The results of the LRRR pointing analysis conducted for the new Apollo 14 lunar landing site (Fra Mauro) are presented. The analysis provides parameters for the design of the LRRR alignment mechanism. The Fra Mauro coordinates, which were the basis for the analysis, are as follows: 17° 29' 3" W, 3° 40' 7" S. Results are presented for monthly launch dates from December 1970 through April 1971. The parameters for a launch about 1 February 1971, based on a tilted gnomon, were selected for the design. Errors resulting from a launch in other months, during the period analyzed, are insignificant and no adjustment of shadow marks is required.

These results supersede those provided in ATM-870 for the former Apollo 14 site (Littrow). The computational procedures described in ATM-870 are still valid and it should be retained for reference purposes.

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The pointing angle requirements for the LRRR reflector array and sun compass at the Fra Mauro site are presented in Section 1. The effect of launch date changes and pointing angle errors are considered in Section 2. The computational procedures employed in determining the pointing angles have been described in ATM-870.

1. Pointing Angles Required for the Fra Mauro Site

The pointing angles required to aim the LRRR array to the mean position of the earth from the Fra Mauro site are summarized in Table 1. The position of the shadow mark line on the sun compass is dependent on the time at which the experiment is emplaced on the lunar surface and on whether the gnomon is vertical or tipped into the plane of the ecliptic. Table 1 contains shadow mark angle specifications for both vertical and tipped gnomons and for emplacement during the months of December through April at a local sun angle of $15^{\circ}$ at the Fra Mauro site. Based on the aiming error analysis presented in Section 2.1, it is recommended that the "February-tipped gnomon" specification be adopted for the flight hardware.

Definitions of the terms employed in specifying the pointing angles are as follows:

Array Tilt Angle - is the angle between the zenith vector and the normal to the reflecting surface of the array.

Shadow Mark Angle - is the angle from the tilt axis of the array to the shadow mark line. The angle is measured counterclockwise in the horizontal plane when viewed from above.

Tilt Axis - is the axis about which the reflector array is rotated to achieve the required tilt angle (from an initially horizontal position). The sense of the tilt axis is defined by the direction of advance of a right-handed screw turned through the tilt angle

Gnomon Tip Angle - is the angle between the zenith vector and the gnomon. For the Fra Mauro site, the gnomon is constrained to be in the plane of the local meridian with the tip of the gnomon displaced to the north.

## 2. Aiming Errors

The accuracy with which the LRRR array will be pointed to the mean position of the earth is considered below. The effect of launch date changes on aiming accuracy and/or sun compass specifications are discussed in Section 2.1. The effects of fabrication tolerances and astronaut emplacement accuracy are included in Section 2.2.

### 2.1 Launch Date Trade-Offs for Sun Compass

The shadow mark specifications given in Table 1 assume that the LRRR experiment will be emplaced at a sun angle of 15° during the corresponding months listed in the table. If emplacement should occur at a different sun angle or in a different month than specified, the array will be misaligned. The aiming errors; i.e., angular displacement between the aim point of the array and the mean position of the earth, that will result from changes in sun angle or launch date are presented in Tables 2 (for vertical gnomon) and 3 (for tipped gnomon). Each column of data in these tables give the aiming error that would result if the shadow mark setting indicated at the top of the column were employed to emplace the experiment at the time indicated in the left most column of the table. The shadow mark settings for the column headings are taken directly from Table 1. For example, Table 2, indicates that with a shadow mark setting of 78.50° (the recommended setting for a January landing), the aiming error for emplacement at a 25° sun angle in February would be -0.47°. The algebraic sign affixed to the aiming error data indicates that the aimpoint lies either to the East or West of the Meridian on the celestial sphere which passes through the mean earth position..

The diagonal elements in each table, enclosed in boxes, give the aiming errors that would result during each month if the shadow mark position is adjusted to the setting appropriate for each month. The data indicate that a 10° variation in sun angle will produce about 1/4° aiming error if the gnomon is vertical and essentially no aiming error ($\lesssim$ .01°) if the gnomon is tipped. The tipped gnomon is therefore preferred on that basis. Also, for a fixed shadow mark setting, the aiming errors produced by a change in launch date tend to be smaller for the tipped gnomon than they are for the vertical gnomon.

