Bendix
Aerospace Systems Division

Apollo 14 LRRR Pointing Analysis

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The results of the Pointing Analysis conducted for the Apollo 14 LRRR Experiment are presented. The analysis provides parameters for the design of the LRRR alignment mechanisms. The Littrow site (21° 44' 10" N, 28° 57' E) was the basis for the analysis.

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The pointing analysis for the Apollo 14 LRRR experiment is presented below. The required pointing angles for the reflector array and sun compass are summarized in Section 1. The effect of pointing angle errors are considered in Section 2 and the computational procedures employed in determining the pointing angles are described in Section 3.

1. Pointing Angles Required for Apollo 14 Site

The pointing angles required to point the LRRR array to the mean position of the earth from the Apollo 14 site are summarized in Table 1 which also includes a sketch of the azimuthal angular relationships between the sun compass and reflector array. As indicated in the table, the experiment is assumed to be emplaced in early October, 1970 when the sun angle at the landing site is ~ 15°. The pointing angles however are insensitive to the actual value of the sun angle at the time of emplacement because the gnomon is tipped into the ecliptic plane. The sun's latitude at the time of emplacement will be 1.002° South. If the landing date is postponed, the sun's latitude, and hence the sun compass settings, will change. Definitions of terms employed in Table 1 are as follows:

Array Tilt Angle is the angle between the zenith vector and the normal to the reflecting surface of the array.

Shadow Mark Angle is the angle from the tilt axis of the array to the shadow mark line. The angle is measured counterclockwise in the horizontal plane when viewed from above.

Tilt Axis is the axis about which the reflector array is rotated to achieve the required tilt angle. The sense of the tilt axis is defined by the direction of advance of a right-handed screw turned through the tilt angle.

Gnomon Tip Angle is the angle between the zenith vector and the gnomon.

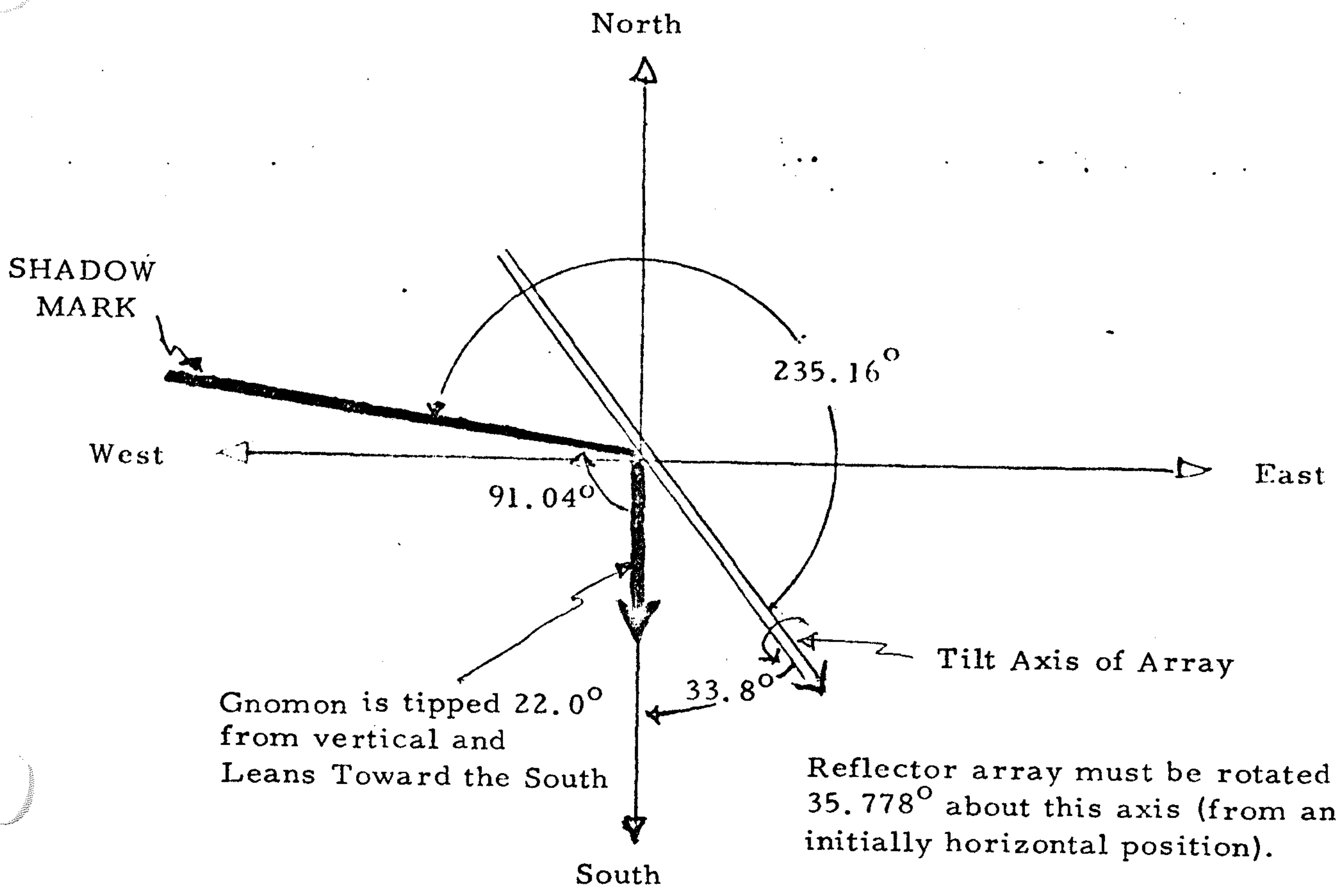
When fabricating the sun compass, it should be noted that the shadow mark angle is more critical than the orientation of the gnomon. Thus, while the

