelerations. First discovered by Pierre Mechain in 1786, Comet Encke was rediscovered by Caroline Herschel in 1795 and discovered yet again by Jean-Louis Pons in 1805 and in 1818. Johann Encke provided the numerical computations to show that the comets discovered in 1786, 1795, 1805, and 1818 were one and the same object returning to perihelion at 3.3-year intervals. After noting that the comet returned to perihelion a few hours earlier than his predictions, *Encke* (1823) postulated that the comet moved under the influence of a resisting medium that he envisaged as an extension of the Sun's atmosphere or the debris of cometary and planetary atmospheres remaining in space. Encke's resisting medium allowed him to successfully predict the perihelion returns for the comet between 1825 and 1858.

Although the resisting medium theory seemed to be the contemporary consensus opinion, *Bessel* (1836) noted that a comet expelling material in a radial sunward direction would suffer a recoil force, and if the expulsion of material did not take place symmetrically with respect to perihelion, there would be a shortening or lengthening of the comet's period depending on whether the comet expelled more material before or after perihelion (see equation (2)). Although Bessel did not identify the physical mechanism with water vaporization from the nucleus, his basic concept of cometary nongravitational forces would ultimately prove to be correct.

In the second half of the nineteenth century, the motion of Comet Encke did not seem to behave in strict accordance with Encke's resisting medium hypothesis, and several alternate mechanisms were introduced to explain the phenomena (see Yeomans, 1991; Sekanina, 1991a). The final blows to the resisting medium came when Kamienski (1933) and Recht (1940) found uniform decreases in the mean motion of period Comets 14P/Wolf and 6P/d'Arrest respectively. With the discovery of mean motions that decreased with time, as well as some that increased, a successful hypothesis had to explain both phenomena. A resisting medium could only cause the latter phenomena.

The breakthrough work that allowed a proper modeling of the nongravitational effects on comets came with Whipple's introduction of his icy conglomerate model for the cometary nucleus (Whipple, 1950, 1951). Part of his motivation for this model was to explain the so-called nongravitational accelerations that were evident in the motion of Comet Encke and many other active periodic comets. That is, even after all the gravitational perturbations of the planets were taken into account, the observations of many active comets could not be well represented without the introduction of additional so-called nongravitational effects into the dynamical model. These effects are brought about by cometary activity when the sublimating ices transfer momentum to the nucleus. The nongravitational effects become most evident as deviations in a comet's perihelion passage when compared with a purely gravitational orbit, and these deviations are typically a fraction of a day per apparition, although for Comet 1P/Halley it is as large as four days.

2.1. Symmetrical Nongravitational Force Model

Whipple noted that for an active, rotating, icy cometary nucleus, a thermal lag between cometary noon and the time of maximum outgassing would introduce a transverse acceleration into a comet's motion. In an attempt to model these effects, Marsden (1968, 1969) first introduced a semi-empirical nongravitational acceleration model using what are now termed Style I nongravitational parameters. Style II parameters were added when Marsden et al. (1973) introduced what has become the standard, or symmetric, nongravitational acceleration model for cometary motions; a rotating cometary nucleus is assumed to undergo vaporization from water ice that acts symmetrically with respect to perihelion. That is, at the same heliocentric distance before and after perihelion, the cometary nucleus experiences the same nongravitational acceleration. The expressions for these nongravitational accelerations can be written

$$A_1 g(r) \mathbf{r}/r + A_2 g(r) \mathbf{t}$$

where

$$g(r) = \alpha (r/r_0)^{-m} (1 + (r/r_0)^n)^{-k}$$

The scale distance r_0 is the heliocentric distance inside which

the bulk of solar insolation goes to sublimating the comet's ices. For water ice, $r_0 = 2.808$ AU and the normalizing constant $\alpha = 0.111262$. The exponents m , n , and k equal 2.15, 5.093, and 4.6142 respectively. The nongravitational acceleration is represented by a radial term, $A_1 g(r)$, and a transverse term, $A_2 g(r)$, in the equations of motion. The radial unit vector (\mathbf{r}/r) is defined outward along the Sun-comet line, while the transverse unit vector (\mathbf{t}) is directed normal to \mathbf{r}/r , in the orbit plane, and in the general direction of the comet's motion. An acceleration component normal to the orbit plane, $A_3 g(r)$, is also present for most active comets, but its periodic nature often makes it difficult to determine because we are usually solving for an average nongravitational acceleration effect over three or more apparitions. If the comet's nucleus were not rotating, the outgassing in this model would always be toward the Sun and the resulting nongravitational acceleration would act only in the antisolar direction. The rotation of the nucleus, however, coupled with a thermal lag angle (η) between the nucleus subsolar point and the point on the nucleus where there is maximum outgassing, introduces a transverse acceleration component in either the direction of the comet's motion or contrary to it — depending upon the nucleus rotation direction.

Equation (2) represents the time derivative of the comet's orbital semimajor axis (a) as a result of radial and transverse perturbing accelerations (R_p , T_p)

$$\frac{da}{dt} = \frac{2[(e \sin v)R_p + (p/r)T_p]}{n\sqrt{1 - e^2}} \quad (2)$$

In this equation, n , e , v , and r denote, respectively, the orbital mean motion, eccentricity, true anomaly, and the comet's heliocentric distance, while p is the orbital semilatus rectum, $a(1 - e^2)$.

Because of the thermal lag angle, a comet in direct rotation will have a positive transverse nongravitational acceleration component, and from equation (2), it is apparent that the comet's orbital semimajor axis will increase with time (its orbital energy will increase). Likewise, a comet in retrograde rotation will be acted upon by a negative T_p and its semimajor axis will decrease with time. Because the nongravitational acceleration is assumed to act symmetrically with respect to perihelion, the time-averaged effect of the periodic radial acceleration cancels out.

When introducing the standard model, Marsden et al. (1973) included possible time dependences in the transverse parameter (A_2). Subsequently, however, the standard nongravitational acceleration model was most often used solving only for the constant radial and transverse parameters (A_1 and A_2) over data intervals short enough that neglected time dependences did not cause systematic trends in the residuals. Solutions for the nongravitational parameters usually require astrometric data from at least three apparitions, and one can empirically determine their change with time by comparing the nongravitational parameters determined from several of these three-apparition solutions.

The standard nongravitational parameters can be expressed as function of time by $A_i(t) = A \cdot C_i(t)$, $i = 1, 2, 3$, where $A = (A_1^2 + A_2^2 + A_3^2)^{1/2}$, and $C_i(t)$ are direction cosines for the nongravitational force acting on the rotating cometary nucleus. The direction cosines C_i , derived by *Sekanina* (1981) have a form

$$\begin{aligned} C_1 &= \cos\eta + (1 - \cos\eta) \cdot \sin^2 I \cdot \sin^2 \lambda \\ C_2 &= \sin\eta \cdot \cos I + (1 - \cos\eta) \cdot \sin^2 I \cdot \sin\lambda \cdot \cos\lambda \\ C_3 &= [\sin\eta \cdot \cos\lambda - (1 - \cos\eta) \cdot \cos I \cdot \sin\lambda] \sin\lambda \end{aligned}$$

where η is the lag angle, I is the equatorial obliquity, $\lambda = v + \phi$, v is the true anomaly of the comet, ϕ = the cometocentric longitude of the Sun at perihelion; the time dependence of $C_i(t)$ is given by the true anomaly $v(t)$. Thus three parameters A_1, A_2, A_3 can be replaced by four parameters A, η, I, ϕ , which should be determined along with the corrections to the six orbital elements in the orbit-determination process. The angles η, I, ϕ describing the direction of the nongravitational force vector in orbital coordinates are presented in Fig. 1.

