Bendix

Aerospace Systems Division

FRAGMENTATION AND CRATERING AND STRIKING PROBABILITY INVESTIGATION LSPE EXPLOSIVE PACKAGE DETONATION	ATM 1079
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This ATM is written in response to AI-286 which requires BxA to investigate the probability of dust or debris from the detonation of an explosive packaging striking the ALSEP central station or the orbiting CSM.

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ABSTRACT

The Lunar Surface Profiling Experiment (LSPE) includes a chain of explosives which are detonated on the lunar surface.

One purpose of this ATM is to determine the probability of a fragment or a lunar soil (debris) particle striking the Command Service Module (CSM) under the contingent situation that the CSM is still in lunar orbit. The calculations show that if the velocities of either fragments or debris are less than 602 meters per second, the CSM hit probability is zero. Even if it is assumed that all of the explosive (chemical potential) energy is converted into kinetic energy, the probability of the CSM being hit by one projectile greater than or equal to .001 pound is less than 7.04×10^{-7} percent (7.04×10^{-9}).

The second purpose of the ATM is to determine the probability of soil debris striking the ALSEP Central Station. This soil debris results from the cratering of soil directly beneath the explosive. A previous memo, ATM 1046, determined the probabilities of LSPE fragments striking ALSEP Central Station. The calculations presented in the later chapters show that the probability of at least one hit by a single soil particle greater than or equal to .001 pound is less than 5.66 percent (.0566). Even combined with the hit probability by fragments of the case, the probability for at least one hit is still small, i.e., 5.837%. With lunar dust particle size of 800 micron, the eight explosive packages give a maximum value of 1.065 particles/ft^2 relative to the location of the Central Station. Therefore, the ALSEP Central Station may have dust impact due to the soil cratering effect; however, no serious impact or penetration problem is foreseen. It is also shown that any lunar surface proturberance between the LSPE explosive charges and the ALSEP Central Station greatly reduces the probability of soil impact on the ALSEP Central Station.

I. INTRODUCTION

Eight explosive packages will be detonated on the lunar surface for the Lunar Seismic Profiling Experiment. Due to the low gravitational field and high vacuum lunar environments, the trajectories of fragments and debris greatly increase in altitude and range. Therefore, two important questions arise:

(1) Contingent situation: The mission is extended such that the eight explosive packages are exploded before the Command Service Module (CSM) leaves its lunar orbit. What is the probability that the CSM will be hit by Explosive Package (EP) fragments and/or lunar surface particles?

(2) Lunar surface particles striking the ALSEP Central Station: What is the probability that the ALSEP Central Station will be hit by debris from cratering and/or fragments from an EP case? The probability of fragments from explosive packages striking the Central Station has been discussed in ATM 1046.)

Due to the complexity and uncertainty of the numerical values of the parameters involved, the above two questions are investigated separately by two different approaches. Each question is analyzed using assumptions which are conservative, i.e., worst case. For example, the assumption is made that all explosive energy is converted into the kinetic energy of the fragments and/or debris.

The second question involves a parabolic trajectory and the first one an elliptic trajectory. The elliptic trajectory is determined by considering the Moon curvature, the initial velocity and the flight angle which actually enter the ballistic phase of motion. The second question is essentially answered by extending the analysis derived in ATM 1046 (ref 5) to include lunar surface particles projected by cratering. The results of that study are supplemented by the results of this study.

In order to check the assumptions made in this study and in Reference 5, prototype model field tests of the LSPE experiment will include the detonation of actual explosive packages. These explosive packages will be covered by hemispherical domes which have an interior four-inch polyurethane foam layer. This layer is intended to trap debris and fragments. By correlating depth of penetration with mass and velocity and by observing the geometry of the debris entrapment, it is expected that the values of the various parameters (velocities, fragment sizes, trajectories) can be determined with some accuracy. It is expected that test results from these prototype tests will demonstrate that the assumptions used in the analysis are conservative.

II. DEBRIS FROM CRATERING

EP fragments and lunar surface debris are projected by the sudden release of large quantities of gases at high pressure and temperature. In addition to the fragments of the explosive package case and timer mechanism, a crater is formed directly beneath the explosive charge. The size and shape of the crater and the distribution of its debris depend on a number of factors as follows:

(1) Location of the charge: An explosion on the surface of the ground will dig a shallow crater but one that may have considerably greater diameter than a charge buried under the surface of the ground.

(2) The character of the ground soil where the explosion takes place: Since the crater is formed by the scouring action of the blast and of the fragments of the casing, a soft, low density, non-cohesive soil will blow away more easily than a firm rocky one and the crater will be larger in every direction.

(3) Type of explosive involved: By exploding on the ground, a brisant explosive like the HNS used in LSPE may dig a larger crater than a slow-acting explosive.

(4) Distribution pattern: From Reference 1, the crater ejecta from the Apollo thumper is estimated using an on-the-ground explosion and 20-30 mesh quartz sand as a base. The particles obtained a velocity of 0.009 to 0.022 km/sec (29 to 72 feet per second) at an approximate angle of 45°. Reference 2 states that a subsurface explosion will accelerate the material above it to a velocity on the order of .06 km per sec. Since the peak gas pressures decay according to the cube of the range from the blast center and do not accelerate the particles further, the mean velocities of debris may be considered as .06 km per sec with uniform distribution from 0° to 90°.