TABLE 1

POINTING ANGLE REQUIREMENTS FOR APOLLO 14 LRRR

Landing Site:	28° 57' 0" E, 21° 44' 10" N
Landing Time:	Emplacement in early October, 1970 Sun Angle ~ 15°, Sun Latitude 1.002° S.
Array Tilt Angle:	35.778° from zenith to normal of array
Shadow Mark Angle:	235.16° from tilt axis to shadow mark
Gnomon Alignment:	Tipped 22° from vertical and tip of gnomon projects toward the South which is -33.80° from tilt axis.

Sketch of Azimuthal Angular Relationships Given Above



sketch in Table 1 indicates that the gnomon should be 91.04° from the shadow mark and 33.80° from the tilt axis, neither of these angles have to be set accurately. However, the sum of the angles (91.04 + 33.80 = 124.84 = 360° - 235.16°) should be controlled as precisely as possible in order to insure the proper orientation of the shadow mark with respect to the tilt axis.

2. Aiming Errors

2.1 Specific Sources of Aiming Error

The effects which specific types of alignment errors (due either to design, fabrication, or astronaut emplacement) will have on the accuracy with which the reflector array is pointed to the mean position of the earth are summarized in Table 2. For each source of error, the aiming errors are tabulated as a function of sun angle at the time of emplacement. The tabulated values for the aiming error give the angular displacement between the mean position of the earth and the aimpoint of the array.

The first entry in the table assumes no alignment errors of any kind and illustrates the dependence of aiming error on sun angle at the time of emplacement of the experiment. With the tilted gnomon, the aiming error produced by a variation in sun angles from 5° to 25° is less than 0.02°. For comparison, the second entry in the table presents corresponding data for a vertical gnomon with no other sources of alignment error except the effect of variable sun angle. The shadow mark, in this case, is assumed to be at the proper position for a vertical gnomon at 15° sun angle (i.e., ≠ 235.16°). The data show that the variation in sun angle could introduce significant aiming error, ∼2.5°, if the gnomon were vertical.