Usually the radial and transverse components of the nongravitational acceleration parameters (i.e., A_1 and A_2) are determined when investigating the motion of short-period comets. In some cases the parameter A_3 also has a meaningful contribution to the successful orbital solution. When investigating the nongravitational motion of comets using the parameters A, η, I, ϕ it is necessary to first determine values

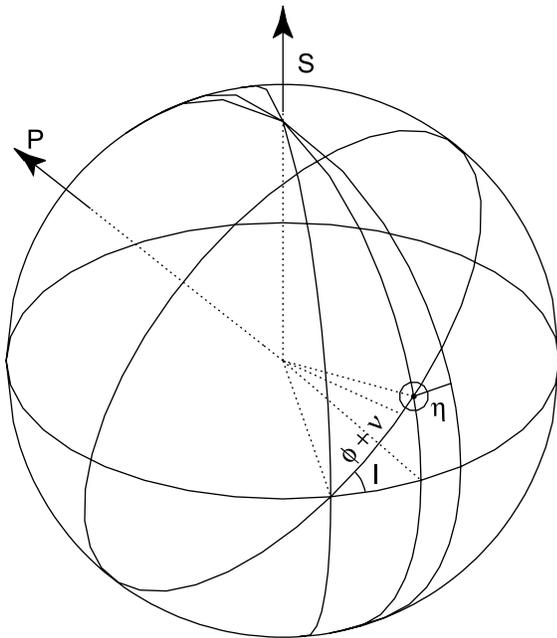


Fig. 1. Orientation of a spherical rotating nucleus. P is the northern orbital pole, S the northern pole of rotation. Angle I is the obliquity of the orbit plane to the comet equator, ϕ the solar longitude at perihelion, v the true anomaly. The maximum outgassing is shifted behind the subsolar meridian by the lag angle η .

of A_1, A_2, A_3 , even if A_3 is poorly determined, to estimate preliminary values of A, η, I, ϕ . These four parameters can then be improved together with six orbital elements by the least-squares correction process (*Sitarski, 1990*).

Largely because of its success in allowing accurate ephemeris predictions, the standard nongravitational force model has been in use for three decades. More recently, it has become understood that, while this model is often successful in representing the astrometric observational data and allowing the computation of accurate ephemeris predictions, the standard model does not provide a completely accurate representation of the actual processes taking place in the cometary nucleus.

Froeschlé and Rickman (1986) and *Rickman and Froeschlé* (1986) used theoretical calculations to examine the secular evolution of the nongravitational parameters as a function of the heliocentric distance for various kinds of short-period comets and different assumed thermal inertias. In general, their values of these parameters did not correspond to those computed from the standard model. In fact, there was such a wide variation in the respective behavior of the A_1, A_2 , and A_3 parameters that no generally applicable model for the nongravitational effects was suggested. They noted that improved models would likely have to include the effects of rotation pole orientation and seasonal heat flows.

For a more comprehensive outline of the earlier work on cometary nongravitational forces, the reader is directed to previously published reviews (*Marsden, 1968, 1969, 1985; Marsden et al., 1973; Yeomans, 1994*).

3. MODIFICATIONS TO THE STANDARD NONGRAVITATIONAL ACCELERATION MODEL

3.1. Normal Nongravitational Parameter A_3

A nongravitational acceleration acting normal to the comet's orbit plane will affect the longitude of the ascending node and the orbital inclination, but neither of these perturbations is secular. Since these perturbations are modulated by either sine or cosine functions of the true anomaly, much of the nongravitational perturbations upon the two orbital elements would average to zero even if the normal perturbative forces remain positive or negative throughout the orbit. *Sekanina* (1993c) noted that a meaningful solution for the normal nongravitational parameter (A_3) would be possible only for the special case where the perturbations upon the ascending node and the inclination yield a similar value of A_3 . Solutions for the A_3 parameter are often not useful but there are some notable exceptions. Meaningful values of A_3 were obtained for the 1808–1988 apparitions of 26P/Grigg-Skjellerup, the 1906–1991 apparitions of 97P/Metcalf-Brewington, and the 1958–1977 apparitions of 22P/Kopff (*Sitarski, 1991, 1992a; Rickman et al., 1987a*). Over the four returns of periodic Comet 71P/Clark, *Nakano* (1992) found a value for A_3 with a formal uncertainty of only 3% and *Sekanina* (1993c) suggested that for this comet, the

effective nongravitational perturbations on the ascending node and the orbital inclination are about equal. The rotation axis at perihelion is located in the plane defined by the Sun-comet line at perihelion and perpendicular to the comet's orbit plane. The rotation axis is inclined about 45° with respect to the orbit plane. As seen from the comet, the rotation axis at perihelion would be pointing in the general direction of the Sun but about 45° above it. In this configuration, the cometary outgassing produces a noncanceling, nongravitational thrust in a normal direction and hence a valid solution for A_3 . In general, the extreme values for A_3 are reached when the single active region is located at a rotation pole and when the obliquity of the orbit plane to the equatorial plane is near 50° or 130° . The principal active vent for Comet 19P/Borrelly is nearly aligned with the rotation pole and Chesley and Yeomans (2002) found that an inclusion of the A_3 parameter in their solutions for this comet's orbit significantly improved the prediction ephemeris for the successful flyby of the *Deep Space 1* spacecraft on September 22, 2001.

3.2. Asymmetric Nongravitational Force Models

From equation (2), we note that if the outgassing is asymmetric with respect to perihelion, a purely radial thrust can introduce a secular change in a comet's semimajor axis. This asymmetric nongravitational thrusting was first suggested by Bessel (1836), and the modern version of this idea has become known as the asymmetric nongravitational force model. Using the asymmetric light curve of Comet Halley, Yeomans (1984) attempted to employ the nucleus rotation parameters introduced by Sekanina (1981) to improve the nongravitational force model for Comet Halley. For this latter model, the outgassing was assumed to result from a subsolar active area, which is defined by its cometocentric solar longitude at perihelion (ϕ), the obliquity of the nucleus equator with respect to the orbit plane (I), and by a thermal lag angle (η) measured from the cometary subsolar point to the point of maximum outgassing. Although the optimum lag angle and obliquity turned out to be small, in apparent agreement with subsequent results, the orbital solution did not improve upon the standard nongravitational force model. In an influential work, Rickman (1986) pointed out that the asymmetric outgassing observed for Comets 1P/Halley and 22P/Kopff were the likely cause for the nongravitational effects noted for these comets.

Photometric observations of comets suggest that the brightness behaviour of comets near the Sun is sometimes strongly asymmetric with respect to perihelion. Festou et al. (1990) established an important statistical correlation between the nongravitational effects and the perihelion asymmetries of the gas production curves. A linear relationship was devised between the nongravitational perturbation of the orbital period, ΔP , and the difference, E , between the integrated gas production rate before and after perihelion. The light curve can, in most cases, be used to indicate whether orbital energy is being added or subtracted as a

result of the cometary outgassing. To include this fact in orbital computations, Sekanina (1988b) proposed taking into account the asymmetric outgassing of the comet's nucleus with respect to perihelion by replacing the $g(r)$ function with $g(r')$ where $r'(t) = r(t - \tau)$. Thus the maximum value of $g(r)$, describing the comet's maximum activity, is shifted by τ with respect to the perihelion time.

Using Sekanina's idea, Yeomans and Chodas (1989) analyzed the influence of asymmetric cometary activity with respect to perihelion upon the orbit-determination process for several comets. They examined the nongravitational motion of a number of periodic comets and found that the asymmetric nongravitational acceleration model usually improved an orbital solution when compared to the standard symmetric model. To find the best solution for an individual comet, the authors varied the value of τ in some ranges to obtain the best fit to the astrometric observations. However, τ may be treated as an additional nongravitational parameter that can be determined along with other parameters by the least-squares method.

