Using the Least Square Method to Obtain the Absolute Magnitude of Ceres from the Light Curve

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Abstract: We used the least square method to obtain the absolute magnitude of Ceres that gave the best fit to the light curve obtained by the Heinrich-Hertz submillimeter telescope taken from January 4, 2003 to May 6, 2004. We obtained an absolute magnitude of 2.75 for this data.

Introduction

Asteroids are clumps of rock, the sizes of which range from less than a kilometer to a few hundred kilometers in diameter. They are generally found in the unusually large gap between Mars and Jupiter. There are probably more than 40,000 asteroids in this gap called the asteroid belt. Two fundamental parameters characterizing an asteroid are its albedo (ratio of reflected or scattered flux over incoming flux) and size (effective diameter). Information on the albedo and size of an asteroid is a vital prerequisite for attempts to investigate their physical nature, mineralogy, taxonomy, and give clues into the history and formation of our Solar system. Since we wanted to establish an asteroid research program in our school, we focused on Ceres, one of the largest asteroids found, with an accepted diameter of 950 km, and forming 25% of the asteroid belt's mass. In this investigation we show how the absolute magnitude can be obtained from the light curve of Ceres. Knowledge of the absolute magnitude can then be used to obtain the size of Ceres.

Figure 1: Artist rendering of NASA Dawn orbiter hovering 240 miles above Ceres on March 15, 2016 [1].

Procedure

When you look at an asteroid through a telescope it looks like an uninteresting point of light. However, by analyzing a number of images important information can be obtained concerning the size of the asteroid and its surface characteristics. The geometry of observation of an asteroid is shown in Figure 2.

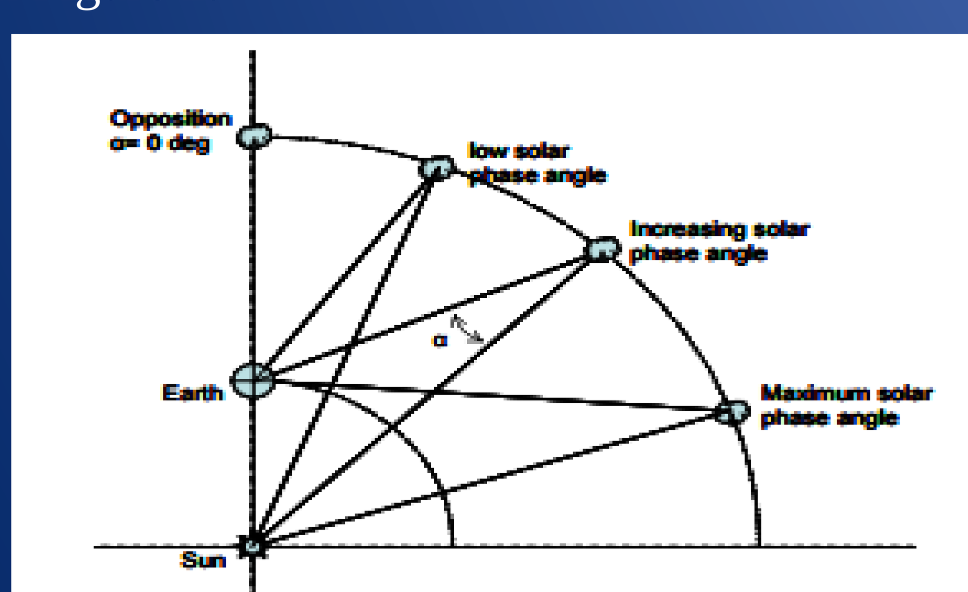

Figure 2: Geometry of orbits: Earth distance, Sun distance, and solar phase angle [2].

The solar phase angle α, is analogous to the moon's phase. When α=0, the asteroid is fully illuminated, corresponding to a full moon's phase. The solar phase angle is given by the cosine rule,

$$1 = r^2 + \Delta^2 - 2r\Delta\cos\alpha \qquad (1)$$

In the above equation r and Δ are the distance of the asteroid from the Sun and the Earth respectively. The absolute magnitude H of an asteroid is the V-band magnitude of the asteroid taken at a solar phase of zero and when the asteroid is 1 AU from the Earth and the Sun [4].