The third entry shows that a ± 5° error in the azimuthal orientation of the gnomon will produce, at most, a 0.05° aiming error whereas entry #4 shows that a ± 5° error in shadow mark can produce a 2.95° aiming error. This illustrates the importance of shadow mark alignment, as compared to gnomon alignment, noted in Section 1. The error in shadow mark alignment could arise from either erroneous marking of the sun compass during fabrication or from astronaut misalignment of the sun shadow. Both sources of error would produce the same degree of aiming error.

The inaccurate setting of the gnomon tip angle considered in item 5, indicates that it has a relatively small effect on aiming error since the aiming error is roughly equal to 0.15 times the error in the tip angle. Item 6 however shows that any error in the array tilt angle produces a corresponding error in the aim. Thus, the tilt angle setting is also more critical than the gnomon tip angle setting. The error in tilt angle also may be caused by either the fabrication process or inaccurate leveling

TABLE 2

EFFECT OF MISALIGNMENTS ON AIMING ACCURACY

Item	Misalignment Contributing to Aiming Error	Magnitude of Misalignment	Resultant Aiming Error for Emplacement at Sun Angle of 5°	15°	25°
1	No alignment errors	-	0.0°	0.0°	0.02°
2	Vertical gnomon with no alignment errors	-	-2.25°	0.0°	2.45°
3	Azimuthal alignment of Gnomon	+5°	0.01	0.01	0.03
		-5°	0.01	0.02	0.05
4	Alignment of, or on, shadow mark	-5	-2.92	-2.92	-2.90
		-1	-0.58	-0.58	-0.56
		+1	0.59	0.59	0.61
		+5	2.93	2.93	2.95
5	Gnomon tip angle	+1	-0.05	-0.16	-0.27
		-1	0.05	0.17	0.31
6	Array tilt angle	+1	1.00	1.00	1.00
		-1	1.00	1.00	1.00
7	Location of Landing site	+1° Longitude	-1.00	-1.00	-0.99
		-1° Latitude	-0.92	-1.01	-1.10
		Both of above	-1.38	-1.50	-1.60

The algebraic signs assigned to the values for aiming error indicate whether aim point is to the East or West of the meridian on celestial sphere which passes through the mean earth point.

of the experiment at the time of emplacement. The data for landing site location in item 7 indicates that the aiming error is approximately equal to the angular displacement, on the lunar surface, from the intended landing site. That is, if the actual landing site differs from the intended site by 1° in both longitude and latitude, the resultant aiming error would be approximately equal to $\sqrt{1^2 + 1^2} = 1.42^\circ$.

2.2 Estimated Nominal and Worst Case Aiming Error

Aiming error produced by specific types of misalignment have been considered above. Estimates of the net aiming error resulting from all sources of error are presented in the following paragraphs.

The two general categories of error contributing to misalignment of the array are those due to: (1) fabrication tolerances anticipated during manufacture, and (2) misalignment errors incurred during emplacement on the lunar surface. For the previous LRRR package, a third category of error due to inherent design limitations was also considered. This source of error is essentially zero for the Apollo 14 mission (see aiming error estimates for item #1 in Table 2) because the landing site and time are more precisely defined. In contrast, the previous package had to be designed for emplacement at any one of five dispersed landing sites and at any time during a period of approximately one year.

Estimates of misalignment errors due to manufacturing and emplacement will be resolved into θ and ϕ components of error where θ represents an error in the polar orientation (tilt angle) of the array and ϕ represents an error in the azimuthal orientation (East-West alignment) of the base pallet. The net aiming error, α, is then given by

$$\alpha = (\theta^2 + \phi_w^{\ 2})^{1/2} \qquad (1$$

where $\phi_w = \phi \cdot \mathrm{Sin}\,(T_A + \theta/2)$ is an appropriately weighted ϕ-component of error. The angle α is the angular deviation between the mean position of the earth and the aim point of the LRRR array. *

Estimates of the probable errors that could result during fabrication and emplacement of the experiment package are presented in Part A of Table 3. Both nominal and worst case estimates of the magnitude of error are indicated. Those sources of error have been combined, using Equation 1 to determine the net aiming error, α, which is tabulated in Part B of the table. In calculating the net aiming error, the individual contributions to the θ and ϕ components of error were assumed to be additive.