It is hard to predict the exact volume of cratering for each different charge weight. A very conservative approximate formula is given in Reference 4. It is the Olsen formula which fits brisant explosive on average soil as follows:

$$V = 0.4Q^{8/7}$$

where V is the expected crater volume, in cubic feet, and Q is the weight of exploding charge.

Based on this formula the crater volumes for the eight explosive packages with six different charge weights are calculated in Table I.

TABLE 1 Crater Volume Using Olsen Formula

Q (#)	$Q^{8/7}$	V (Ft^3)	Vx(Volume* of Charge ft^3)	$N = \frac{V}{Vx}$
.125	.096	.0384	.00115742	33.177
.25	.205	.0802	.0023149	34.65
.5	.452	.1808	.0046297	39.05
1.0	1.0	.4	.0092594	43.2
3.0	3.51	1.404	.02778	50.54
6.0	7.75	3.10	.0555563	55.799

*HNS Density = 1.73 g/cc = 107.999 $\#/ft^3$

*Assumed Lunar Soil Density = 1.2 g/cc = 74.91 $\#/ft^3$

III. FRAGMENT VELOCITIES

The purpose of this chapter is to contrast explosive phenomena in air and in vacuum. The conclusion will be made that peak velocities in air (and in the vented domes used for the LSPE prototype tests) are greater than peak velocities in vacuum. This conclusion establishes that the use of test data from atmospheric tests are conservative.

As explained in reference 3, terrestrial airblast overpressures generally vary inversely with the first power of distance from the blast center. On the other hand, lunar blast overpressures resulting from the expansion of vented explosion gases should vary inversely proportional to the cube of the distance from their source. The physical explanation for this phenomenon is that the distance between gas molecules increases more rapidly in vacuum as the gas expands. When the collision between molecules and particles effectively ceases, no more energy can be transferred to the particles to increase their speed. Thus, in vacuum, the net force available to accelerate fragments is smaller.

In Reference 3, it is stated that it is theoretically possible for particles to gain lunar orbital velocity (1680 m/sec) and go into circular orbit around the moon. These particles would be small, 0.3 mm (0.3×10^{-3} meters) or less for a 2-metric-ton charge. The likelihood of circular orbit is negligible, though, because it requires that the particle have the correct speed and inclination and that it not strike any lunar surface feature. By using the SRI [1] data, the maximum velocity (600 m/sec) is less than the orbital velocity (1680 m/sec) required. Based on that comparison, it is clear that no fragments will be in orbit. Therefore, the hit probability on the Command Service Module is zero.

In Chapters IV and V a different approach using a very conservative assumption is used to see what change the CSM has of being hit by at least one fragment. The assumption is that the explosive energy is wholly transferred to the kinetic energy of the fragments which then take an elliptic trajectory.

IV. PROBABILITY OF HITTING THE COMMAND SERVICE MODULE

1. Introduction

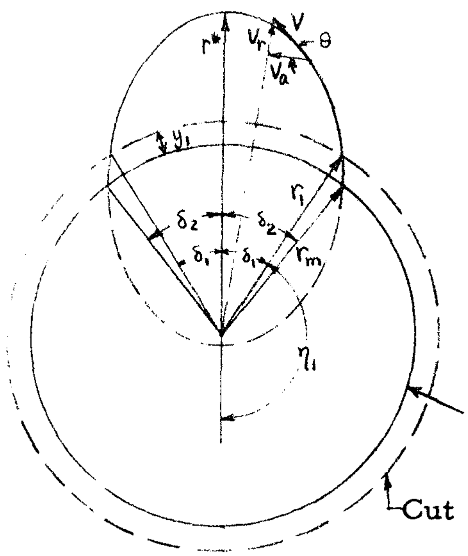
Chapters II & III provide estimates of fragment and soil debris parameters due to explosive package detonation of different charge weights. The purpose of this chapter is to estimate the probability that the CSM could be hit by a fragment or by soil debris. The factors involved in a hit possibility are fragment velocity, projected area of space vehicle, type of orbit, orbit period and altitude, terrain clearance, number of charges exploded and the time interval between the explosions. The space vehicle takes a circular lunar orbit at 60 nautical miles with a period of 2 hours. The minimum vertical component velocity to achieve this lunar height is calculated to be 602 meters/second. Reference 1, the SRI report, presents test data that establishes a fragment velocity range of from 200 to 600 meters/second. Based on this report, the hit probability is zero because it cannot quite achieve the altitude of 60 nautical miles.

The remainder of this chapter develops the equations and numerical results necessary to determine CSM hit probability for the very conservative assumption that explosive energy is completely converted into projectile kinetic energy.

2. Derivation of Trajectory Equations

During the lunar seismic explosions experiment, the projectiles, either the fragments from the explosive package or the debris from cratering, follow elliptic paths. Elliptic trajectories take into account the moon's curvature. By contrast for a very short flight range it is permissible to regard the moon's surface as flat. In this flat surface case the free flight path must follow portions of a parabolic trajectory. These two concepts serve their specific purposes: the elliptical flight path provides information on the probability of hitting the Command Service Module and the parabolic flight path predicts the probability of impacting the ALSEP central station. Since the flight range is the key element in specifying the flight path and is dependent on the initial velocity and flight angle, a derivation relating these factors, which can be found in any text on ballistics, e.g. Ref. (2), has been modified and presented here for completeness.

The elliptic trajectory represents the portion of an ellipse which is most remote from the center of attraction, i.e., the region around the apogee. The largest portion of the ellipse lies inside the Moon and therefore has no practical significance. The basic terminology used here is shown in Figure 1.