It is related to the reduced magnitude H(α) taken at another solar phase angle,

$$H = H(\alpha) + 2.5\log[(1-G)\Phi_1(\alpha) + G\Phi_2(\alpha)] \qquad (2)$$

In the above equation *G* is the slope of *H* versus *α* curve and is related to the opposition effect (surge in brightness when the phase angle approaches zero). In this work we used the value of G given in [5] which is 0.12. In general an asteroid surface will contain ice, dirt, and rock. The surface is not smooth and will not reflect light like a mirror. Rather there will be multiple scattering and the functions *Φ* describes single and multiple scattering at the asteroid's surface. They are defined as,

$$\Phi_1(\alpha) = \exp[-3.33(\tan\frac{\alpha}{2})] \qquad (3a)$$

$$\Phi_2(\alpha) = \exp[-1.87(\tan\frac{\alpha}{2})] \qquad (3b)$$

The reduced magnitude is related to the apparent visual magnitude V, observed by a telescope by,

$$H(\alpha) = V - 5\log(r\Delta) \qquad (4)$$

Thus if we know the absolute magnitude of an asteroid and its distances from the Sun and the Earth at various intervals in time we can use (1)-(4) to generate a light curve. We wrote a program in Visual Basic that solves these equations. The absolute magnitude is also related to the diameter, D (km), of an asteroid through the formula,

$$D = \frac{1329}{\sqrt{p_V}}10^{-H/5} \qquad (5)$$

In (5) p_V is the geometric albedo.

Results and Discussion

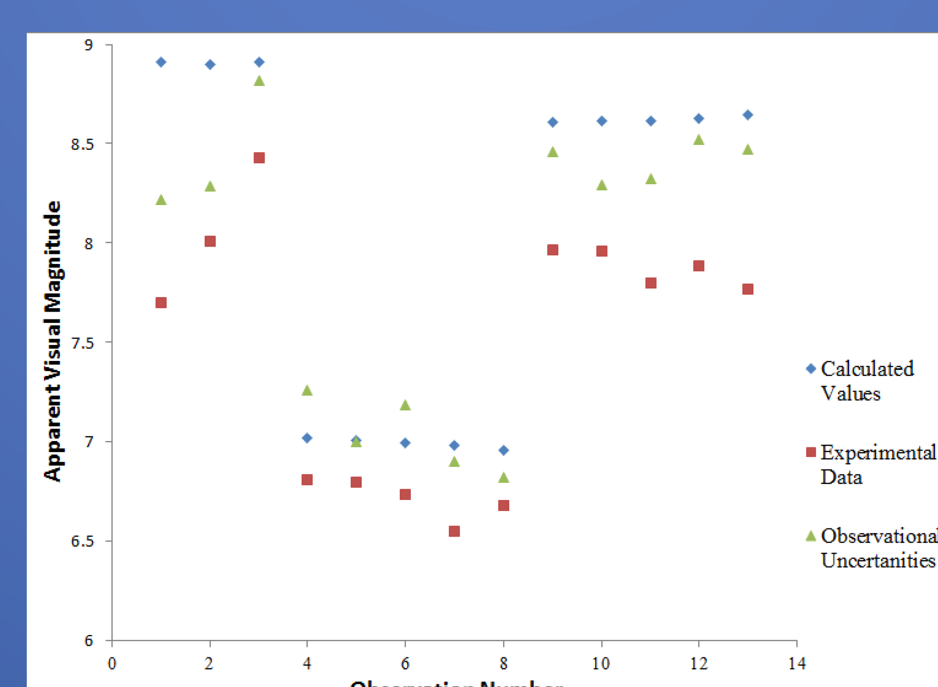

Figure 3 shows experimental data taken by the Heinrich-Hertz submillimeter telescope taken from January 4, 2003 to May 6, 2004[3]. Also shown are the observational uncertainties and our calculated values using 3.34 as the absolute magnitude.

Figure 3 shows experimental data taken by the Heinrich-Hertz submillimeter telescope from January 4, 2003 to May 6, 2004 [3]. The observations were not evenly spread during this period but clustered around a few dates. Observations were taken from January 4, 2003 to January 6, 2003, then January 4, 2004 to January 10, 2004 and then May 1 to May 6, 2004. There were a total of 13 observations. To facilitate comparison between calculated and observed results these observations were equally spaced on the x-axis. The smallest phase angle in the data was between five to six degrees and most values were around twenty degrees. Shown also in Figure 3 are the observational uncertainties. Assuming an absolute magnitude of 3.34 as reported in [5] does not give a great fit to the data as there are a few points that lie outside the observational uncertainties. If we assume that the error is mainly due absolute magnitude we can use the least squares method to obtain a value that is a better fit for the data. In this method the total error is defined as,

$$\text{Total Error} = \sum_{i=1}^{i=13}(\text{Experimental value- calculated value})^2 \qquad (7)$$

The total error using 3.34 is 5.7129 and this value is greater than the error associated with experimental uncertainties which is 2.6156. Figure 4 shows a graph of the total error versus absolute magnitude. We found that an absolute magnitude of 2.75 gave the best fit to the data. On either side of 2.75 the error increases rather quickly giving us confidence in our results. Using this value for the absolute magnitude and assuming that the geometric albedo remains 0.09 we obtain a diameter of 1248.5 km compared to the value 951.5 km reported in [5].

