Bendix

Aerospace Systems Division

Alignment of ALSEP at Latitudes Off the Lunar Equator

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This ATM presents methods of computing alignment errors and typical errors at latitudes off the lunar equator using the present method of ALSEP alignment. Other methods of alignment are recommended at latitudes which exceed the capabilities of the present system.

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Plans to deploy future ALSEP systems at latitudes off the lunar equator have raised questions regarding the adequacy of the present alignment concepts for the ALSEP packages. Misalignment can have adverse thermal and scientific effects on the central station and experiments. These effects are not covered in this study. The following analysis is limited to showing the method of computing alignment errors and the errors which result at off latitudes with the present method of alignment. Other methods of alignment are recommended at latitudes which exceed the capabilities of the present system.

CONCLUSIONS AND RECOMMENDATIONS

1. The present method of alignment is adequate up to 10° latitude if ALSEP is deployed at very low sun angles (15° or less) and during or near a lunar equinox.

2. At latitudes greater than 10°, or at higher sun angles, alignment should be accomplished by setting the gnomon shadow to a discrete point on a compass rose, the value of which is determined by the latitude and the sun angle at the time of deployment. (See Figures 3 and 4.)

3. The use of tilting gnomons would eliminate the sun angle as a variable and require only the alignment of the gnomon shadow to a reference mark once the gnomon is tilted to the proper angle to compensate for latitude, i.e., in the same manner as present equatorial deployment.

DEFINITIONS

Equinox = The lunar equator and subsolar point coincide.

Summer Solstice = The subsolar point is at its maximum distance north of the lunar equator.

Winter Solstice = The converse of summer solstice.

Ecliptic Plane = This would be the lunar equatorial plane if the moon's axis of rotation were not tilted. The tilt of the lunar orbital plane is negligible.

β = Lunar latitude in degrees.

δ = Sun angle in the ecliptic plane, or lunar time of day, where dawn is 0°, noon is 90°, and sunset is 180° for any given location on the lunar surface. Not to be confused with actual sun angle above the horizon for a given site.

θ = Angle of inclination of the lunar equatorial plane to the ecliptic plane.

α = Angular alignment error in degrees of azimuth from a west direction in the plane normal to a local vertical. This is also the desired angular setting on a compass rose to obtain desired alignment relative to the west direction.

α' = Same as α, except in the plane normal to the ecliptic plane.

ANALYSIS

1. DEPLOYMENT ON THE LUNAR EQUATOR

Sun compasses, or gnomons, are used extensively for aligning ALSEP packages and experiments during deployment on the lunar surface. The gnomon consists of a straight post which is aligned to local vertical and casts its shadow on a surface which is normal to local vertical and is also the base of the gnomon. The gnomon shadow is aligned to a reference mark on its base surface thus aligning the package in the correct direction. The reference mark location is predetermined for the desired package alignment. The package must be level during the alignment operation.

The above method will result in precise alignment relative to an east-west direction when the packages are deployed exactly on the equator and at the time of the lunar equinox, assuming precise leveling and shadow alignment to the reference mark by the astronaut. Note, however, that a package precisely aligned to the sun line (ecliptic) is also precisely aligned relative to east-west (equator) only at the equinox. Later, at the lunar solstices, the east-west direction (and the package) will be misaligned 1-1/2^{o} to the ecliptic in each direction depending on which solstice is occurring. This small amount of misalignment to the ecliptic does not present any problems to the ALSEP packages.

The next case considered is the deployment of packages on the lunar equator at the time of a solstice. Assuming the same gnomon geometry and alignment method as was used previously, the package would be aligned precisely to the ecliptic but would be misaligned 1-1/2^{o} to the equator (position 1 in Figure 1b). Note also that at the opposite solstice the package would be misaligned 3^{o} to the ecliptic as show n by position 2. The 1-1/2^{o} initial misalignment applies if the package is deployed at lunar dawn (on the terminator). The alignment error becomes progressively less if deployment is done at higher sun angles, until theoretically for lunar noon deployment there would be no error from this effect.