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*T_A is the array tilt angle.

TABLE 3

ESTIMATES OF NOMINAL AND WORST CASE AIMING ERROR

		Estimate of Error*	
		Nominal	Worst Case
A)	Contributions to Aiming Error		
	1) Fabrication Process:		
	a) Alignment of tilt angle relative to reference plane of bubble level	± 0.18°	± 1.5°
	b) Alignment of shadow mark relative to tilt axis	± 0.12°	± 1.0°
	c) Positioning of Gnomon	± 0.03°	± 0.25°
	d) Alignment of corner reflectors relative to array	± 0.04°	± 0.25°
	2) Emplacement on Lunar Surface		
	a) Alignment of shadow on shadow mark	± 1.5°	± 5.0°
	b) Accuracy in leveling experiment	± 1.5°	± 5.0°
B)	Net Aiming Error, α		
	1) Fabrication Process only	± 0.25°	± 2.1°
	2) Emplacement Process only	± 1.75°	± 5.9°
	3) Both Fabrication and Emplacement	± 2.0°	± 8.0°

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*The terms "Nominal" and "Worst Case" can be interpreted as "one-sigma" and "three-sigma" estimates of error, respectively.

3. Computational Method

Two selenocentric coordinate systems are employed in the calculation of tilt angle and shadow mark location. One represents a cartesian coordinate system at the subearth point and the other a cartesian coordinate system at the local landing site. In each system, the z-axis is defined to be colinear with the local zenith vector and the x-axis is constrained to lie in the plane of the local meridian and point toward the lunar south pole. The coordinate notation (x, y, z) therefore corresponds to the directions (south, east, zenith) at the respective lunar sites.

The first step in the calculation is to determine the direction cosines to the mean earth position and to the sun, with respect to the coordinate system at the subearth point. By definition, the direction cosines to the mean earth position are (0, 0, 1). The direction cosines, SUBSUN, to the sun are:

$$\text{SUBSUN} = (-\text{Sin } T,\ \text{Cos } T \cdot \text{Sin } L,\ \text{Cos } T \cdot \text{Cos } L) \qquad (1$$

when the sun is at the selenographic longitude L (measured positive to the East) and latitude T (measured positive to the North).

The direction cosines with respect to the subearth point are then transformed to the landing site coordinate system using the rotation matrix SHIFT where

$$\text{SHIFT} = \begin{pmatrix} \text{Cos } \tau & \text{Sin } \tau \cdot \text{Sin } \lambda & \text{Sin } \tau \cdot \text{Cos } \lambda \\ 0 & \text{Cos } \lambda & -\text{Sin } \lambda \\ -\text{Sin } \tau & \text{Cos } \tau \cdot \text{Sin } \lambda & \text{Cos } \tau \cdot \text{Cos } \lambda \end{pmatrix} \qquad (2$$

and λ is the selenographic longitude of the landing site and τ is the latitude. Thus, the direction cosines, EARTH, to the mean position of the earth with respect to the local coordinate system at the landing site, are

$$(\text{EARTH}) = (\text{SHIFT}) \cdot (\text{ZENITH}) \qquad (3$$

where ZENITH is the vector whose direction cosines are (0, 0, 1). Similarily, SUNPOS, the position of the sun with respect to the local landing site coordinate system is

$$(\text{SUNPOS}) = (\text{SHIFT}) \cdot (\text{SUBSUN}) \qquad (4$$

3.1 TILT ANGLE

The tilt angle, T_A, that must be built into the array in order that it point to the mean position of the earth from the landing site is obtained from the vector dot-product of the vectors to the local zenith and to the mean position of the earth. Thus

$$\text{Cos } T_A = \overline{\text{ZENITH}} \cdot \overline{\text{EARTH}} \tag{5}$$