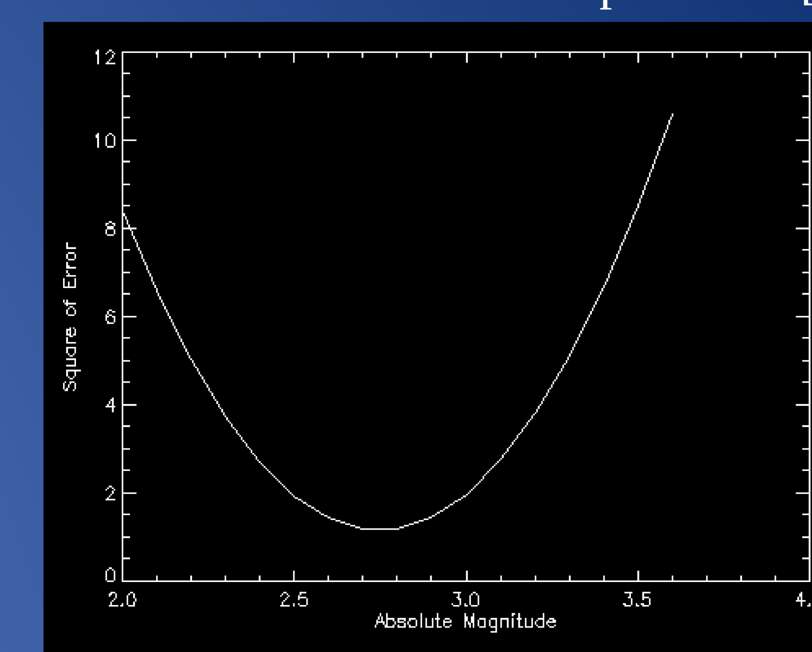

Figure 5: Total error vs. Absolute Magnitude. The minimum error is obtained when the absolute magnitude is 2.7.

Conclusion

In this work we took observational data on Ceres and used the least squares method to find the absolute magnitude that best fit the light curve. Using this method the absolute magnitude obtained was 2.75 compared to 3.34 reported in [5]. We believe that the main reason for this discrepancy was the quality of our observational data. A good experimental phase curve should have angles from around 20 degrees to nearly zero degrees. The observations we used had a few phase angles between 5 to 6 degrees and most were clustered around 20 degrees. Our future efforts will be directed to obtaining more observational data and finding the absolute magnitude that best fits all the additional data.

Program in Visual Basic

```
Sub spaceangels1()

'Constants
H = 3.3
r = Cells(1, 2)
D = Cells(2, 2)

ActiveCell.Offset(0, 1).Value = "Angle"
ActiveCell.Offset(0, 2).Value = "I1(A)"
ActiveCell.Offset(0, 3).Value = "I2(A)"
ActiveCell.Offset(0, 4).Value = "H(A)"
ActiveCell.Offset(0, 5).Value = "V"

'Finding Angle
y = ((r ^ 2 + D ^ 2 - 1) / (2 * r * D))
A = Atn(Sqr(-y * y + 1) / y)
ActiveCell.Offset(1, 1) = A

'Finding I1(A)
I1A = Exp((-3.33 * (Tan((A / 2))) ^ (0.63)))
ActiveCell.Offset(1, 2) = I1A

'Finding I2(A)
I2A = Exp((-1.87 * (Tan((A / 2))) ^ (1.22)))
ActiveCell.Offset(1, 3) = I2A

'Finding H(A)
HA = H - (2.5 * ((Log((0.88 * I1A) + (0.12 * I2A))) / 2.303))
ActiveCell.Offset(1, 4) = HA

'Finding V
V = HA + (5 * ((Log((r * D)))) / 2.303)
ActiveCell.Offset(1, 5) = V

End Sub
```

References

1. http://dawn.jpl.nasa.gov/multimedia/images/ceres.html
2. Buchheim_SAS-2010.147123427.pdf
3. http://sbn.psi.edu/pds/resources/submmic.html.
4. J. Br. Astron.Assoc. 117, 6, 2007
5. neo.jpl.nasa.gov/orbits

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