Sitarski (1994a) elaborated upon the method of determining the values of τ as an additional parameter in the solution and for some comets repeated the cases done by Yeomans and Chodas (1989), confirming their conclusions. The asymmetric model of the comet's outgassing can considerably change the values of A_1 and A_2 computed using the standard symmetric model. In long intervals of time the shift τ may change its value and sign. In the case of Comet 6P/d'Arrest, it is about 40 d and is stable in many observed apparitions of the comet, but in the case of Comet 21P/Giacobini-Zinner $\tau = +14$ days during 1959–1973 and $\tau = -13$ d in 1972–1987. Similarly in the case of Comet 22P/Kopff, τ changed its sign within the interval 1906–1990. Assuming that $\tau = \tau_0 + d\tau/dt \cdot (t - t_0)$, where the new parameter $d\tau/dt$ denotes the daily change of τ , Sitarski found it possible to link all the apparitions of Comet Kopff over the interval 1906–1990, determining, along with the orbital elements, five nongravitational parameters: $A_1, A_2, A_3, \tau_0, d\tau/dt$.

Comet 6P/d'Arrest exhibits nearly time-independent nongravitational effects and its visual light curves show an extraordinary asymmetry with respect to perihelion with a peak about 40–50 d after perihelion. Replacement of the standard function $g(r)$ by the observed light curve led to a satisfactory orbital linkage of the positional observations over the intervals 1910–1989 (Szutowicz and Rickman, 1993) and 1851–1995 (Szutowicz, 1999b).

3.3. Linear Precession Model (Spherically Symmetric Nucleus)

One of the limitations of the standard nongravitational acceleration model is the lack of any long-term time dependence that would allow the nongravitational parameters (A_1, A_2, A_3) to change with time. Traditionally, this limitation has been handled by solving for the nongravitational parameters using limited sets of observations that cover consecutive time intervals. Three apparitions is usually the

minimum number of apparitions that allow a meaningful solution for the nongravitational parameters, so consecutive sets of three apparitions are often used for the orbit solutions when these nongravitational parameters are changing with time.

Partly to account for the long-term variations in the nongravitational parameters, *Whipple and Sekanina* (1979) and *Sekanina* (1981, 1984) introduced a model in which sublimating ices would provide a nongravitational force that was not aligned with the nucleus' center-of-mass and hence would (except for locations on the equator and poles) exert a precessional torque on the rotation pole. Coupled with a lag angle between cometary noon and the time of the peak outgassing, this model can introduce a time-varying nongravitational effect in a natural manner.

While the model was first applied to fit the secular decrease of the nongravitational acceleration of Comet 2P/Encke (*Whipple and Sekanina*, 1979), a slightly modified version of this model was developed later and applied to a number of other comets (*Sekanina*, 1984, 1985a,b; *Sekanina and Yeomans*, 1985).

Making the assumption that the spin axis of the rotating nucleus should precess, one can take into account linear terms of the precessional motion of the spin axis by assuming that $\phi(t) = \phi_0 + d\phi/dt \cdot (t - t_0)$ and $I(t) = I_0 + dI/dt \cdot (t - t_0)$, where $d\phi/dt$ and dI/dt are constant. Thus, six nongravitational parameters (A , η , I , ϕ , $d\phi/dt$, dI/dt) must be determined along with corrections to the six orbital elements within the orbit-determination process. On that assumption, *Bielicki and Sitarski* (1991) linked four apparitions of Comet 64P/Swift-Gehrels in the interval 1889–1991. For the standard model with A_1 , A_2 , A_3 they got the mean residual *a posteriori* $\mu_{\text{apo}} = 1''.94$ and for the model with linear precession $\mu_{\text{apo}} = 1''.75$, whereas the mean residual *a priori* $\mu_{\text{apr}} = 1''.67$. For Comet 26P/Grigg-Skjellerup, *Sitarski* (1991) obtained a satisfactory result linking 16 apparitions of the comet over the 1808–1988 interval using the model of a rotating cometary nucleus with linear precession, and also including a displacement of the photometric center from the center-of-mass. In that case $\mu_{\text{apr}} = 1''.31$ but $\mu_{\text{apo}} = 1''.60$.

3.4. Linear Precession Within an Asymmetric Nongravitational Acceleration Model

While using the linear precession model, one may also include a shift τ as an additional nongravitational parameter to account for an asymmetry of the comet's outgassing. This approach made it possible to link seven apparitions of Comet 45P/Honda-Mrkos-Pajdušáková over the interval 1948–1990 and the seven returns to the Sun of Comet 51P/Harrington in 1953–1994 (*Sitarski*, 1995, 1996). For Comet 22P/Kopff, *Sitarski* (1994b) found a successful solution when linking 13 apparitions of the comet over the 1906–1990 interval. However, he had to take into account a complicated function of the shift $\tau(t)$ that, according to the earlier investigations, had changed its sign within the interval considered (*Yeomans and Chodas*, 1989; *Sitarski*, 1994a). Thus,

ten nongravitational parameters (A , η , I_0 , dI/dt , ϕ_0 , $d\phi/dt$, τ_0 , $d\tau/dt$, t_1 , and t_2) were determined along with six corrections to the orbital elements. Among those parameters it was possible to determine two critical moments for $\tau(t)$:

$$t_1 = 1936.40 \pm 0.32 \quad \text{and} \quad t_2 = 1970.92 \pm 0.24$$

when

$$\tau_1 = +27.63 \text{ d for } t \leq t_1 \quad \text{and} \quad \tau_2 = -39.65 \text{ d for } t \geq t_2$$

but

$$\tau = \tau_0 + d\tau/dt \cdot (t - t_{\text{osc}})$$

when

$$t_1 < t < t_2 \quad \text{and} \quad t_0 = (-2.27 \pm 0.57) \text{ d}$$

for the osculating epoch $t_{\text{osc}} = 1951$ December 20.0.

There are examples of successful linkages of many apparitions of short-period comets using nongravitational models of motion that include the rotating and precessing cometary nucleus assuming a constant secular change of I and ϕ (*Sitarski*, 1991, 1994b). However, the linear model for the precession of the spin axis of the nucleus should be considered as only a first approximation and it is often unsuitable for extrapolations over long time intervals, especially when dI/dt is assumed to be constant.

The numerical models of nongravitational motion successfully linking many apparitions of short-period comets are verified if extrapolations of their motions can be used to recover the comets close to the predictions at subsequent apparitions.

3.5. Forced Precession Model (Nonspherical Nucleus)

Various interpretations have been proposed in an effort to understand long-term variations in nongravitational perturbations. One of them is based upon the concept of a forced precession of a nonspherical cometary nucleus, caused by torques associated with the jet force of outgassing. The phenomenon of the spin-axis precession of the cometary rotating nucleus could explain the variations of A_2 with time for Comet 22P/Kopff as found by *Yeomans* (1974), who investigated the long-term nongravitational motion of the comet. *Sekanina* (1984) used values of A_2 obtained by *Yeomans* (1974) to determine a forced precession model for the rotating oblate nucleus of the comet. Thus *Sekanina* (1984) showed a relationship between the physical parameters of the nucleus and its nongravitational behavior.

Assuming that the angles I and ϕ are functions of time as a result of the forced precession due to the asymmetric gas ejection, *Sekanina* (1984) derived formulae for changes of the spin-axis orientation of the cometary nucleus. The

following formulae for the time-dependence of I and ϕ were adopted for use in orbital computations (Królikowska et al., 1998a)

$$I = I_0 + \int_0^t dt \cdot \dot{\phi} \cdot \cos(\alpha + \eta)$$

$$\phi = \phi_0 - \int_0^t dt \cdot \dot{\phi} \cdot \sin(\alpha + \eta) / \sin I$$

$$\dot{\phi} = A \cdot f_p \cdot g(r) \cdot (2 - s) \cdot \sin \psi \cdot \cos \psi \cdot (1 - S_1 \sin^2 \psi)^{1/2} (1 - S \sin^2 \psi)^{-3/2}$$

where $\dot{\phi}$ is the precession rate of the spin axis, and ψ and α are the cometographic latitude and longitude of the sub-solar point respectively. They are given by $\sin \psi = \sin I \cdot \sin \lambda$, and $\tan \alpha = \tan \lambda \cdot \cos I$, respectively. S and S_1 are defined as $S = s \cdot (2 - s)$, $S_1 = S \cdot (2 - S)$, and s denotes the nucleus oblateness ($s = 1 - R_b/R_a$, where R_a and R_b are the equatorial and polar radii of the nucleus respectively).