There is another effect, however, when deploying during a solstice which produces insignificant errors at low sun angles, but sizeable errors at very high sun angles. Referring to Figure 1b it can be seen that during a solstice, a gnomon deployed on the lunar equator will be at some equivalent latitude off the ecliptic equator depending on the time of lunar day (sun angle). This equivalent latitude, β_e, (or gnomon tilt) can be computed from the formula:

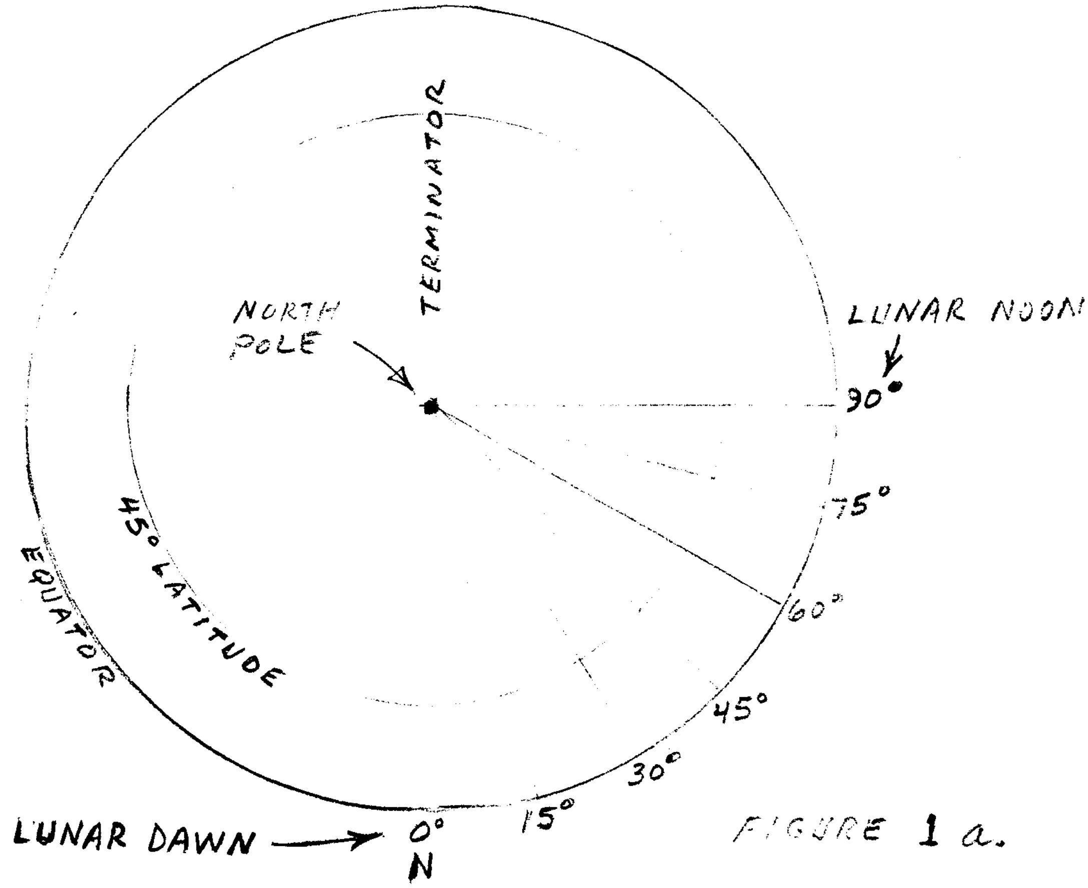

FIGURE 1 a.