In the case of the tipped gnomon data presented in Table 3, the gnomon tip angle should also be adjusted for each new shadow mark position. However, the adjustment in tip angle is small and, considering the difficulties inherent in making the change, it would be preferable if adjustment could be omitted.

The last column of data in Table 3 labeled "adjusted, 3.62°N" give the aiming errors that would result if the shadow mark is adjusted for the appropriate month but the gnomon tip angle is permitted to remain at the February design point of 3.62°N. These aiming errors are small and indicate that it would not be necessary to adjust the gnomon tip angle. Data not included in the table indicate that it would not be necessary to change the gnomon tip angle even if it is designed for some month other than February, as assumed here.

It is recommended that the design specifications for a tipped gnomon with landing in February be adopted for the flight hardware because, with the tipped gnomon, the alignment of the array will be insensitive to sun angle at the time of emplacement, and the errors incurred by landing in other months, without changing the shadow mark line, will be minimal.

## 2.2 Estimated Nominal and Worst Case Aiming Error

Three general categories of error which contribute to the misalignment of the array are: (1) trade-offs made in the design specifications for the sun compass, (2) fabrication tolerances anticipated during manufacture, and (3) misalignment errors incurred during emplacement on the lunar surface. Effects of sun compass trade-offs on aiming accuracy has been discussed above. Estimates of the probable errors that could result during fabrication and emplacement of the experiment package are presented in Part A of Table 4. Both nominal and worst case estimates of error are presented. These terms can be interpreted as one-sigma and three-sigma estimates of error, respectively.

The contributions to the net aiming error can be resolved into $\theta$ and $\phi$ components where $\theta$ represents an error in the polar orientation (e.g., tilt angle) of the array and $\phi$ represents an error in the azimuthal alignment (e.g., East-West alignment) of the base pallet. The net aiming error, $\alpha$, is then given by

$$\alpha = (\theta^2 + \phi_w^2)^{1/2} \quad (1$$

where $\phi_w = \phi \cdot \text{Sin}(T_A + \theta/2)$ is an appropriately weighted $\phi$-component of error. The angle $\alpha$ is the angular displacement between the mean position of the earth and the aimpoint of the LRRR array. $T_A$ is the tilt angle of the reflector array. For the Fra Mauro site, the weight function, $\text{Sin}(T_A + \theta/2)$ is approximately equal to 0.31, but is weakly dependent on the $\theta$-component of error.

The contributions to the aiming error given in Part A of Table 4 have been combined using Equation 1 and the net aiming errors $\alpha$ are tabulated in Part B of the table. In calculating the net aiming error, the individual contributions to the $\theta$ and $\phi$ components of error were assumed to be additive.

TABLE 1

POINTING ANGLE REQUIREMENTS FOR FRA MAURO LRRR

Landing Site: 17° 29' 3" W, 3° 40' 7" S

Array Tilt Angle: 17.933° from zenith to normal of array

Shadow Mark Angle: (Angle from tilt axis to shadow mark)

| Month | Vertical Gnomon | Tipped Gnomon | Gnomon Tip Angle |
|---|---|---|---|
| December | 78.01 | 77.12 | 3.30°N |
| January | 78.50 | 77.58 | 3.42°N |
| February | 79.30 | 78.33* | 3.62°N* |
| March | 80.15 | 79.13 | 3.83°N |
| April | 80.81 | 79.74 | 4.00°N |

*Recommended Specification for Flight Hardware

Orientation of Tilt Axis: The tilt axis will point 11.48° to the West of North at the landing site.

Gnomon Orientation: For the recommended specification indicated above, the gnomon should be tipped 3.62° from the vertical toward the North.

Latitude of the Sun: The latitude of the sun changes from month-to-month as follows: December, 1.44°S; January, 0.97°S; February 0.20°S; March, 0.63°N; April, 1.27°N.