The direction cosines $C_i(t)$ for $i = 1, 2, 3$ have a more complex form than those derived by Sekanina (1981) for the spherically symmetric rotating nucleus since they are modified by terms containing the oblateness s . Variations of I and ϕ depend on s and on the precession factor f_p , which is connected with the torque factor f_{tor} , introduced by Sekanina (1984), by the relation $f_p = s \cdot f_{\text{tor}}$. Preliminary estimates of A , η , I_0 , ϕ_0 have to be determined from the standard constant parameters A_i , while an initial estimate of the precession factor f_p can be determined by setting $f_p = 0$ and $s = 0$; the values of the six parameters A , η , I_0 , ϕ_0 , f_p , and s can then be determined along with the six corrections to the orbital elements in the iterative orbit improvement process. The time shift τ could also be included as an additional parameter taking the more universal function $g(r')$, $r'(t) = r(t - \tau)$ instead of the symmetric function $g(r)$.

Sekanina's forced precession model has been used for investigations into the long-term nongravitational motion of Comets 26P/Grigg-Skjellerup and 45P/Honda-Mrkos-Pajdušáková (Sitarski, 1992b, 1995). In both cases values of the oblateness were determined. It was found that $s = +0.437 \pm 0.014$ for Honda-Mrkos-Pajdušáková, and $s = -0.373 \pm 0.065$ for Grigg-Skjellerup. This implies that the nucleus of Honda-Mrkos-Pajdušáková is oblate but for Grigg-Skjellerup the nucleus is a prolate spheroid rotating around its longer axis. Solutions for the forced precession models can be extended to determine the time variations of the angles I and ϕ . Figure 2 presents plots of $I(t)$ for both comets. The noticeable peaks for Comet Grigg-Skjellerup correspond to rapid changes of $I(t)$ during its perihelion passages. The sudden changes of $I(t)$ for Comet Honda-Mrkos-Pajdušáková are due to close approaches of the comet to Jupiter (e.g., in March 1983 to within 0.111 AU). For Comet Honda-Mrkos-Pajdušáková, Sitarski (1995) compared two solutions for the rotating nucleus, with the linear precession and with the forced precession: Whereas the linear preces-

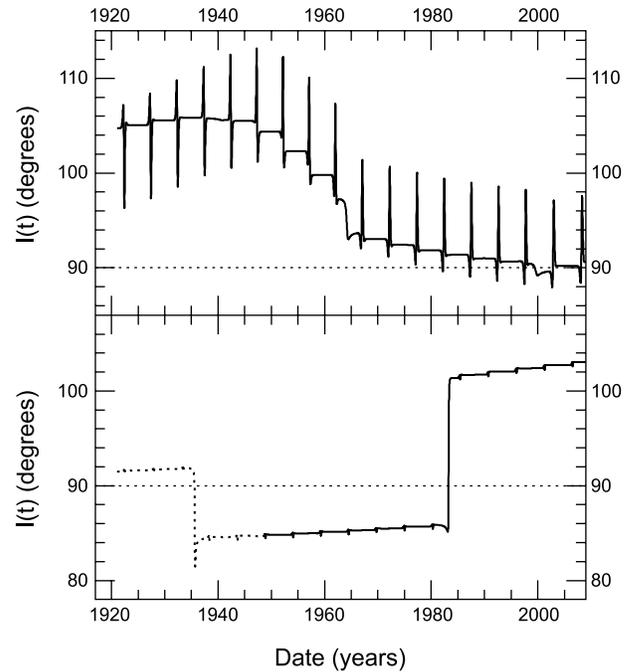


Fig. 2. Temporal variation of angle I due to the spin-axis forced precession of the comet's nucleus for two short-period comets: 26P/Grigg-Skjellerup (top), and 45P/Honda-Mrkos-Pajdušáková (bottom). The Grigg-Skjellerup forced precession model was constructed on the basis of all positional observations taken during 1922–1991 (Sitarski, 1992b), and the model for Comet Honda-Mrkos-Pajdušáková was derived from the time interval 1948–1990 (Sitarski, 1995). Dashed parts of the curve for Honda-Mrkos-Pajdušáková indicate variation of I before the comet's discovery. Dotted horizontal lines divide models with prograde rotation ($I < 90^\circ$) from models with retrograde rotation ($I > 90^\circ$). The same scale on the vertical axes allows one to compare the dramatically different amplitudes of the I variations due to the forced precession alone. Models of these two comets are very different. Forced precession models give a fast precession of the slightly prolate nucleus of Grigg-Skjellerup and a very slow precession for the considerably oblate nucleus of Honda-Mrkos-Pajdušáková (see also Table 1).

sion model could link all the observations with an rms residual of $2''.21$, the forced precession solution gave an improved rms residual of $1''.41$.

Królikowska et al. (1998b) applied the forced precession model of the rotating cometary nucleus to examine the nongravitational motion of three comets (30P/Reinmuth 1, 37P/Forbes, and 43P/Wolf-Harrington) over similar intervals of about 70 years. For Comets Reinmuth 1 and Wolf-Harrington they found satisfactory solutions for the pure forced precession model, but for Comet Forbes they had to include an additional parameter, the time shift $\tau = -9.75 \pm 0.76$ d. They concluded that the nucleus of Comet Reinmuth 1 is oblate ($s = +0.198 \pm 0.013$) while the nuclei of Comets Forbes and Wolf-Harrington are prolate ($s = -0.047 \pm 0.005$

TABLE 1. Orbital elements and nucleus physical parameters arising from the forced precession models for six short-period comets.

	Prolate Spheroid Models			Oblate Spheroid Models		
	37P/Forbes	26P/Grigg-Skjellerup	43P/Wolf-Harrington	46P/Wirtanen	21P/Giacobini-Zinner	45P/Honda-Mrkos-Pajdušáková
Orbital elements						
T	19990504.3758	20021129.7266	19970929.4341	20020826.6370	19981121.3214	20010415.4468
q	1.44602	1.11787	1.58183	1.056769	1.03375	0.53063
e	0.56811	0.63271	0.54398	0.65780	0.70647	0.82476
ω	310°.70	1°.62	187°.13	356°40	172°.54	323°.69
Ω	334°.37	211°.74	254°.76	82°.17	195°.40	91°.36
i	7°.16	22°.35	18°.51	11.74	31°.86	3°.89
Forced precession parameters						
A	0.5584	0.01746	+0.3634	0.6780	0.3944	0.3068
η	11°.89	28°.67	6°.36	15°.48	6°.53	12°.16
I_0	121°.65	95°.63	130°.32	145°.15	29°.81	102°.92
ϕ_0	12°.68	338°.20	136°.84	357°.58	260°.85	158°.96
f_p	-0.4623	-2.422	-0.3136	0.1234	0.3460	0.04210
s	-0.0589	-0.0588	-0.0451	0.1019	0.2926	0.4396
τ	-8.33	—	-11.69	-23.48	-49.38	-13.65

Orbital elements are given for the epoch of the last perihelion passage T; angular elements ω , Ω , and i refer to the equinox J2000.0. Nongravitational parameter A is in units of 10^{-8} AU/day², the precession factor f_p is in units of 10^7 d/AU, and the time shift τ is in days.

and $s = -0.195 \pm 0.032$ respectively). The orbital elements and nucleus physical parameters arising from the forced precession model for six short-period comets are presented in Table 1.