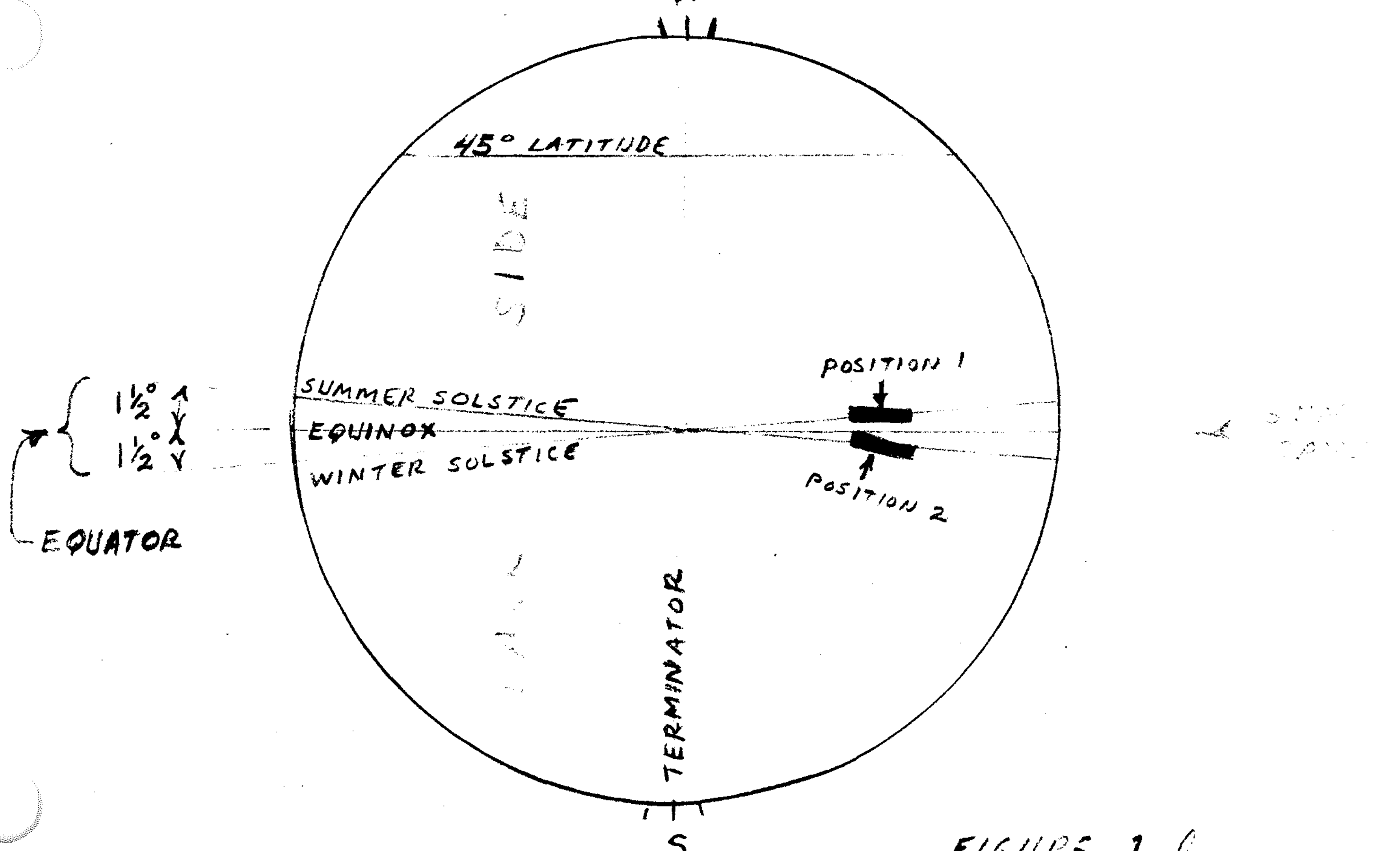

FIGURE 1 b.

$$\beta_e = \theta \sin \phi$$

where θ = 1-1/2°.

Table 1 contains some computed values of β_e.