A parallax correction, ΔT_A, is added to the calculated tilt angle to account for the fact that the vector EARTH is referenced to a selenocentric coordinate system rather than one with the origin of coordinates on the surface of the moon. The parallax correction for transforming from the selenocentric coordinates to the surface is

$$\Delta T_A = 4.5218 \times 10^{-3} \cdot \text{Sin } T_A \tag{6}$$

where ΔT_A is in units of radians. In degrees, this correction is approximately $\Delta T_A \approx 0.26^\circ \cdot \text{Sin } T_A$

3.2 Shadow Mark Location

The first step in defining the shadow mark location is to determine the position of the shadow cast by the gnomon. For the high latitude Apollo 14 site, the gnomon will be tipped into the plane of the ecliptic in order to minimize the effect of variable sun angle on the position of the shadow. The direction cosines of the gnomon stem can therefore be specified as

$$\text{GNOMON} = (\text{Sin } \theta \cdot \text{Cos } \phi, \text{ Sin } \theta \cdot \text{Sin } \phi, \text{ Cos } \theta)$$

where θ is the angle at which the gnomon is tipped from the z-axis and ϕ is its azimuthal orientation with respect to the x-axis. If the gnomon is of unit length and its base is at the origin of coordinates, the components of GNOMON will also be the cartesian coordinates of the tip of the gnomon. A sun ray which grazes the tip of the gnomon will also form the tip of the shadow cast by the gnomon so the position of the shadow can be determined by use of the relation

$$\overline{X}_f = \overline{X}_i + \overline{\alpha} \cdot \ell \tag{7}$$

where $\overline{X}_i$ and $\overline{X}_f$ are the initial and final coordinates of a position vector, $\overline{\alpha}$, are the direction Cosines of the vector connecting $\overline{X}_i$ to $\overline{X}_f$ and ℓ is the distance between $\overline{X}_i$ and $\overline{X}_f$. Since the sun shadow is cast in the x-y plane where z = 0, it follows that the distance from the tip of the gnomon to the tip of its shadow is ℓ where

$$0 = \text{GNOMON}(3) + \ell \cdot (-\text{SUNPOS}(3))$$

or

$$\ell = \text{GNOMON}(3) \div \text{SUNPOS}(3) \qquad \text{(8}$$

and the notation GNOMON (3) indicates the third or z- component of GNOMON, etc. It follows that XS and YS, the x and y coordinates of the tip of the gnomon shadow, are

$$\begin{aligned} XS &= \text{GNOMON}(1) - \text{GNOMON}(3) \cdot \text{SUNPOS}(1) \div \text{SUNPOS}(3) \qquad \text{(9} \\ YS &= \text{GNOMON}(2) - \text{GNOMON}(3) \cdot \text{SUNPOS}(2) \div \text{SUNPOS}(3) \end{aligned}$$

Since the shadow line begins at the base of the gnomon, which is also the origin of coordinates, the direction cosines of the shadow line are

$$\text{SHADOW} = (N \cdot XS,\ N \cdot YS,\ 0) \qquad \text{(10}$$

where N is the normalization constant. Parallax corrections to the position of the sun caused by shifting the origin of coordinates from the center of the moon to the base of the gnomon at the lunar surface are very small ($\lesssim 0.0007^{\circ}$) so they have not been included in the calculation.