$r_1 \cong r_m$ in this study

Figure a Elliptic Trajectory

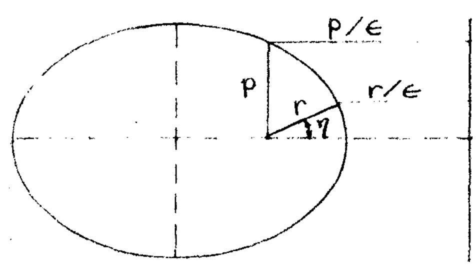


Figure b Elliptic Geometry

Figure 1 ELLIPTIC TRAJECTORY AND GEOMETRY

The equation of the ellipse in polar coordinates is

$$r = \frac{p}{1 + \epsilon \cos \eta} = \frac{p}{1 - \epsilon \cos \delta} \tag{1}$$

where p is the semilatus rectum and ϵ is the eccentricity.

whence

$$\epsilon \cos \eta = \frac{p}{r} - 1 = q - 1 \tag{2}$$

q can also be expressed in the form

$$q = \frac{V^2 \cos^2 \theta}{K/r} = \left(\frac{V}{V_c}\right)^2 \cos^2 \theta = \nu^2 \cos^2 \theta \tag{3}$$

where $\nu = V/V_c$ with V_c = circular velocity and the numerical eccentricity, in terms of q and θ is

$$\epsilon = \sqrt{(1 - q)^2 + q^2 \tan^2 \theta} \tag{4a}$$

whence

$$\epsilon = \sqrt{[1 - \nu^2 \cos^2 \theta]^2 + \nu^4 \cos^2 \theta \sin^2 \theta} \tag{4b}$$

Differentiation of Eq. (2) with respect to time yields

$$\epsilon \sin \eta \frac{d\eta}{dt} = \frac{p}{r^2} \frac{dr}{dt} = \frac{p}{r^2} V_r$$

Dividing this expression by Eq. (2) one obtains

$$\tan \eta \frac{d\eta}{dt} = \frac{\frac{p}{r^2} V_r}{q - 1} \tag{5}$$

By definition

$$r^2 \frac{d\eta}{dt} = C = rV_\theta = rV \cos \Theta \tag{6}$$

where C is the constant of the elliptic orbit according to the second Kepler law, representing the total angular momentum per unit mass of the projectile. For $\tan \eta$ in Eq. (5) one can substitute

$$\tan \eta = \frac{pV_r}{C(q-1)} = \frac{q \tan \theta}{q-1}$$

using Eq. (6) and the relation $V_r = V \sin \theta$. Because of $\tan \eta_1 = -\tan \delta_1$ ($\eta_1 = 180 - \delta_1$), the half center angle of the elliptic trajectory becomes:

$$\tan \delta_1 = \frac{q_1 \tan \theta_1}{1 - q_1} = \frac{\nu_1^2 \sin \theta_1 \cos \theta_1}{1 - \nu_1^2 \cos^2 \theta_1} = \frac{\nu_1^2 \sin 2\theta_1}{2 - 2\nu_1^2 \cos^2 \theta_1} \qquad (7)$$

where the subscript 1 refers to the conditions at the point where the ballistic phase is entered. For $\theta_1 \to 0$, one finds $\delta_1 \to 0$, and for $\theta_1 \to 90°$, $\tan \delta_1 \to 1/\tan \theta_1 \to 1/\infty \to 0$. In between, δ_1 is a maximum for a given ν_1.

Solving Eq. (7) for the initial velocity parameter ν_1 yields

$$\nu_1^2 = \frac{2 \tan \delta_1}{2 \cos^2 \theta_1 \tan \delta_1 + \sin 2\theta_1} \qquad (8a)$$

which provides an explicit expression for the initial velocity ν_1 in terms of cut-off altitude r_1 (here $r_1 = r_m$) semi-range angle δ_1 and departure path angle θ_1,

$$V_1^2 = \frac{K}{r_m} \frac{r_m}{r_1} \cdot \frac{2 \tan \delta_1}{2 \cos^2 \theta_1 \tan \delta_1 + \sin 2\theta_1} \qquad (8b)$$

where r_m is the Moon's radius. (r_m = 1741 km), K = 4913.7984 km^3/sec^2 whence the square of the circular velocity at the radial distance r_m from the Moon's center is ($\epsilon_1 = 0$)

$$V_c^2 = \frac{K}{r_m} \cdot \frac{r_m}{r_m} \cdot \frac{2 \tan \delta_1}{2 \tan \delta_1 + 0} = \frac{K}{r_m} = \frac{4913.7984}{1741} = 2.8224$$

$$V_c = 1.68 \ ^{km}/sec$$

For very small ranges ($2\,\delta_1 \leq 2^o$) the term $2 \cos^2 \theta_1 \tan \delta_1$ becomes negligible compared to $\sin 2\theta_1$, $r_m/r \rightarrow 1$ and $2 \tan \delta_1 \rightarrow 2\,\delta_1$, whence

$$V_1^2 \approx \frac{K}{r_m} \cdot \frac{2\delta_1}{\sin 2\theta_1} \tag{8c}$$

or, considering that $\delta_1 = X_1/r_m$, where X_1 is the horizontal range, one obtains for the flat Moon the familiar parabolic expression

$$V_1^2 \approx \frac{K}{r_m^2} \frac{X_1}{\sin 2\theta_1} = \frac{g_m X_1}{\sin 2\theta_1} \tag{8d}$$

where g_m is the gravitational acceleration at the Moon's surface ($g_m = 1.63 \ ^m/sec$).