TABLE I

Sun Angle, ϕ	0°	15°	30°	45°	60°	75°	90°
Equivalent Latitude, β_e	0°	0° 23'	0° 45'	1° 04'	1° 18'	1° 27'	1-1/2°

Angular alignment errors, α, resulting from these tilt angles, β_e can be computed from the following formula, whose derivation is discussed later:

$$\tan \alpha = \sin \beta_e \tan \phi$$

Computed values of α are shown in Table II and are negligible up to 30° sun angle.

TABLE II

Sun Angle, ϕ	0°	15°	30°	45°	60°	75°	90°
Alignment Error, α	0°	0° 06'	0° 26'	1° 04'	2° 15'	5° 24'	90°

Since ALSEP deployments will occur at low sun angles, these latter effects can be neglected. It is recommended, however, for deployments at or near a solstice that a correction offset of 1-1/2° be applied to obtain a more accurate alignment relative to east-west. That is, when the sun is north of the lunar equator (summer solstice) the gnomon shadow should be placed 1-1/2° to the north of the reference mark. Apollo 13 landing in March 1970 will occur close to a summer solstice. Deployments during a winter solstice can be compensated for by placing the shadow 1-1/2° to the south of the reference mark.

No corrections are required at or near an equinox. If misalignments up to 3-1/2° to the eliptic are acceptable, then no corrections for solstice deployment would be required on the lunar equator.

2. DEPLOYMENT AT LATITUDES OFF THE LUNAR EQUATOR

The foregoing analysis considered deployment only on the lunar equator and the effect of the 1-1/2° tilt of the lunar equatorial plane to the ecliptic during lunar solstices.

The next step is to determine the alignment errors as a function of latitude off the lunar equator using the present method of ALSEP alignment. Small increments of latitude (3°) will be investigated first to determine where the alignment errors start to become prohibitive.

Referring to Figure 2, the derivation of the formula for alignment error, α, is discussed. As can be seen:

$$\tan \alpha = \frac{X}{Z}$$

and $$X = H \tan \beta$$

$$Z = Y \cot \phi, \text{ where } Y = \frac{H}{\cos \beta}$$

by substitution:

$$Z = \frac{H \cot \phi}{\cos \beta}$$

substituting for X and Z gives:

$$\tan \alpha = \frac{H \tan \beta}{\frac{H \cot \phi}{\cos \beta}} = \frac{\tan \beta \cos \beta}{\cot \phi} = \frac{\sin \beta}{\cot \phi}$$

