File Bob Musser

RECEIVED
MAR 25 1970

ALSEP S-BAND DATA TRANSMITTER SHORT TERM STABILITY

ATM-861

19 MARCH 1970

Prepared by: John T. Reeves
J. Reeves

Approved by: Warren Tosh
W. Tosh, Supervisor
ALSEP Systems Engineering

1.0 INTRODUCTION

Interest has been expressed in the short term stability characteristics of the ALSEP Data Transmitters. If this stability is adequate, it becomes possible to use the carrier for certain moon/earth relationship measurements. The purpose of this ATM is to define the carrier stability and to provide samples of typical stability data applicable to ALSEP's 1 through 4. The data presented should permit those interested in such potential use to determine if the stability is adequate for their needs.

2.0 SUMMARY

Short term crystal oscillator stability is variously referred to as "rms fractional frequency deviation", "normalized standard frequency deviation", "rms frequency deviation", "rms phase deviation" and others. It is generally agreed that short term crystal oscillator stability is defined as the normalized standard deviation of the frequency or phase about the average center frequency or phase.

The specified value of short term stability for the ALSEP transmitters used in ALSEP's 1 through 4 (#2 is EASEP) was better than 2.2×10^{-10} parts per second. Measurements on all ALSEP transmitters gave results better than the specification with typical values of flight units showing 0.67×10^{-10} parts per second.

3.0 TEXT

Crystal oscillator short term stability notational symbols are as varied as is the nomenclature presented in the SUMMARY. Some of the more commonly used symbols will be used in the presentation of relationships below:

a) S = short term stability to be determined

b) Δf or $\Delta\phi$ = statistically derived standard deviation (sigma, σ) of a large sample of frequency or phase deviations from the average nominal

c) f_o = average frequency of the signal being measured

d) ω_o = $2\pi f_o$

e) τ_{av} = average time of n averaging periods

f) τ_i = period of the i-th measurement

g) $\delta\tau = \tau_i - \tau_{av}$ = deviations of the i-th measurement

h) σ_τ = standard deviation of n periods around the average period.

i) f_b = average beat frequency

j) δf_i = deviation of the beat frequency during the i-th measurement

k) c = = number of periods counted in τ_{av}

The commonly used basic stability definition is as follows:

1) $$S = \frac{\Delta f}{f_o} = \frac{\Delta\phi}{\omega_o \tau_{av}}$$

If "rms fractional frequency deviation $\frac{\Delta f}{f_o}$ is given and phase stability is desired, rearrangement of 1) provides the relationship.

2) $$\Delta\phi = 2\pi f_o \left(\frac{\Delta f}{f_o}\right) \tau_{av} = 2\pi \Delta f \tau_{av}$$

Measurements of frequency or phase deviation are typically made by beating the output of the oscillator circuit under test with a known highly stable oscillator output and measuring the deviations of the lower frequency beat which would ideally be the same as the desired averaging period τ. Typical methods used to measure the beat deviations vary. The gated counter method is used for the ALSEP transmitter short term oscillator stability measurements. The measurement period τ, chosen to be approximately one second, is measured by a gated counter. Thirty (30) consecutive samples are averaged to determine the average period (τ_{av}). The standard deviation of the individual periods (σ_τ) for the average lot of 30, is obtained from the standard deviation formula:

3) $$\sigma_\tau = \left[\frac{1}{n} \sum_{i=1}^{n} (\tau_i - \tau_{av})^2 \right]^{1/2}$$

= standard deviation around the average period

4) $$\tau_{ave} = \frac{1}{fb} = \frac{1}{n} \sum_{i=1}^{n} \tau_i$$

= average period

The equation to use period measurements, instead of frequency measurements is derived below.

5) $$\tau_i = \frac{1}{fb + \delta fi} = \frac{fb}{fb\,(fb + \delta fi)} = \frac{fb + \delta fi - \delta fi}{fb\,(fb + \delta fi)}$$

$$\tau_i = \frac{fb + \delta fi}{fb\,(fb + \delta fi)} - \frac{\delta fi}{fb\,(fb + \delta fi)}$$

$$\tau_i = \frac{1}{fb} - \frac{\delta fi}{fb\,(fb + \delta fi)}$$

δf_i is much smaller than fb may be ignored in the denominator

6) $$\tau_i = \left[\frac{1}{fb} - \frac{\delta fi}{(fb)^2}\right]$$

$$\Delta\tau_i \quad \left[\frac{1}{fb} - \frac{\delta fi}{(fb)^2}\right] - \frac{1}{fb} \quad (\tau_i - \tau_{ave})$$

7) $$\Delta\tau_i \quad - \frac{\delta fi}{(fb)^2}$$

Since $\tau_i - \tau_{ave} \quad \delta_{\tau_i} \quad \frac{\delta fi}{(fb)^2}$ substitute in equation 3) and obtain:

$$\sigma\tau = \left[\frac{1}{n}\sum_{i=1}^{n}\left(-\frac{\delta fi}{(fb)^2}\right)^2\right]^{1/2}$$