3.6. Erratic Comets

Comet 32P/Comas-Solá belongs to a group of comets dubbed “erratic” by *Marsden and Sekanina* (1971). For these comets, long-term nongravitational effects are irregular and sometimes their values change rapidly. Nongravitational effects in the motion of Comet 21P/Giacobini-Zinner show an irregular behavior in time if we observe values of the parameter A_2 as determined by linking three consecutive apparitions of the comet: In the period 1900–1999, A_2 changed its sign after 1959 (*Yeomans*, 1971). *Sekanina* (1985a) examined the nongravitational motion of Comet Giacobini-Zinner, trying to explain its erratic character by the precessional motion of the spin axis of the comet’s nucleus. However, he had to assume an unrealistically large oblateness for the nucleus equal to 0.88. *Sekanina* (1985b) found a similarly unacceptable solution for Comet Comas-Solá, although in this case the irregular behavior of the comet was less dramatic than for Comet Giacobini-Zinner.

Królikowska et al. (2001) investigated the motion of six erratic comets: 16P/Brooks 2, 21P/Giacobini-Zinner, 31P/Schwassmann-Wachmann 2, 32P/Comas Solá, 37P/Forbes, and 43P/Wolf-Harrington. They showed it was possible to link all apparitions of each comet on the basis of a forced precession model with physically reasonable parameters. Hence, one may conclude that the forced precession model of the rotating nonspherical cometary nucleus can explain

variations of the nongravitational effects observed in erratic comets (see Fig. 3). However, it was sometimes necessary to introduce several additional parameters — which to some extent simulated the wild behavior of the comet — to obtain a reasonable solution for the numerical model of the comet’s motion. For example, to link the observations of Comet Giacobini-Zinner over the 1900–1999 interval, the authors had to include $A^{(1)}$ before 1956 and $A^{(2)}$ after 1956 (instead of one parameter A), τ_1 before 1956, τ_2 between 1956 and 1969, τ_3 between 1969 and 1989, and τ_4 after 1989 (see Fig. 4). Eleven nongravitational parameters [$A^{(1)}$, $A^{(2)}$, η , I_0 , ϕ_0 , f_p , s, τ_1 , τ_2 , τ_3 , and τ_4] were therefore necessary to link 1589 astrometric observations over a 100-yr interval. It should be noted that the determined oblateness $s = +0.2926 \pm 0.0055$ for the nucleus of Comet Giacobini-Zinner now seems physically reasonable.

3.7. Case Study of Comet Comas Solá

Królikowska et al. (1998a) studied the nongravitational motion of Comet 32P/Comas Solá during the 1927–1996 interval. This comet had been investigated earlier by *Sekanina* (1985b), who applied the forced precession model and found that this comet precessed more rapidly than any other known comet. His model required a large oblateness of the nucleus, $s = 0.57$, and according to *Sekanina* gave “intolerably large perturbations” in the spin-axis obliquity, which changed rapidly by about 90° after 1952. *Królikowska et al.* (1998a) used new observations from the comet’s apparitions in 1987 and 1996 and employed the forced precession model fit to 582 observations. They found three almost equivalent models, two with the oblate nucleus and one with

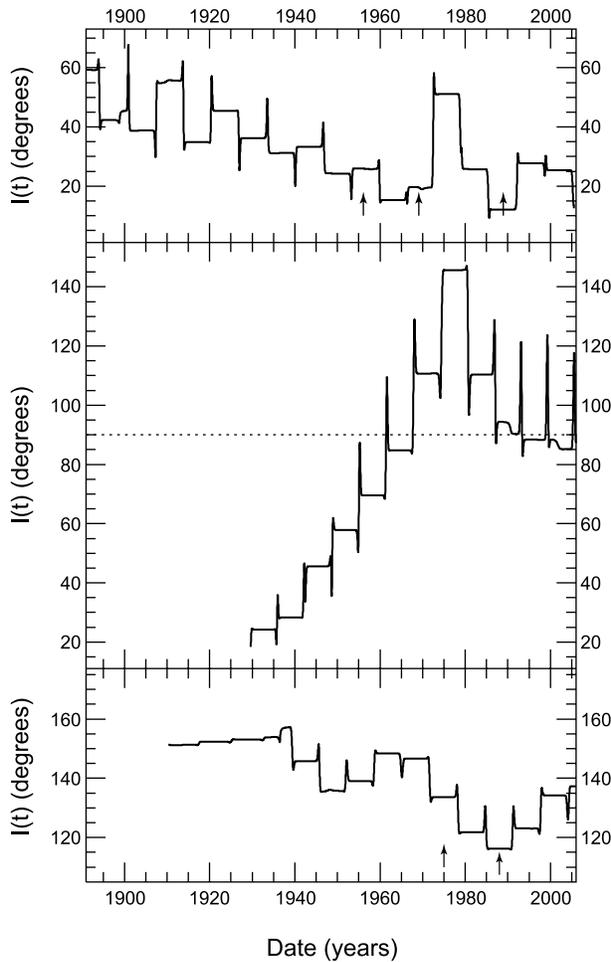


Fig. 3. Temporal variations of the angle I due to the spin-axis forced precession of the comet's nucleus for three erratic comets: 21P/Giacobini-Zinner (top), 37P/Forbes (middle), and 43P/Wolf-Harrington (bottom). The forced precession models were constructed on the basis of all the positional observations taken before the year 2000. There are 9 apparitions of Comets 37P/Forbes and 43P/Wolf-Harrington, and 13 apparitions (almost 100 years) for Comet 21P/Giacobini-Zinner. The same scale on the vertical axes allows one to compare the amplitudes of the I variations. The dotted horizontal line divides the models with prograde rotation ($I < 90^\circ$) from the models with retrograde rotation ($I > 90^\circ$). The forced precession models give a slightly prolate shape for the nucleus of Forbes and Wolf-Harrington, and quite an oblate shape for the nucleus of Giacobini-Zinner (see also Table 1). The upright arrows have the same meaning as in Fig. 4.

the prolate nucleus. The oblate solutions ($s = 0.35$) were to some extent similar to Sekanina's solution, but $I(t)$ now showed rather moderate variations without rapid jumps. In all cases it was necessary to include two additional values of the time shift, τ_1 before 1940, and τ_2 after 1940. The best solution appeared to be that for the prolate nucleus where $s = -0.105 \pm 0.024$ and $\tau_1 = -55.05 \pm 3.79$ d. A solution for $\tau_2 = +8.13$ d was found by changing its value to find the best fit to the observations. Three solutions could be compared by their rms mean residuals. In the oblate nucleus