or, $$\tan \alpha = \sin \beta \tan \phi.$$

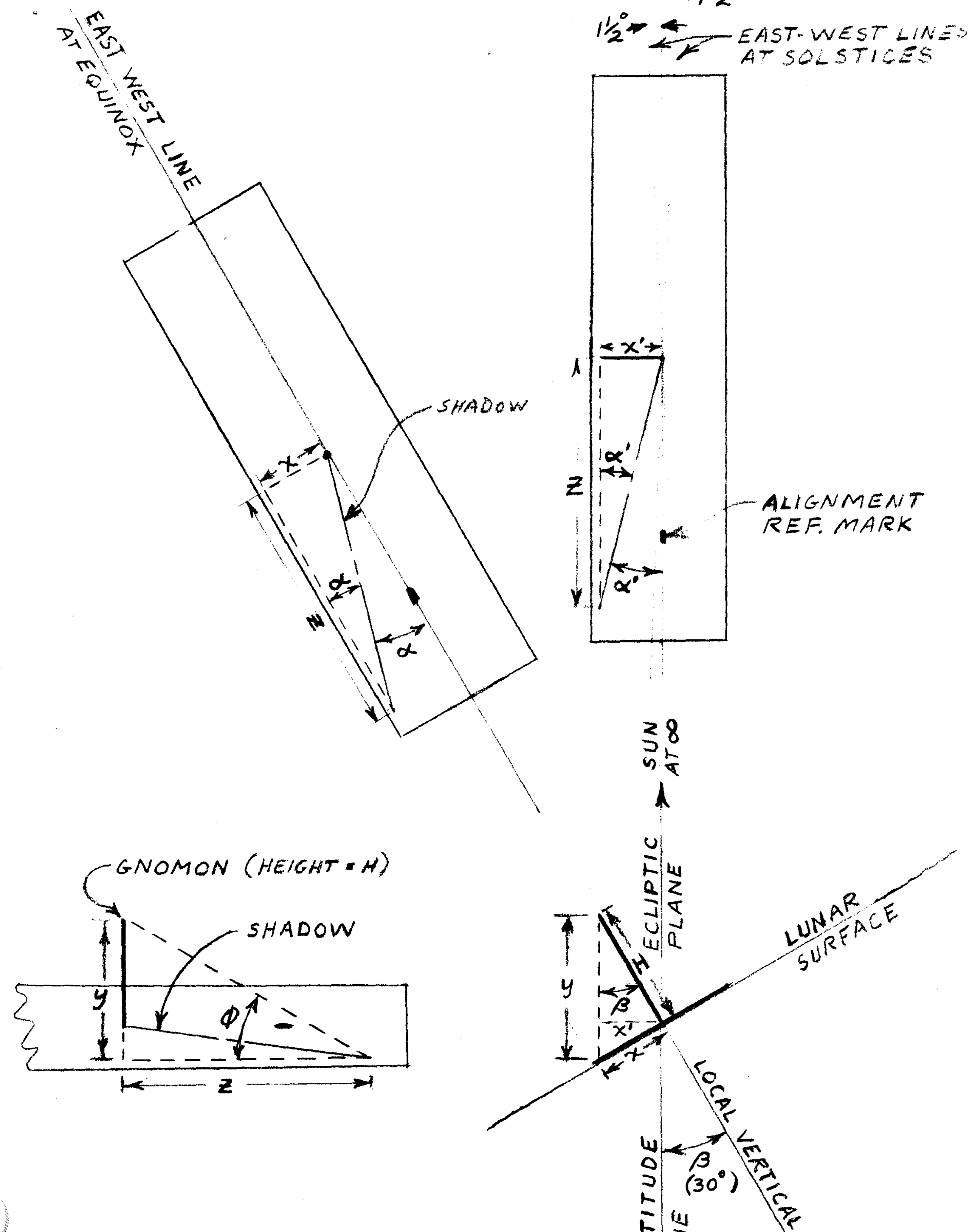
EAST WEST LINE
AT EQUINOX
SHADOW
x
z
α
1½°
1½°
EAST-WEST LINES
AT SOLSTICES
x'
α'
ALIGNMENT
REF. MARK
SUN
AT ∞
GNOMON (HEIGHT = H)
SHADOW
y
Φ
z
ECLIPTIC
PLANE
LUNAR
SURFACE
H
β
x'
x
LOCAL VERTICAL
LATITUDE
LINE
β
(30°)

FIGURE 2

Note that the angular error, α, is the angle measured in the plane normal to local vertical and is the alignment error which would result if the reference mark were placed under the shadow. Or, as shown in Figure 2, this is the angle at which the shadow should fall relative to a line through the base of the gnomon and the reference mark to align the package with the ecliptic. The misalignment angle, α', in the plane normal to the ecliptic plane is:

$$\tan \alpha' = \tan \alpha \cos \beta$$

Table III is a tabulation of alignment errors for 3° increments of latitude off the lunar equator up to 15° latitude. The effect of sun angle is also shown. (Deployment at equinox is assumed.) Figure 3 is a plot of the same data and shows that the alignment errors are small for low sun angles, even at 15° latitude.

Alignment errors at greater latitudes versus time of day are shown in Table IV. Alignment error is a misnomer since this can also be the desired azimuth angle, or offset, at which the gnomon shadow should be placed on a compass rose in order to attain the desired package alignment. Figure 4 is a plot of the tabulated data from Table IV.