TABLE 2

SUN COMPASS AIMING ERRORS WITH VERTICAL GNOMON

| Time of Emplacement | | Shadow Mark Set at: 78.01 | 78.50 | 79.30 | 80.15 | 80.81 |
|---|---|---|---|---|---|---|
| December | 5° Sun Angle | .19 | .34 | .59 | .85 | 1.05 |
| | 15 | .0 | .15 | .40 | .66 | .86 |
| | 25 | -.20 | -.04 | .20 | .46 | .67 |
| January | 5 | .05 | .20 | .45 | .71 | .91 |
| | 15 | -.15 | .0 | .25 | .51 | .71 |
| | 25 | -.36 | -.21 | .04 | .30 | .50 |
| February | 5 | -.19 | -.04 | .21 | .47 | .68 |
| | 15 | -.40 | -.25 | .0 | .26 | .47 |
| | 25 | -.62 | -.47 | -.23 | .03 | .24 |
| March | 5 | -.44 | -.29 | -.04 | .22 | .42 |
| | 15 | -.66 | -.51 | -.26 | .0 | .20 |
| | 25 | -.90 | -.75 | -.51 | -.25 | -.04 |
| April | 5 | -.64 | -.49 | -.24 | .02 | .22 |
| | 15 | -.86 | -.71 | -.46 | -.20 | .0 |
| | 25 | -1.12 | -.97 | -.72 | -.46 | -.26 |

TABLE 3

SUN COMPASS AIMING ERRORS WITH TIPPED GNOMON

| Time of Emplacement | | Shadow Mark and Gnomon Tip Angle Set at: 77.12° 3.30°N | 77.58° 3.42°N | 78.33 3.62°N | 79.13 3.83°N | 79.74 4.00°N | Adjusted* 3.62°N |
|---|---|---|---|---|---|---|---|
| December | 5° Sun Angle | .01 | .15 | .39 | .64 | .83 | .02 |
| | 15 | .0 | .15 | .40 | .66 | .86 | .03 |
| | 25 | .01 | .16 | .42 | .70 | .91 | .05 |
| January | 5 | -.14 | .01 | .25 | .50 | .69 | .01 |
| | 15 | -.15 | .0 | .25 | .51 | .71 | .02 |
| | 25 | -.16 | -.01 | .26 | .54 | .75 | .03 |
| February | 5 | -.37 | -.23 | .01 | .26 | .45 | .01 |
| | 15 | -.40 | 0.25 | .0 | .26 | .47 | .0 |
| | 25 | -.42 | -.27 | -.01 | .27 | .48 | -.01 |
| March | 5 | -.63 | -.48 | -.25 | .01 | .20 | -.00 |
| | 15 | -.66 | -.51 | -.26 | .0 | .20 | -.02 |
| | 25 | -.71 | -.55 | -.29 | -.01 | .20 | -.04 |
| April | 5 | -.83 | -.68 | -.45 | -.19 | .00 | -.01 |
| | 15 | -.87 | -.71 | -.47 | -.20 | .0 | -.03 |
| | 25 | -.92 | -.76 | -.50 | -.23 | -.01 | -.07 |

*The data in the last column assume the shadow mark position is adjusted to the preferred location for each month (e.g., 77.12° for December landing, 77.58° for January, etc.) but it is also assumed that the gnomon tip angle remains fixed at the preferred value for the February design date, 3.62°N.

TABLE 4

ESTIMATES OF NOMINAL AND WORST CASE AIMING ERROR

| | Estimate of Error | |
|---|---|---|
| | Nominal | Worst Case |
| **A) Contributions to Aiming Error** | | |
| 1) Fabrication Process: | | |
| a) Alignment of tilt angle relative to reference plane of bubble level | ± 0.18° | ± 1.5° |
| b) Alignment of shadow mark relative to tilt axis | ± 0.12 | ± 1.0 |
| c) Positioning of Gnomon | ± 0.03 | ± 0.25 |
| d) Alignment of corner reflectors relative to array | ± 0.04 | ± 0.25 |
| 2) Emplacement on Lunar Surface | | |
| a) Alignment of shadow on shadow mark | ± 1.5 | ± 5.0 |
| b) Accuracy in leveling experiment | ± 1.5 | ± 5.0 |
| **B) Net Aiming Error, $\alpha$** | | |
| 1) Fabrication Process only | ± 0.25 | ± 2.0 |
| 2) Emplacement Process only | ± 1.58 | ± 5.3 |
| 3) Both Fabrication and Emplacement and no design trade off error | ± 1.8 | ± 7.3 |
| 4) Both Fabrication and Emplacement and including a worst case 0.45° error in sun compass design trade-off. | ± 2.0 | ± 7.5 |