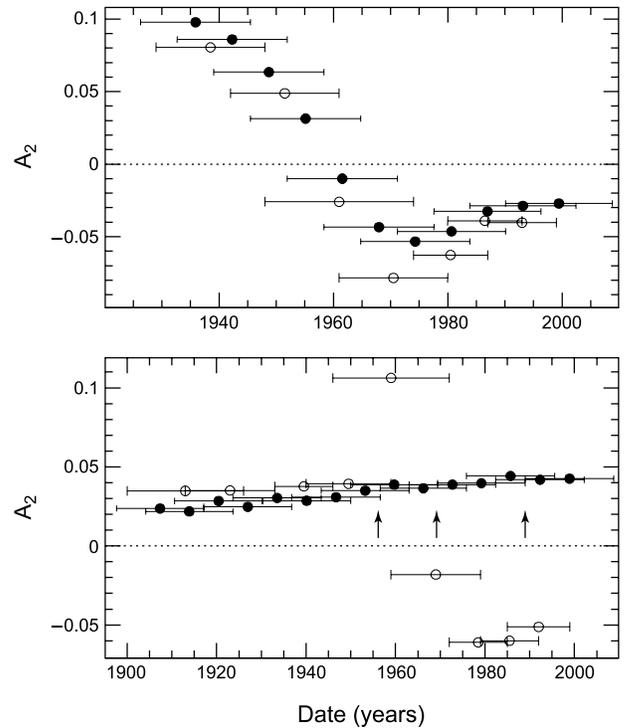


Fig. 4. Temporal variations in the nongravitational parameter A_2 for two erratic comets: 37P/Forbes (top) and 21P/Giacobini-Zinner (bottom). The open circles represent values of A_2 determined as constants within sets of at least three consecutive apparitions. These circles lie in the middle of the time intervals (shown as thin solid horizontal lines) taken for the calculations. The solid circles are mean values of A_2 (averaged over three consecutive revolutions around the Sun) resulting from the forced precession models. The thick horizontal lines represent the time intervals taken into account for these averaged A_2 values. Upright arrows (Giacobini-Zinner case) indicate the assumed moments of changes for the derived parameters: τ [time shift of the maximum of $g(r)$ with respect to the perihelion time] and A [level of activity; for a detailed description of the Giacobini-Zinner model, see Królikowska et al. (2001)]. These postulated moments for the discontinuities of τ (and A) were necessary to obtain a satisfactory forced precession model for Comet Giacobini-Zinner, and they model the actual changes of displacement of maximum activity with respect to perihelion (and the level of activity) for almost 100 years of observations.

cases it amounted to $2''.11$, whereas it dropped somewhat to $2''.05$ for the prolate case: The *a priori* mean residual was $1''.41$.

3.8. Case Study of Comet Wirtanen

Investigations into the motion of Comet 46P/Wirtanen are especially interesting because this comet is one of a few that have been considered for space flight rendezvous missions. Until early 2003, it was the target body for the European Space Agency's *Rosetta* comet rendezvous mission. The comet, discovered in 1948, had only 67 positional ob-

servations through 1991. In that period, the comet experienced two close approaches to Jupiter, to within 0.28 AU in 1972 and to within 0.47 AU in 1984, and both encounters changed the comet's orbit considerably. *Królikowska and Sitarski* (1996) undertook a preliminary investigation of the comet's nongravitational motion, applying the model of the rotating and precessing cometary nucleus with linear precession of the spin axis. They found that the comet's nucleus should be oblate (f_p was positive), but the poor observational material did not allow a determination for the value of s . The 1995–1997 apparition of Comet Wirtanen yielded 247 new positional observations, and *Królikowska and Szutowicz* (1999) again studied the comet's motion based on Sekanina's forced precession model of the rotating cometary nucleus. They were able to determine the value of the oblateness $s = +0.1019 \pm 0.0342$. However, to satisfactorily adjust the solution to all the observations, they had to introduce some additional parameters to the numerical model of the comet's motion: the time shift $\tau = -23.48 \pm 1.25$ d, and instead of the single parameter A , two parameters were required, $A^I = +0.802 \pm 0.009$ before 1989.0, and $A^{II} = +0.678 \pm 0.007$ after 1989.0 (in the units of 10^{-8} AU/d²). This was the best solution (among others considered in their paper) representing the observations with an rms residual equal to 1".59 (the mean residual *a priori* was 1".38). Figure 5 shows the time dependence of $I(t)$ and $\phi(t)$ as well as the components of the nongravitational force per unit mass $F_i(t)$, extrapolated to 2015.

3.9. Nongravitational Accelerations Due to Discrete Source Regions

The traditional view of nearly uniform outgassing from any part of the nucleus surface when it is exposed to solar insolation contrasts with the concept of localized active regions. Closeup images of the nucleus of Comet 1P/Halley taken by the spacecraft *Giotto* in March 1986 revealed a few distinct dust jets, emanating from the sunlit side. Similar distinct dust jets were evident in Comet 19P/Borrelly when the *Deep Space 1* spacecraft flew past this comet on September 22, 2001 (*Soderblom et al.*, 2002). The local outgassing restricted to a few "active regions" evolving into craters has recently been incorporated into the physical models of comets (*Colwell et al.*, 1990; *Colwell*, 1997). The distribution of jets around the nucleus and their contribution to the total production rate also has implications for the nongravitational effects in a comet's orbital motion. For a nucleus with discrete outgassing regions (spotty nucleus), the maximum sublimation rate will take place when the subsolar point is closest to an active region, which may not occur at perihelion.

The effects of discrete outgassing on the shape of the gas production curve and on the nongravitational parameters were discussed in detail by *Sekanina* (1991b, 1993a,c). In *Sekanina's* model (1988a) the absorbed solar energy is spent on sublimation and thermal re-radiation, but not on

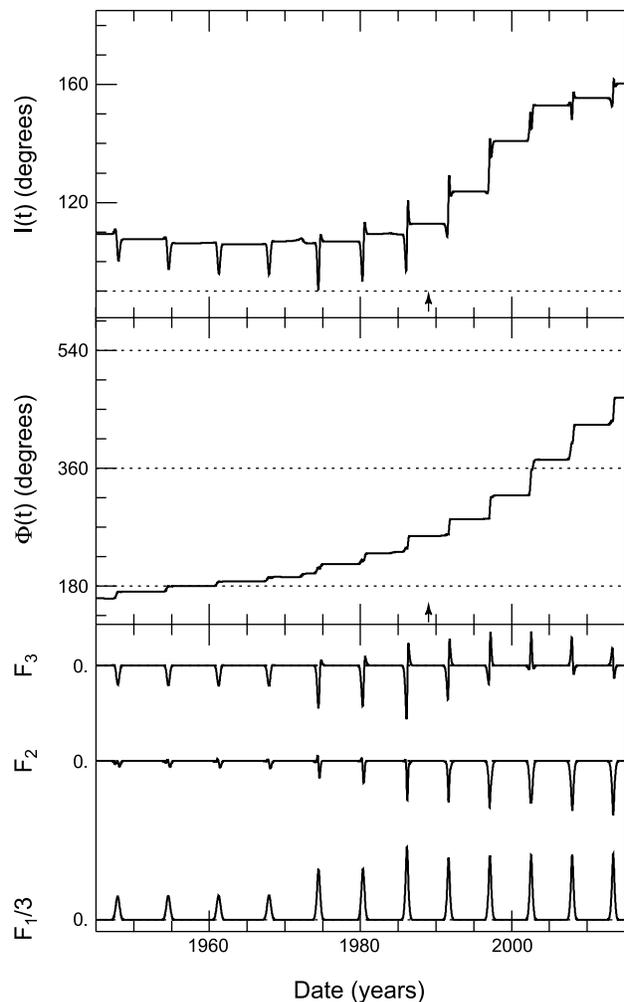


Fig. 5. Forced precession model of Comet 46P/Wirtanen based on an almost 50-yr interval of positional observations. For model details see *Królikowska and Szutowicz* (1999). Temporal variations are presented for the angle I (top), ϕ (middle), and components F_1 , F_2 , F_3 of the nongravitational force per unit mass \mathbf{F} (bottom) due to the spin-axis forced precession of the comet's nucleus. These latter three components are only meant to convey the qualitative changes in their magnitudes and directions near each perihelion passage. This model gives an oblate shape of the nucleus with $P_{\text{rot}}/R_a = 4.9 \pm 1.4$ h/km, where R_a denotes the equatorial radius. Assuming 6.0 ± 0.3 h for the rotational period P_{rot} (*Lamy et al.*, 1998), the range of 0.9–1.7 km for the nucleus radius (consistent with the photometric observations) is obtained. The vertical arrow denotes the time (1989) when the modeled levels of outgassing activity and perihelion asymmetry are assumed to change.

heat conduction into the nucleus. Hence there is no need for a thermal lag angle. The sublimation rate from a unit surface area on the nucleus is expressed as a function of heliocentric distance and the Sun's zenith distance. The model, as applied to Comet 2P/Encke (*Sekanina*, 1988a,b, 1991a), allowed him to interpret its observed sunward fan-

like coma as an effect of the northern and southern localized vents on the comet's nucleus ($+55^\circ\text{N}$, -75°S); the comet's spin axis was fixed. *Sekanina* (1991b, 1993a) noted examples of the rotation-averaged sublimation rates of point-like sources at various locations on the nucleus surface calculated for a spherical nucleus with various axial positions in an unperturbed heliocentric orbit. For the spotty model of the nucleus, the meaning of the nongravitational parameters is different than for the standard model. For the spotty model, there is a correlation between the sign of A_2 , the asymmetry of the production curve, and the location of the active regions. In the standard model the sense of rotation was directly correlated with the sign of A_2 but this need not be true for the spotty model. Furthermore, a negative value of A_1 , unrealistic in the standard model, can correspond to a circumpolar or high-latitude active source and certain combinations of the spin-axis orientation. Erratic discontinuities in the nongravitational perturbations for three comets and their long-term changes in A_2 were interpreted by *Sekanina* (1993b) as the initiation of new active areas or the deactivation of existing ones on the nucleus surface. Thus the proper modeling of the nongravitational effects should contain information on the true characteristics and locations of the comet's active areas.