In order to obtain accurate alignment relative to east-west, the gnomon shadow should be placed at the indicated angle on a compass rose. The shadow will be to the north of the reference line for deployment at north latitudes, and in the southwest quadrant of the compass rose at south latitudes, assuming lunar morning deployments.

Corrections for solstices can also be made when deploying at latitudes off the lunar equator. During a winter solstice, the $1\text{-}1/2^{\circ}$ correction is added to the azimuth angle for north latitude deployments, and subtracted for south latitude deployments. For a summer solstice, the converse applies.

A tiltable gnomon would obviate the requirement for compass roses. Tilting the gnomon toward the equator the same number of degrees as the latitude at which it is deployed puts the gnomon parallel to the plane of the ecliptic and alignment would be accomplished in the same manner as for equatorial deployment.

TABLE III

	Sun Angle, ∅ 15°		30°		45°		60°	
Latitude, ℓ	α	α+1-1/2°	α	α+1-1/2°	α	α+1-1/2°	α	α+1-1/2°
0° (Equator)	0	1° 30'	0	1° 30'	0	1° 30'	0	1° 30'
3°	0° 48'	2° 18'	1° 44'	3° 14'	3° 0'	4° 30'	5° 11'	6° 41'
6°	1° 36'	3° 06'	3° 27'	4° 57'	5° 58'	7° 28'	10° 16'	11° 46'
9°	2° 24'	3° 54'	5° 10'	6° 40'	8° 53'	10° 23'	15° 10'	16° 40'
12°	3° 11'	4° 41'	6° 51'	8° 21'	11° 45'	13° 15'	19° 49'	21° 19'
15°	3° 58'	5° 28'	8° 30'	10° 0'	14° 31'	16° 01'	24° 08'	25° 38'

The values for α assume deployment at the equinox and α + 1-1/2° is the misalignment to the ecliptic at the next solstice in the same hemisphere. For deployment during a solstice add 1-1/2° to the above values (for low sun angles) when the solstice is in the opposite hemisphere to the deployment latitude.