The radial ("vertical") velocity component is given by

$$V_r = \dot{r} = V \sin \theta = V_a \frac{\epsilon}{q} \sin \theta \tag{9}$$

and the azimuthal velocity component

$$V_a = r\dot{\eta} = V \cos \theta = \frac{a}{r}\sqrt{\frac{K}{a}}\sqrt{1-\epsilon^2} \tag{10}$$

The trajectory angle at any point of the flight path is given by

$$\tan\theta = \frac{V_r}{V_a} = \frac{\epsilon \sin\delta}{1 - \epsilon\cos\delta} \tag{11}$$

The instantaneous flight velocity is

$$V = \sqrt{V_r^{\,2} + V_a^{\,2}} \tag{12}$$

or, from Eq. (6)

$$V = \frac{C}{r\cos\theta} = \sqrt{\frac{Kp}{r\cos\theta}} \tag{13}$$

The velocity at the summit point follows from Eq. (13) with $\eta = 180^o$ or $\delta = 0^o$,

$$V* = V_{a1}\frac{1-\epsilon}{q_1} = V_{a1}\frac{r_1}{r*} = V_c\sqrt{q_1}\;\frac{r_1}{r*} \tag{14}$$

where r* is the distance of the summit point from the center of the moon.

If the orbital energy of a body, which is dissociated from the Moon, is defined as zero, then the energy of a body moving in an elliptic orbit within the gravity field of the Moon is negative and given by

$$h_e = V^2 - V_p^{\,2} = V^2 - \frac{2K}{r} = \frac{K}{r}(\nu^2 - 2) \tag{15}$$

where v_p is the parabolic or minimum "escape" velocity from the gravity field of the Moon. Recalling Eq. (12) and taking into account that at the summit point $V_r = 0$ by definition, so that $V* = V_n$, it follows from Eqs. (12) and (15),

$$V* = \sqrt{V_1^2 - \frac{2K}{r_1} + \frac{2K}{r*}} \qquad (16)$$

This is an alternate expression to Eq. (14). The summit distance follows from Eq. (10) to be

$$r* = r_1\left(\frac{q_1}{1 - \epsilon}\right) \qquad (17)$$

whence the summit altitude is

$$y* = y_1 + r* - r_1 = y_1 + r_1\left(\frac{q_1}{1 - \epsilon} - 1\right) \qquad (18)$$

the range between cut-off point and summit point, measured on the cut-off circle which is the lunar surface in this case ($y_1 = 0$), is given by

$$x_1* = \frac{\pi}{180} \; r_1 \; \delta_1(0) = r_1 \; \delta_1(r) \qquad (19a)$$

and, if measured along the surface,

$$x_1* = \frac{x*_m}{r_1/r_m} = r_m \; \delta_1(r) \qquad (19b)$$

From Eq. (10) follows a convenient relation for the eccentricity ϵ. Since, for $\delta = \delta_1$ we have $r = r_1$, hence $q_1 = 1 - \epsilon \cos \delta_1$ and therefore

$$\epsilon = \frac{1 - \nu_1^2 \cos^2 \theta_1}{\cos \delta_1} = \frac{1 - \delta_1}{\cos \delta_1} \qquad (20)$$

In Eq. (18), let $y_1 = 0$ which is the case of on the ground explosion. Then:

$$y^* = r_1\left(\frac{q_1}{1 - \epsilon} - 1\right) \tag{18a}$$

by using Eq. (3) and Eq. (20), the above equation becomes

$$1 + \bar{C} = \frac{\nu_1^2 \cos^2 \theta_1 \cos \delta_1}{\cos \delta_1 - (1 - \nu_1^2 \cos^2 \theta_1)} \tag{21}$$

where $\bar{C} = {}^{y^*}/r_1$

Using Eq. (7), $\cos \delta_1$ has the form

$$\cos \delta_1 = (1 - \nu_1^2 \cos^2 \theta_1) \Big/ \sqrt{1 - 2\nu_1^2 \cos^2 \theta_1 + \nu_1^4 \cos^2 \theta_1} \tag{22}$$

Substituting Eq. (22) into Eq. (21) and using algebraic manipulation yields the relationships between the initial velocity and initial flight angle with fixed summit altitude

$$\nu_1^2 = \frac{2\bar{C}(1 + \bar{C})}{(1 + \bar{C})^2 - \cos^2 \theta_1} \tag{23a}$$

or

$$\cos^2 \theta_1 = (1 + \bar{C})^2 - \frac{2\bar{C}(1 + \bar{C})}{\nu_1^2} \tag{23b}$$

By plotting Eq. (23b), the lower bound of initial flight angle which will reach the specified altitude with a fixed initial velocity can be defined precisely in Figure 2.