8) $$\sigma\tau \quad \frac{1}{fb^2}\left[\frac{1}{n}\sum_{i=1}^{n}(\delta fi)^2\right]^{1/2}$$

since $$\left[\frac{1}{n}\sum_{i=1}^{n}(-\delta fi)^2\right]^{1/2} \quad \Delta f$$

then $$\sigma\tau = \frac{\Delta f}{(fb)^2}$$

and $$\Delta f = \sigma\tau\,(fb)^2$$

9) $$\therefore \quad S = \frac{\Delta f}{fo} = \frac{\sigma\tau\,(fb)^2}{fo}$$

In use of the beat frequency method, a one cycle beat is not practical to adjust and maintain. Typically, a 1 KHz beat is used and the gated counter is set to measure 1000 periods to arrive at one second. The one second period measurements are therefore an algebraic accumulation of the individual 1 millisecond periods and if divided by 1000, represent the average 1 millisecond variation based on 1 second. In order to use equation 9), this 1000 to 1 relationship must be considered. The statistical sigma for 30 samples of one second periods must be divided by 1000 (same as dividing each 1 sec sample by 1000 prior to obtaining the sigma). The stability factor is actually that of the 1 second periods, but for purposes of using the equation with a fb of 1000, the 1-second sigma must be divided by 1000 to be consistent with the beat frequency. If it was desired to obtain S for 1/10 second periods, the gated counter would be set for 100 counts and the 1/10 second period sigma would be divided by 100 before insertion into equation 9.

For applications using a 1000 Hz beat and a 1000 sample (1 second) averaging time, equation 9) becomes

9) $$S = \frac{\sigma\tau\,(fb)^2}{fo}$$

10) $$S = \frac{\frac{\sigma\tau}{C}\,(fb)^2}{fo}$$

where τ_{ave} is very close to 1.000 and (fb) is very close to 1000, C becomes 1000 and 10) simplifies to

11) $$S = \frac{\sigma\tau\;10^3}{fo}$$

The measurements for the ALSEP Flight transmitter local oscillators provided the following data:

$fo = 142 \times 10^6$

$\tau_{ave} = 1$ sec

$fb = 1000$ Hz

No. Samples = 30

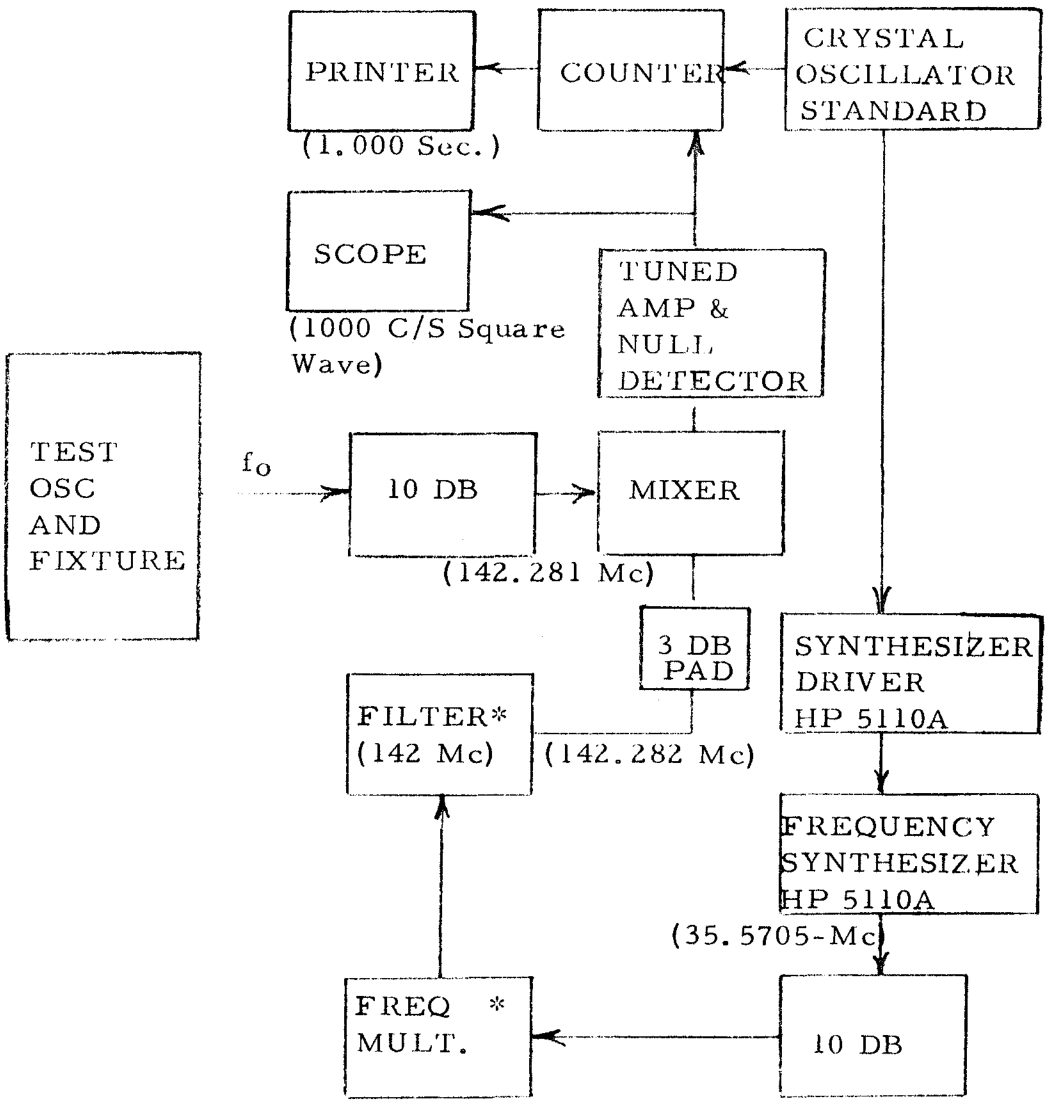
$$\text{Typical } \sigma_T = 95 \times 10^{-7}$$