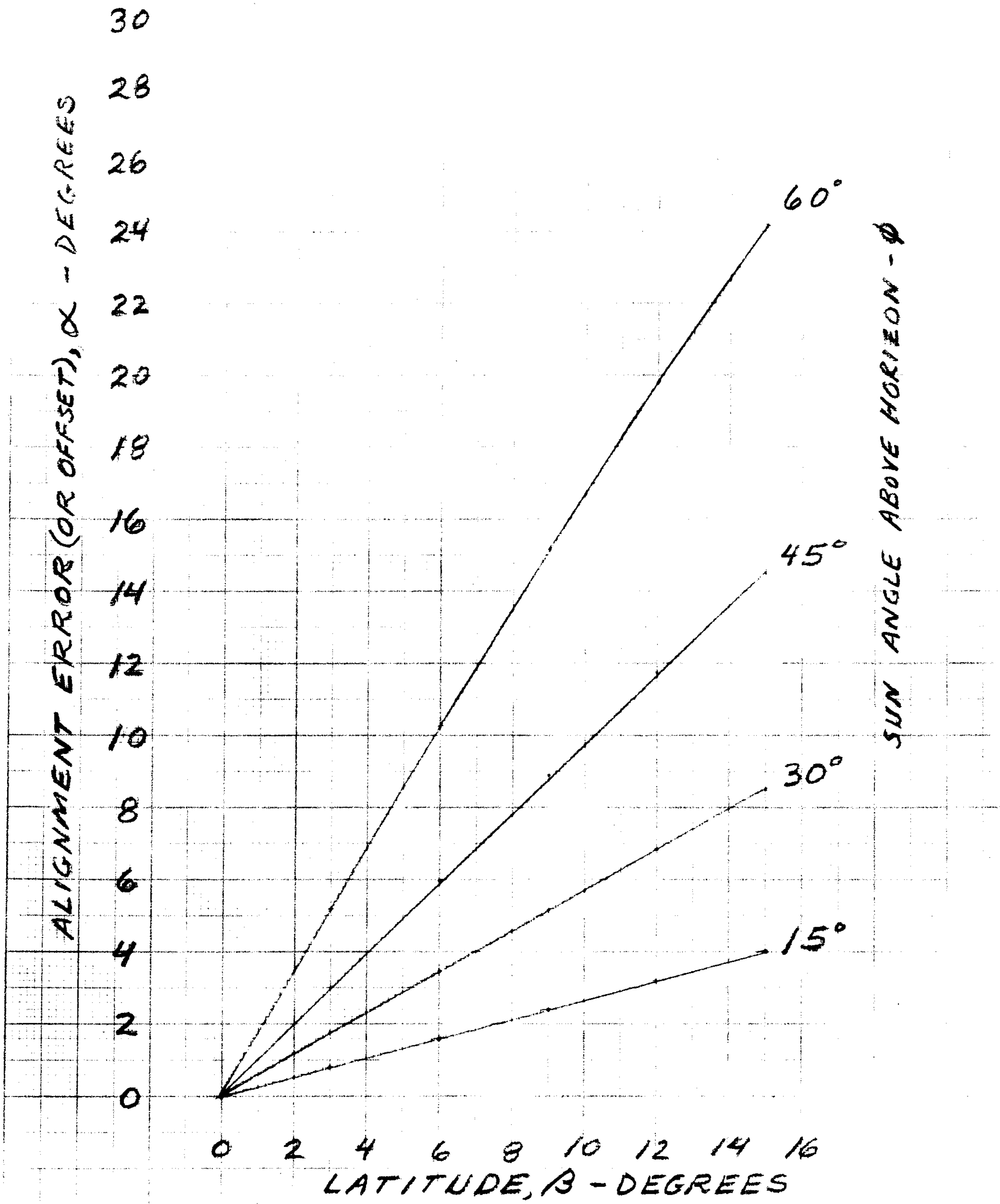
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
ALIGNMENT ERROR (OR OFFSET), α - DEGREES
0 2 4 6 8 10 12 14 16
LATITUDE, β - DEGREES
60°
45°
30°
15°
SUN ANGLE ABOVE HORIZON - φ

FIGURE 3

TABLE IV

Sun Angle Above Horizon - ϕ

Latitude β	10°	15°	20°	25°	30°	45°	60°
0° (Equator)	0	0	0	0	0	0	0
15°	2° 37'	3° 58'	5° 23'	6° 52'	8° 30'	14° 31'	24° 08'
20°	3° 35'	5° 14'	7° 05'	9° 03'	11° 10'	18° 53'	30° 39'
25°	4° 16'	6° 28'	8° 44'	11° 09'	13° 44'	22° 55'	36° 12'
30°	5° 02'	7° 38'	10° 18'	13° 07'	16° 07'	26° 34'	40° 56'
35°	5° 47'	8° 44'	11° 47'	14° 58'	18° 19'	29° 50'	44° 48'
40°	6° 28'	9° 47'	13° 13'	16° 40'	20° 23'	32° 44'	48° 05'
45°	7° 06'	10° 44'	14° 25'	18° 14'	22° 12'	35° 16'	50° 46'
50°	7° 42'	11° 36'	15° 34'	19° 39'	23° 52'	37° 27'	53° 0'