If the altitude and initial flight angle θ_1 are known, δ_1 can be defined by substituting Eq. (23a) into Eq. (7).

$$\delta_1 = \tan^{-1} \frac{2\bar{C}(1+\bar{C}) \sin\theta_1 \cos\theta_1}{(1+\bar{C}) - \cos^2\theta_1 - 2\bar{C}(1+\bar{C})\cos^2\theta_1} \qquad (24)$$

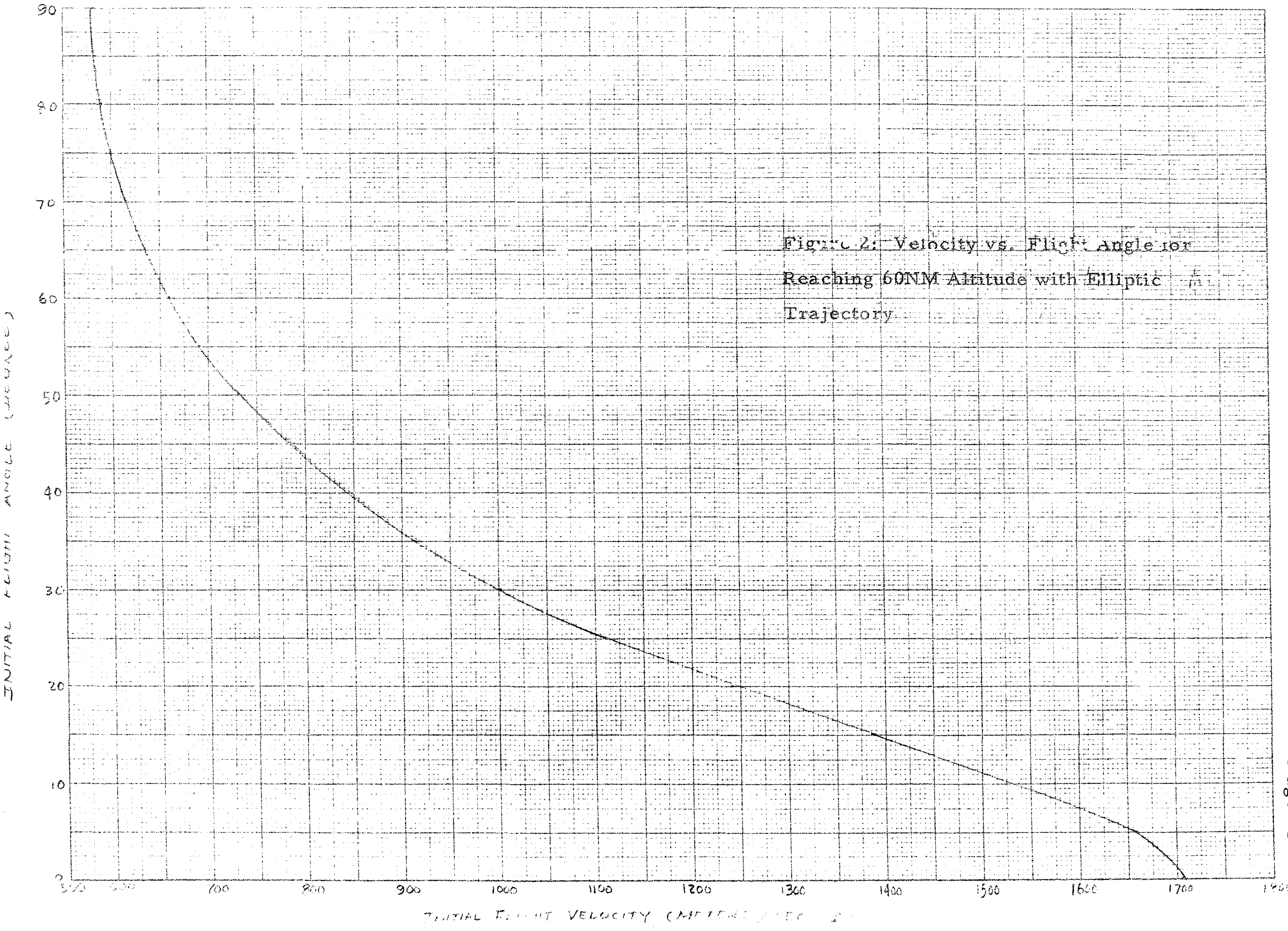


Figure 2: Velocity vs. Flight Angle for Reaching 60NM Altitude with Elliptic Trajectory

3. Probability Formulation

Based on the previous assumption that the explosive energy is wholly transferred into the kinetic energy, it is interesting to check another conversion into potential energy. This potential energy is assumed to lift the case weight upward so as to reach altitude h. The energy conversion relationships are stated as follows:

Explosive Energy = Kinetic Energy = Gravitational Potential Energy

or

$$E = 1/2\, MV_1^2 = Mgh \qquad (25)$$

Where M is the mass of case, V_1 the initial velocity and E can be calculated from the energy per unit weight specified by the manufacturer. For HNS explosive, we have:

$$E = (1.5 \times 10^6 \cdot \frac{\text{ft - lb}}{\text{lb}}) \times (\text{Charge Weight in lbs}) \qquad (26)$$

Since the CSM is in a circular lunar orbit with a two hour period equivalent to an altitude H of 60 NM (.3705 x 10^6 ft), the energy conversion provides a comparison between h and H as shown in Table 2.

TABLE 2: ENERGY CONVERSION AND ALTITUDES COMPARISON

Charge Weight (lbs)	Case Weight (Mg) (lbs)	Explosive Energy E (10^6 ft-lb)	Altitude h Reached (10^6 ft)	Comparison h vs. H	Barometer of Energy Ratio H/h (%)
1/8	2.563	.1875	.07316	h<H	506.4
1/4	2.561	.375	.14643	h<H	253.02
1/2	2.696	.75	.2782	h<H	133.2
1	2.737	1.5	.548	h>H	67.6
3	2.811	4.5	1.6	h>H	23.2
6	2.814	9.0	3.198	h>H	11.6