The rotational-averaged orbital components of the nongravitational acceleration for a nucleus with active regions were adopted for orbital computations and introduced directly into the equations of a comet's motion by *Szutowicz* (2000). The lifetime of each active region was limited by time because of its activation and deactivation. The parameters of the model are three angles η , I , ϕ characterizing the nucleus, the cometocentric latitude of the j^{th} active region β_j , and the constants A_j are proportional to S_j/M , where S_j is the outgassing area of the j^{th} source and M is the nucleus mass. These parameters can be determined along with the osculating orbital elements in the orbit-determination process. The first attempt to introduce the spotty nucleus model into orbital calculations was made to explain a dramatic jump of the nongravitational effects in the motion of Comet 71P/Clark (*Szutowicz*, 1999a). The spotty nucleus model was successfully used to link all observations of Comet 46P/Wirtanen spanning the interval 1947–1997 (*Szutowicz*, 1999b) and of 43P/Wolf-Harrington over the period 1925–1997 (*Szutowicz*, 2000), with the rms residuals equal to $1''.57$ and $1''.39$ respectively. For the former comet, the temporal variations of the activity level are responsible for its nongravitational behavior, whereas the orbital solutions for the latter one involved a redistribution of the active areas. From orbital solutions for 43P/Wolf-Harrington, it follows that the northern region ($\sim 38^\circ\text{N}$) was persistently active and the profile of the comet's activity was modified by the initiation and disappearance of two southern regions ($\sim -4^\circ\text{S}$, -51°S) in 1965, 1978, and 1991. According to the solutions, the time variation of the modeled sublimation rates is accompanied by parallel changes in the visual light curve. A contradictory result was obtained in the case of Comet 46P/

Wirtanen. The modeled gas production rate peaks before perihelion in accordance with the nongravitational acceleration ($A_2 < 0$), but this is not consistent with the observed light curve that peaks about one week after perihelion (*Rickman and Jorda*, 1998). Clearly there is not a one-to-one correspondence between a comet's visual light curve and its gas production rates and it is the latter that controls the nongravitational effects.

Chesley (2002) used the discrete jet model, averaged over one rotation period, to verify the two main active source regions of 2P/Encke that *Sekanina* (1988a) had suggested (two sources at latitudes 55°N and -75°S). He found that these two source regions, together with *Sekanina*'s pole direction, provided a significant improvement over the standard nongravitational acceleration model in both the goodness of the orbit fit as well as in the orbit's predictive capability. Going further, *Chesley* (2002) found that a pole position around $\text{RA} = 220^\circ$ and $\text{Dec.} = +40^\circ$ provided an even better orbit fit and prediction than those obtained using the pole suggested by *Sekanina*.

It is a rare opportunity when one can test a cometary nongravitational acceleration model with the help of astrometric imaging data provided by spacecraft. *Chesley* (2002) noted that since the principal active vent for Comet 19P/Borrelly was aligned with the rotation pole, this pole direction could be independently determined using the values of A_1 , A_2 , A_3 , τ ; the determined pole direction (RA and $\text{Dec.} = 208^\circ$, -4°) agreed with rotation pole values determined from long-term photometric studies (e.g., *Farnham and Cochran*, 2003) and had reasonable agreement with the pole position determined from the spacecraft images themselves (*Soderblom et al.*, 2002).

4. INFERRING MASSES AND BULK DENSITIES OF THE NUCLEUS USING NONGRAVITATIONAL EFFECTS

It is important to note that there are no direct determinations for the mass or density of any comet and this is likely to remain the situation until a spacecraft rendezvous mission is carried out. Nevertheless, there have been many studies suggesting that comets are rather low-density and porous structures.

Yau et al. (1994) found that the observations of Comet 109P/Swift-Tuttle in 69 B.C. and in A.D. 188, 1737, 1862, and 1992–1993 were consistent with the complete absence of nongravitational effects in this comet's motion and that there have been no obvious changes in this comet's absolute magnitude over two millennia. Because Comet Swift-Tuttle's absolute magnitude has not changed significantly and there is a lack of significant nongravitational effects over the same period, constraints can be placed upon the model for this comet's nucleus. At 1 AU from the Sun, the outgassing activity of Swift-Tuttle is comparable with that of Comet Halley at the same heliocentric distance. Yet Comet Halley experiences an increase in its orbital period

of four days per revolution due to nongravitational effects while Swift-Tuttle has no perceptible change in its period. If the mass of Swift-Tuttle were significantly larger than Halley's, however, one would not expect to be able to detect a nongravitational acceleration in its orbital motion. Based upon an analysis of their respective meteor stream characteristics, *Hughes and McBride* (1989) concluded that the mass of Comet Swift-Tuttle is about 10 times larger than Comet Halley.

Rickman (1986) pointed out that radial outgassing forces that act asymmetrically with respect to perihelion were the likely cause of the nongravitational effects upon Comets Halley and Kopff, and he went on to make estimates of their masses and bulk densities. He noted that the water production curves for Halley and other comets show an asymmetry with respect to perihelion and therefore the effect of the radial component, integrated over one orbital period, would be nonzero. The nucleus masses were estimated by comparing nongravitational parameters with the rocket-like forces expected from the gas-production curves. In turn, the gas-production curves were determined from the light curves using an empirical relationship developed by M. Festou. The bulk density for Comet Halley was estimated to be 0.1–0.2 g/cm³ and that for Kopff was lower still. *Rickman et al.* (1987b) continued this type of analysis and estimated masses for 29 short-period comets. The change in the total orbital period per revolution results from the sum of the contributions from the radial and transverse rocket effects. The mass of each comet was determined as a function of its estimated thermal and rotational properties. While bulk densities for individual comets are very uncertain, the bulk densities of these objects as a group were estimated to be less than 0.5 g/cm³, suggesting that the cometary nucleus is a very porous structure. This type of analysis depends upon the assumption that there is a correlation between the light curve and the assumed gas-production curve, that thermal lag angles are present, and that the surface of each object has an unmantled free sublimating area. Using a similar approach for Comet Halley, *Sagdeev et al.* (1988) estimated a bulk density of 0.6 g/cm³ with error bars of +0.9 and –0.4 g/cm³. After a rather complete discussion of the method and the uncertainties involved in this type of analysis, *Peale* (1989) concluded that it is difficult to provide a meaningful constraint for the bulk density of Comet Halley.