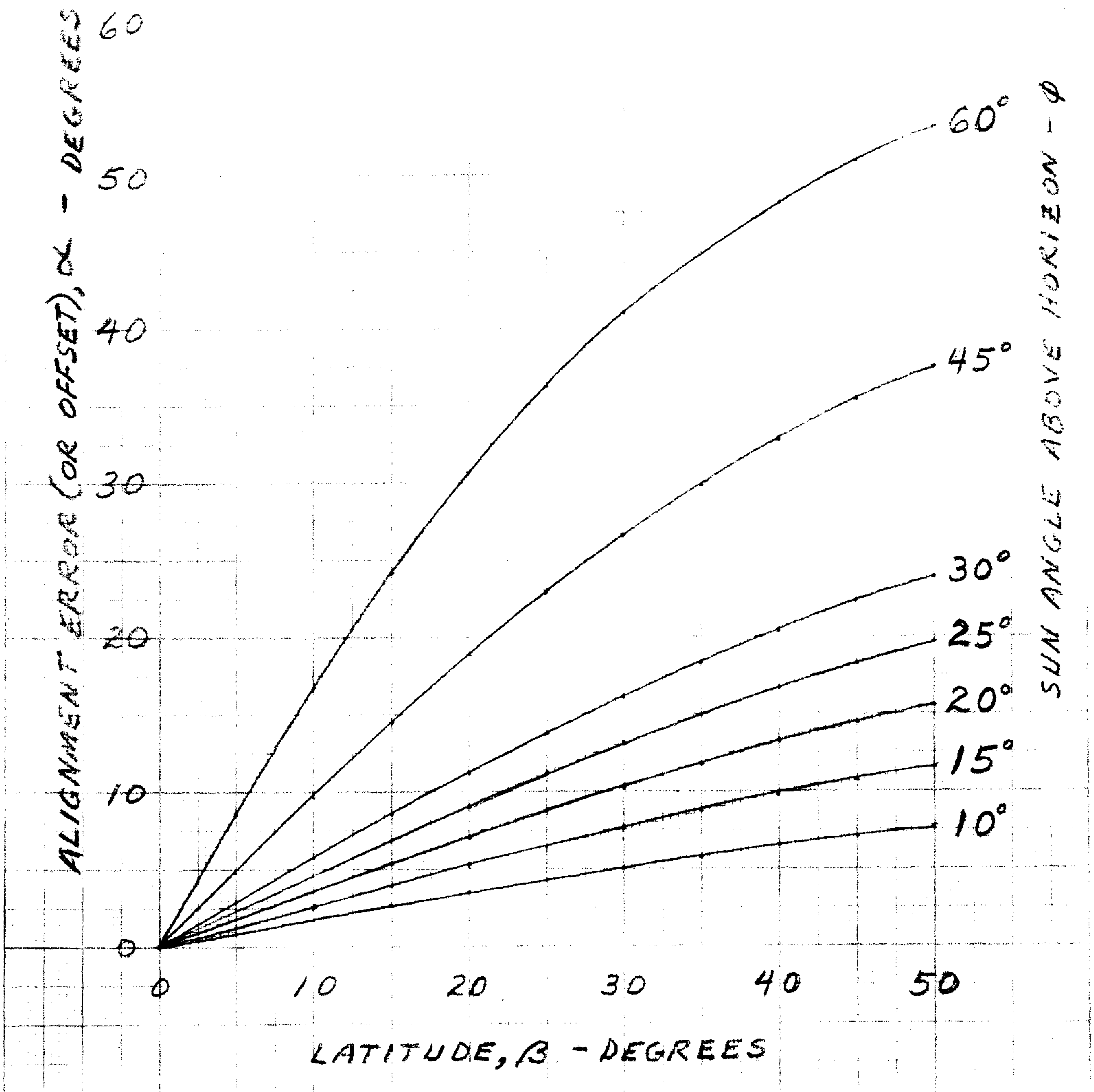
ALIGNMENT ERROR (OR OFFSET), α - DEGREES
60
50
40
30
20
10
0
0
10
20
30
40
50
LATITUDE, β - DEGREES
60°
45°
30°
25°
20°
15°
10°
SUN ANGLE ABOVE HORIZON - φ

FIGURE 4