Table 2 indicates that only three different charges (#1, #3 and #6) can possibly lift the fragments to the CSM altitude. By knowing the initial velocity V_1, the minimum initial flight angle θ_1 which can reach the CSM altitude can be found from Eq. (23b) or Figure 1. Then the half center angle δ_1, of the elliptic trajectory can be defined by using Eq. (7). Since the explosive packages are detonated on the ground surface, the debris from cratering cannot reach the CSM altitude due to small velocities and low flight angles as stated in Chapter 2. Therefore the projectile that may hit the CSM can only come from the fragments of the case. The segment area of sphere covered by the fragments which reach the CSM altitude is defined as

$$A_s = 2\pi \, (r_m + H)^2 \, (1-\cos \delta_1) \tag{27}$$

Inside this segmen surface area, the fragments are assumed to have a uniform distribution from 0^o to 90^o. Each fragment covers an area A_f.

$$A_f = \frac{A_s}{N_f \times \frac{(90^\circ - \theta_1)}{90^\circ}} \qquad (28)$$

Where N_f is the total number of fragments.

The projected area (Ac) of the CSM will assume a normal impact with the fragments. Therefore the probability of the CSM being hit in this specified region is defined as p_o.

$$p_o = A_c/A_f = \frac{A_c}{A_s} \times \left(\frac{90^\circ - \theta_1}{90^\circ}\right) N_f \qquad (29)$$

Since the CSM does not stay in this region all the time, the probability of its being there is defined as $\overline{p}$.

$$\overline{p} = \frac{A_s}{A} \qquad (30)$$

Where A is the surface area of sphere with altitude 60 NM, or

$$A = 4\pi \; (r_m + H)^2 \qquad (31)$$

Counting the characteristic of elliptic trajectory, the fragments will penetrate the 60 NM surface area twice, once outward and once inbound. The probability of hitting the CSM by one specific charge should therefore be:

$$p_i = 2\, p_o\, \overline{p} = 2\frac{A_c}{A_s} \left(\frac{90 - \theta_1}{90}\right) \times \frac{A_s}{A} = \frac{2Ac}{A} \left(\frac{90 - \theta}{90}\right) \qquad (32)$$

The possible detonating time interval for the eight explosive packages varies from a minimum time span between detonation of 6 minutes to a maximum of 114 minutes. Since the CSM has a period of two hours, the location of the CSM relative to the exploding charge is quite random. Conveniently, the striking probabilities are independent of each other. Therefore, the results of the #1, #3 and #6 charges can be summed up as follows:

$$p = \sum_{i=1}^{3} p_i \qquad (33)$$

4. Numerical Data

Since the initial velocity and the minimum flight angle are hard to define precisely, the energy ratio tabulated in Table 2 will be used to estimate the minimum flight angle to reach the CSM altitude according to the equation:

$$\theta_1 = \sin^{-1} \sqrt{\frac{H}{h}} \quad [H \leq h] \qquad (34)$$

Once θ_1 is specified, δ_1 can be calculated by using Eq. (24). At altitude 60 NM, the surface area of this sphere A is calculated.

$$A = 4\pi r^{*2} = 4\pi[(938.75 + 60) \times (6.175 \times 10^3)]^2 = 477.968 \times 10^{12} \text{ ft}^2$$

The projected area A_c of the CSM is calculated using Apollo 15 Model of Figure 3.

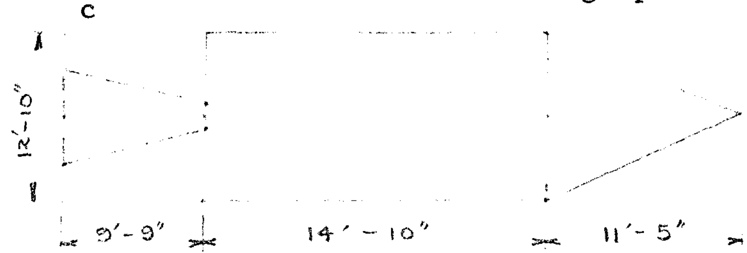


Figure 3 Dimension of the Command Service Module

A_c = 1/2 (12 5/6)·(11 5/12) + (12 10/12)·(14 10/12) + 1/2 (12 10/12)·(9 9/12) = 326.2 ft^2.

With 60 NM altitude, $\overline{C} = \frac{H}{r_m} = \frac{60}{938.75} = .06383$, the probability of hitting the CSM is calculated in Table 3. The resulting probability is very small ($p = 7.041 \times 10^{-9}$).

(1) Charge Weight (lbs)	(2) Energy Ratio H/h (From Table 2)	(3) Minimum Flight Angle θ_1 (Degree)	(4) Half Center Angle δ_1 (Degree)	(5) $1-\cos\delta_1$	(6) $\frac{90° - \theta_1}{90°}$	(7) Segment of Sphere A_s (ft^2) 10^{12}	(8) Case Weight (lbs)	(9) Fragments Based on .001 #/ Fragment N_i	(10) Area Per Fragment A_f (ft^2/Fragment)	(11) Regional Probability $\bar{p}$	(12) Regional Hit Probability p_j	(13) Specific Charge Hit Probability p_i
1	.676	55.31°	4°45'	.00344	.38544	.8221	2.737	2737	7.7928×10^8	.00172	$.4186 \times 10^{-6}$	144×10^{-9}
3	.232	28.79°	12°28'	.02357	.6801	5.6328	2.811	2811	2.9464×10^9	.011785	$.1107 \times 10^{-6}$	2.609×10^{-9}
6	.116	19.91°	18°48'	.05335	.7788	12.7498	2.814	2814	5.8177×10^9	.02668	$.05608 \times 10^{-6}$	2.992×10^{-9}