The orbital nongravitational perturbations combined with the observed gas production rates were also employed to estimate the masses of the two long-period comets: C/1995 O1 Hale-Bopp and C/1996 B2 Hyakutake (*Szutowicz et al.*, 2002a,b). The observed water production rates as a function of the heliocentric distance were included into the nongravitational model represented by parameters: A (A is proportional to Q_m/M), η , I , ϕ , where Q_m is the observed sublimation rate of water at 1 AU. The derived masses divided by the volumes of both comets give bulk densities as low as 0.1 g/cm³ for Comet Hale-Bopp and 0.2 g/cm³ for Comet Hyakutake. This kind of solution is limited by the thermal properties of the nucleus (i.e., the mean outflow velocity of the molecules), the excess of the production rate

due to the ice halo in the coma, and some stochastic perturbations like splitting or outbursts noted for both comets. The latter may cause stronger nongravitational effects than expected from the water sublimation rate alone.

Farnham and Cochran (2002) estimated the mass of Comet 19P/Borrelly by separately computing the nongravitational force and acceleration acting upon the comet's nucleus. The force was computed from the observed gas production rate and the emission velocity while the nongravitational acceleration was forthcoming from the orbital computations. Dividing the comet's determined mass by the volume estimated from the *Deep Space 1* spacecraft imaging provided a bulk density for the comet's nucleus of 0.49 (–0.20, +0.34) g/cm³.

Davidsson and Gutiérrez (2003) deduced a bulk density for 19P/Borrelly of 0.10–0.30 g/cm³ by modeling the comet's observed water production rates with nucleus surface activity maps based upon sophisticated thermophysical models. By requiring that the model reproduce the nongravitational secular changes in the argument of perihelion and the longitude of the ascending node, they were able to tighten the bulk density range to 0.18–0.30 g/cm³.

5. NONGRAVITATIONAL EFFECTS AND THE SOURCE REGION FOR LONG-PERIOD COMETS

Marsden and collaborators first reported the detections of nongravitational forces in the motion of long-period comets in the late 1960s and early 1970s. In the *Catalogue of Cometary Orbits*, *Marsden and Williams* (2001) gave nongravitational parameters A_1 and A_2 for 23 long-period comets. To determine these parameters, the authors applied the standard nongravitational model. It became evident that nongravitational accelerations affecting the orbital motion of long-period comets (at 1 AU from the Sun) are a few to 10 times larger than the similar accelerations detected for short-period comets (*Marsden et al.*, 1973). In the last decade two famous long-period comets (1995 O1 Hale-Bopp and 1996 B2 Hyakutake) offered unique opportunities for detailed investigation of the nongravitational effects on their orbital motion. The nongravitational effects play an essential role in the dynamical evolution of both comets. In particular, the future orbital evolution, including nongravitational effects, gives a significantly higher probability of these comets being ejected from the solar system than does the pure gravitational orbital motion (*Szutowicz et al.*, 2002a,b; *Królikowska*, 2002).

Nongravitational effects also play a role in the identification of hyperbolic comets. The problem of a negative tail in the distribution of the reciprocals of original semimajor axis ($1/a_{ori}$) has been discussed in detail by many authors. *Marsden et al.* (1973) speculated that neglecting the nongravitational effects tends to produce more hyperbolic original orbits than really is the case. Subsequently, many authors (*Yabushita*, 1991; *Bolatto et al.*, 1995) considered the nongravitational perturbation in a comet's energy per orbital revolution and concluded that these perturbations are

too small to explain the negative excess of original binding energy of “hyperbolic” comets. Misleading results can be obtained when the same osculating orbit is used as the initial orbit for backward integrations using nongravitational effects and without these effects. Let us consider that the same osculating orbit is integrated backward twice, first by setting the nongravitational terms equal to zero and then by including the nongravitational accelerations. The differences in the resulting original reciprocals will then be significantly smaller than 10^{-4} AU $^{-1}$. At first look, this seems to suggest that nongravitational effects provide only modest changes in the true original value for $1/a_{\text{ori}}$. However, it is important to realize that an osculating nongravitational orbit determined from a set of observations would not be the same orbit as one determined from these same observations but under the assumption of purely gravitational motion. Królikowska (2001) investigated the problem of the original orbits of 33 comets considered as “hyperbolic” comets (in the pure gravitational case). For the 16 cases for which solutions for nongravitational effects could be carried out, she showed that for almost all cases, the original orbits changed from hyperbolic to elliptic. For the two comets for which the original orbits remained hyperbolic (1996 E1, 1996 N1), their original orbits became less hyperbolic as a result of solving for nongravitational effects and the resulting negative original $1/a_{\text{ori}}$ values were rather modest. The tendency for negative, original $1/a_{\text{ori}}$ values to become positive when nongravitational effects are taken into consideration is due primarily to changes in the orbital eccentricity. Królikowska concluded that the nongravitational effects could significantly affect the Oort peak for comets with perihelion distance smaller than 3 AU.

At the other extreme from the nearly parabolic orbits from the Oort cloud comet lays 2P/Encke with its shortest known cometary orbital period of 3.3 yr. The current orbit of 2P/Encke is completely interior to Jupiter’s orbit and is gravitationally decoupled from that planet. In trying to explain how Comet Encke arrived at this stable orbital position, Steel and Asher (1996) noted that nongravitational effects on the comet, some four times larger than those that have recently been operative, would be enough to evolve the comet from its current orbit into Jupiter-crossing orbits and hence the same mechanism could have dropped the comet into its current orbit. The nongravitational effects can cause the comet to drift across the jovian and saturnian mean-motion resonances in the asteroid belt and even if these nongravitational effects do not act in the same direction for extended periods of time, they can still strongly modify the orbit of an Encke-like object.

6. SUMMARY

Cometary orbit-determination problems are dominated by the proper modeling of the so-called nongravitational perturbations that are due to the rocket-like thrusting of the outgassing cometary nucleus. Modern astrometric positions, particularly those that are referenced to Hipparcos-based star catalogs and where the brightest pixel is employed as

the true position of the cometary nucleus, are usually accurate to the subarcsecond level. Yet multiple apparition orbital solutions for active short-period comets cannot often provide a root mean square (rms) residual (observed minus computed observational position) that is subarcsecond. It is the improper modeling of the nongravitational effects that is the largest problem by far.

Beginning with Encke’s first suggestion of an interplanetary resisting medium to explain the anomalous motion of the comet that bears his name, there have been many different models put forward to explain the accelerations in the motions of active comets that are not due to the gravitational perturbations of neighboring planets or asteroids. Although the notion of an icy conglomerate model for a cometary nucleus (Whipple, 1950, 1951) is still in basic agreement with the observations, there have been a number of recent modifications and refinements to this model. Largely as a result of the impressive images of Comet 1P/Halley’s nucleus taken by the *Giotto* spacecraft and those taken more recently of 19P/Borrelly by the *Deep Space 1* spacecraft, a “vent” model whereby the outgassing activity takes place from discrete active areas has replaced the picture of an outgassing sunlit hemisphere.

It seems likely that each comet has its own set of peculiar jets located at various places on its surface and operating at different strengths so that a completely accurate model for a particular comet’s nongravitational effects would require a detailed knowledge of the comet’s surface outgassing features and rotation characteristics. Since this knowledge is available only for those few comets that are visited by spacecraft, orbit practitioners will have to be content with generic models that approximate the true situation. In this regard the recent advances in bringing forth the asymmetric nongravitational acceleration models, the forced nucleus precession models, and especially the discrete active source models are promising.

Many of the existing models solve for physical characteristics of the cometary nucleus (e.g., oblateness, thermal lag angles, positions of sources, and the rotation pole). However, the formal uncertainties computed for these quantities that arise from the orbit-determination process alone must be considered lower limits rather than realistic values. Whenever possible, these quantities should be confirmed using spacecraft observations or a long series of photometric groundbased observations. For example, the oblateness values and rotation pole positions derived from the orbit-determination process should be checked against the aspect ratios and pole positions determined from groundbased photometric studies. As the astrometric datasets improve and lengthen and as the modeling of the cometary nongravitational effects becomes more realistic, there remains the strong possibility that some physical characteristics of comets will soon be accurately determined from the orbit-determination process alone. We are already beginning to see signs that this is the case.

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