Equation 10 defines the position of the shadow line, and hence the required shadow mark location, in terms of the local coordinate system. For fabrication of the experiment package, it is more convenient to specify the shadow mark location with respect to the tilt axis of the reflector array. The position of the tilt axis can be determined from the following vector cross-product

$$\overline{\text{AXIS}} = \overline{\text{ZENITH}} \times \overline{\text{EARTH}} \qquad \text{(11}$$

and the components of AXIS must be renormalized to form a unit vector. The shadow mark location will be specified by the angle S measured from the tilt axis to the shadow line. Therefore

$$\begin{aligned} \text{Cos } S &= \overline{\text{AXIS}} \cdot \overline{\text{SHADOW}} \\ \text{and} \qquad & \qquad\qquad\qquad \text{(12} \\ \text{Sin } S &= \left| \overline{\text{AXIS}} \times \overline{\text{SHADOW}} \right| \end{aligned}$$

Since the sine and cosine of the shadow mark location are defined by Equation 12, its value is uniquely determined.

3.3 Equations for Estimating Design Parameters

The results presented in Section 1 of this memo were obtained with a computer program which employs the procedures described above to calculate the required tilt angle and shadow mark location. Specific formulas for calculating tilt angle and shadow mark location have also been developed from the procedures. For example Equation 5 for the tilt angle can be reduced to the form

$$\text{Cos } T_A = \text{Cos } \tau \cdot \text{Cos } \lambda \qquad (13$$

The parallax correction in Equation 6 should also be added to estimates of T_A obtained with this formula.

An expression for the shadow mark location can be obtained by introducing the angle ψ which is defined to be the azimuthal position of the shadow mark measured clockwise from the West (or -y) direction. It follows from this definition that

$$\tan \psi = \frac{-XS}{-YS} \qquad (14$$

where XS and YS are the coordinates of the end point of the gnomon shadow as defined in Equation 9. Substituting the expressions for XS and YS into Equation 14 and assuming that:

1) $\phi = 0^\circ$ i.e., the azimuthal orientation of the gnomon is such that it always lies in the plane of the local meridian

2) T, the sun's selenographic latitude, is small so that Cos T = 1 and Sin T = T

it can be shown that

$$\tan \psi = \tan H \cdot \left\{ \text{Sin } \tau - \tan \theta \cdot \text{Cos } \tau \right\} - T \cdot \text{Sec } H \cdot \left\{ \tan\theta \cdot \text{Sin } \tau + \text{Cos } \tau \right\} \qquad (15$$

where H is the sun angle (i.e., $L = \lambda + \pi/2 - H$), θ is the angle at which the gnomon is tipped from the vertical and τ is the latitude of the landing site.

Because the lunar equator and ecliptic are not coplanar, the gnomon can not be positioned so that it will always lie in the ecliptic plane. Therefore, the criterion that has been adopted for determining the gnomon tip angle θ is that $d(\tan \psi)/dH$ should be equal to zero at the estimated time of emplacement. With this criterion, the gnomon tip angle must be set at

$$\theta = \tau - T \cdot \operatorname{Sin} X \qquad (16$$

where X is the sun angle at which the emplacement of the experiment is expected to occur. Since $d(\tan \psi)/dH = 0$ for this value of θ, the gnomon shadow will not be in motion at the expected time of emplacement. Substituting Equation 16 into Equation 15 gives the following expression for the shadow mark position

$$\tan \psi_T = \frac{T \cdot (\operatorname{Sin} x \cdot \tan H - \operatorname{Sec} H)}{\operatorname{Cos} \tau + T \cdot \operatorname{Sin} x \cdot \operatorname{Sin} \tau} \qquad (17$$

where the subscript T on ψ indicates that the gnomon is tipped at the angle specified by Equation 16. If the gnomon is not tipped but remains vertical, the position of the shadow mark is given by

$$\tan \psi_v = \operatorname{Sec} H \left\{ \operatorname{Sin} H \cdot \operatorname{Sin} \tau - T \cdot \operatorname{Cos} \tau \right\} \qquad (18$$

where the subscript v indicates that the expression is valid for a vertical gnomon. The position of the shadow cast by a vertical gnomon continuously changes with sun angle.

The equations above have been used to check the results of the computer program. The program, however, has more general validity since it is not based on the assumptions that $\phi = 0$, T is small, etc.