$$p = \Sigma p_i = 7.041 \times 10^{-9}$$

TABLE 3 PROBABILITY OF HITTING THE COMMAND SERVICE MODULE

V. PROBABILITY OF STRIKING THE ALSEP CENTRAL STATION

In this chapter the probability of striking the ALSEP Central Station is extended to include the soil debris from cratering. The volume of the crater is estimated conservatively by the Olsen formula and is tabulated in column 3 of Table 1. Since the charges are detonated on the lunar surface, the explosion gases themselves constitute the overpressure with no air shock radiated from the blast source. The debris not falling back into the crater is flying outward at low flight angles. According to the test stated in Reference 1 for the on-the-surface explosion, the debris has velocities varying from .009 to .022 km/sec at an approximate angle of 45°. These velocities can be used at the start of the ballistic phase in the theoretical consideration given by Reference 2. Since the equations mentioned in Reference 2 are for detonation beneath the ground surface, modification is needed for the on-the-surface explosion. The total expansion volume at the start of the ballistic phase is assumed to be ten times the true crater volume. Therefore the equations can directly apply to this study and establish the upper bound velocity. The initial ballistic velocity, Vo, of the debris is given as follows.

$$V_o = C_v \left[\frac{V_i + \beta (C_v - V_i)}{C_v + \beta (C_v - V_i)} \right] \qquad (35)$$

with

$$\beta = \frac{\rho_x}{2 \rho_p} \left(\frac{V_x}{10 V_t} \right)^{2/3} \left(\frac{D}{V_x^{1/3}} \right)^{-1} \qquad (36)$$

where

C_v = gas velocity just after venting (assumed C_v = 1500 m/sec)

V_i = mound velocity at the start of the ballistic phase (assumed 9 m/sec ~ 22 m/sec)

ρ_x = initial explosive density (ρ_x = 1.73 g/cc)

ρ_p = debris particle density (ρ_p = 1.2 g/cc)

V_t = true crater volume (given in column 3 of Table 1)

V_x = explosive volume (given in column 4 of Table 1)

D = mean debris dimension

In Chapter 4 and ATM 1046[5], it is indicated that the debris takes a parabolic trajectory when the Moon's surface is assumed to be flat. The hit angle θ as shown in Figure 4 can be defined from Reference 5.

$$\tan \phi = \frac{V_o^2}{gx} \pm \left\{ \left(\frac{V_o^2}{gx}\right)^2 - \left(\frac{2 V_o^2 y}{gx^2} + 1\right) \right\}^{1/2} \qquad (37)$$

where

x = horizontal distance of the station from the blast

y = height of the station

g = lunar gravity

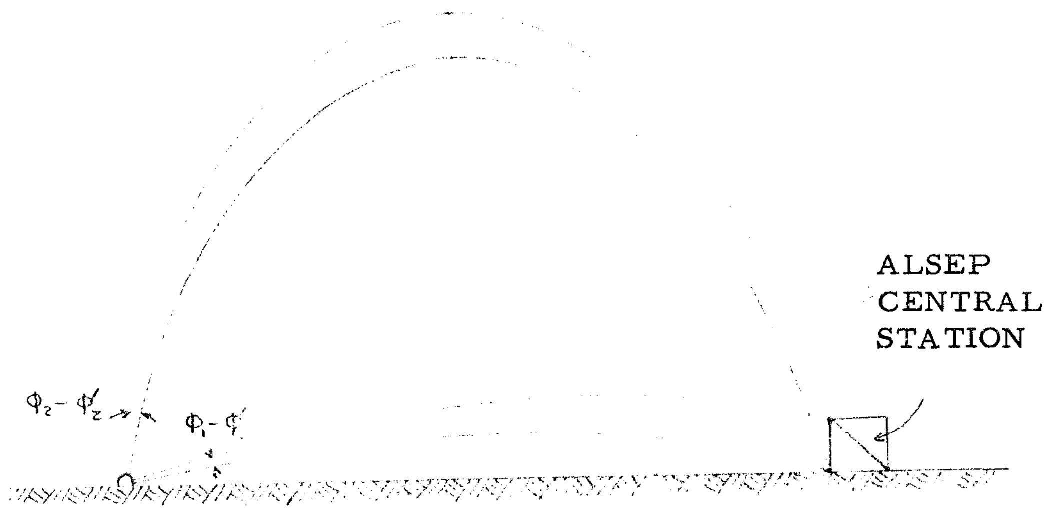


Figure 4: Angles Hitting the Central Station

The unknown on-site soil characteristic makes it hard to predict the break-up size and the distribution pattern of the debris. Since the debris from cratering has a low flight angle and a lot of fall back, some ideal but conservative assumptions are adopted for the probability computation as follows:

(1) No terrain obscurity: The debris follows an exact parabolic trajectory. Any lunar surface protuberances would reduce this probability.

(2) No fall back of debris into the crater.

(3) Uniform distribution of debris between 0° and 90°.