$$S = \frac{95 \times 10^{-7} \times 10^{3}}{142 \times 10^{6}} = .67 \times 10^{-10}$$

Figure 1 shows the test configuration.

The reference oscillator has an $S = 1 \times 10^{-11}$ for $\frac{\Delta f}{fo}$ and 1 sec averaging period or nearly an order of magnitude better than the unit under test. This accuracy is just sufficient to contribute very little error to the measurements of the test items.

SHORT TERM STABILITY



NOTE: The Test Fixture and unit under test must be enclosed in a temperature shield when short-term stability measurements are to be recorded.

*This equipment is not necessary if the harmonic output of the synthesizer is able to drive the mixer.

Figure 1 Short Term Stability Test Setup

BIBLIOGRAPHY

Short Term Frequency Stability

1. Chapin, Roy L., "A Technique for Short-Term Oscillator Stability Measurement", Proc. IRE, vol. 48, pp. 1914-1915, November, 1960.

2. Morgan, A.H. and Barnes, J.A., "Short-Time Stability of a Quartz Crystal Oscillator as Measured with an Ammonia Maser", Proc. IRE, vol. 47, p. 1782, October, 1959.

3. Rarity, J., Saporta, L. and Weiss, G., "Study of the Short Term Stability of Crystal Oscillators:, 1st Quarterly Redort, July 1, 1960 to September 30, 1960, Contract No. DA 36-039-sc-85376.

4. Rarity, J., Saporta, L. and Weiss, G., "Study of the Short Term Stability of Crystal Oscillators", 2nd Quarterly Report, October 1, 1960 to December 31, 1960, Contract No. DA 36-039-sc-85376.

5. Rarity, J., Saporta, L. and Weiss, G., "Study of the Short Term Stability of Crystal Oscillators", 3rd Quarterly Report, January 1, 1961, to March 31, 1961, Contract No. DA 36-039-sc-85376.

6. Steinberg, H., Schultheiss, D.M., Wagrin, C.A. and Zweig, F., "Short-Time Frequency Measurements of Narrow-Band Random Signals by Means Zero Counting Process", J. Appl. Phys., vol. 26, pp. 195-201, February, 1955.

7. Tanzman, H.D., "Short-Term Frequency Stability Measurements", Proc. 13th Annual Symposium on Frequency Control, p. 384, May, 1959.

8. Winkler, G.R., "Short-Time Stability Measurements on Frequency Standards and the Notion of Resolution", Proc. IRE, vol. 47, p. 100, January, 1959.

9. Short-Term Stability Study - Model NC-1001 Atomichron, Final Report, National Company, Inc., Malden 48, Mass.

10. Victor, Walter K., "The Evaluation of Phase Stable Oscillators for Coherent Communications Systems", JPL-EP337.

11. Hafner, E., "Stability of Crystal Oscillators", Proceedings of the 14th Annual Symposium on Frequency Control, 1960, pp. 192-199.

BIBLIOGRAPHY (CONT'D.)

12. Brown, D.A. and Searle, C.L., "Comparison of Performance Criteria of Frequency Standards", Proceedings of the 16th Annual Symposium on Frequency Control, 1962, USASRADL, Fort Monmouth, N.J.

13. DelGuercio, James V. "Short Term Stability Measurements", Philco-Ford Western Development Labs, WDL-TR-E323, Jan, 1963.

14. Computer Program, Oscillator Short Term Stability by the Gated Counter Method, Philco-Ford Western Development Labs, WDL-KB-165595, 12 November 1965.