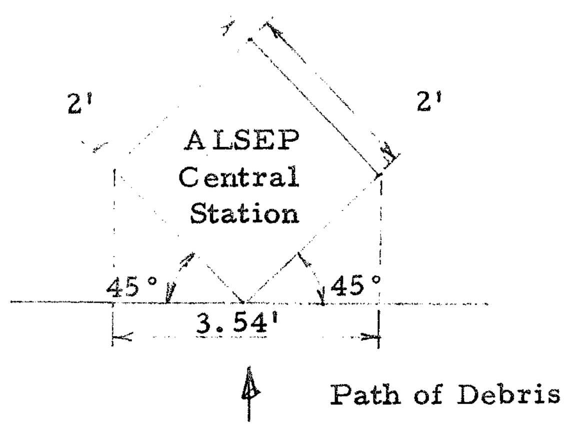
(4) Soil density of 1.2 g/cc used: The top lunar soil has a density varying between 0.7 g/cc to 1.7 g/cc. Since the cratering depth is small, average density of 1.2 g/cc (74.91 $\#/ft^3$) is used.

As discussed in Reference 5, a hit on top of the ALSEP Central Station is presumed to have no significance because of the honeycomb panel.

Based on the set of assumptions listed above the probability of debris striking (the sides of) the ALSEP Central Station is:

$$p_i = N_\alpha \frac{(\phi_1 - \phi'_1) + (\phi_2 - \phi'_2)}{\pi/2} \cdot \frac{3.54 \cdot (.3048)}{2 \quad x} \qquad (38)$$

The value of 3.54 feet comes from the maximum exposure range of side curtains perpendicular to the fragment path shown below.



Since there are eight explosive packages, the probability of at least one hit on the Central Station is summed over the individual probabilities.

$$P = \sum_{i=1}^{8} P_i$$

The probability is calculated and tabulated in Table 4 by using a particle weight of .001 lbs (.4536 grams). The probability for at least one hit is estimated to be less than 5.66%. Even combined with the probability from fragments of the case, the probability for at least one hit is still small (at 5.837%). A close look at those flight angles hitting the Central Station is given in columns 11 and 12 of Table 4. The major part of the striking probability is due to particles launched at low flight angles. It is clear that any lunar surface protuberance could reduce the probability to an even smaller value since it would obstruct particles launched at low angles.

The preceding conclusions are based on a soil particle weight greater than or equal to .001 lb. However, as particle size approaches zero, the number of soil particles approaches infinity and striking of the ALSEP Central Station becomes certain. Now a damage criterion is needed, because particles of very small mass should produce negligible damage. Figure 6 of Reference 6, the Apollo Science Report, indicates that typical lunar soil particles may be as small as 0.8 mm (800 microns). A conservative estimate of dust shower intensity can be made by assuming that the entire crater volume is converted into a shower of spherical soil particles whose diameter is 800 microns. Using a soil density of 1.7 g/cc, the weight of each particle can be calculated, viz. 1.00472 x 10^{-6} lb. (4.557 x 10^{-4} gm). Column 8 of table 5 lists (in millions) the resulting numbers of soil particles. Column 10 of table 5 presents the particle distribution per each package. The eight packages give a maximum value of 1.065 particles per square feet. This amount of dust will have a negligible effect on the thermal control of the ALSEP Central Station or experiments.

(1) CHARGE WEIGHT (LBS)	(2) CRATER VOLUME (FT^3)	(3)* WT. OF DEBRIS (LBS)	(4) DIST. X (METERS)	(5)* MIN. VEL. @ 45° (M/SEC)	(6)* V_x CHARGE VOL. (cm^3)	(7)* $D/V_x^{1/3}$	(8) β EQ.(36)	(10) V_0 EQ.(35) (M/SEC)	(11) ϕ_1/ϕ_1' (DEGREE)	(12) ϕ_2/ϕ_2 (DEGREE)	(13)* N_α	(14) P_i (%)	(15)* $(P_f)_i$ (%)
1/8	.0384	2.8766	125	14.2	32.586	.22603	.06334	79.1	1.2561 / .9065	89.0932 / 89.0931	2877	1.5352	.068
1/4	.082	6.143	250	20.15	65.172	.17932	.0776	91.0	1.60822 / 1.4326	88.5667 / 88.5663	6143	0.8238	.0172
1/2	.1805	13.522	500	28.52	130.344	.1423	.0903	107.2	2.4105 / 2.323	87.677 / 87.677	13522	0.4523	.0045
1	.40	29.965	1000	40.35	260.69	.1128	.10627	114.1	3.8909 / 3.849	86.153 / 86.153	29965	0.25189	.00148
3	1.404	105.177	3500	75.52	782.07	.0783	.140	139.8	8.4203 / 8.9076	81.592 / 81.592	105177	0.07484	.00019
6	3.1	232.978	3500	75.52	1564.14	.0622	.16306	156.5	6.4108 / 6.3981	83.602 / 83.602	232978	0.16318	.00029

*NOTE
COL.(3) = COL.(2) · ρ_s WHERE $\rho_s = 1.2\ g/cc = 74.91\ LBS/FT^3$
COL.(5) = MIN. VEL. AT 45° TO TRAVEL X AND HIT THE CENTRAL STATION
COL.(6) . USING $\rho_x = 1.73\ g/cc$
COL (7) : USING SPHERICAL PARTICLE OF .001 LBS D = .898 cm
COL (13) = COL (3) / .001
COL. (15): FROM COL. (7) OF TABLE II OF ATM1046

$$P = \sum_{i=1}^{8} P_i = 2\times(1.5352+.8238)+.4523+.25189+.07484+.16318 = 5.66\ \%$$

$$P_f = \sum_{i=1}^{8} (P_f)_i = 2\times(.068+.0172)+.0045+.00148+.00019+.00029 = .1769\ \%$$

$$P = P + P_f = 5.837\ \%$$

TABLE 4: HIT PROBABILITY FROM DEBRIS AND FRAGMENTS

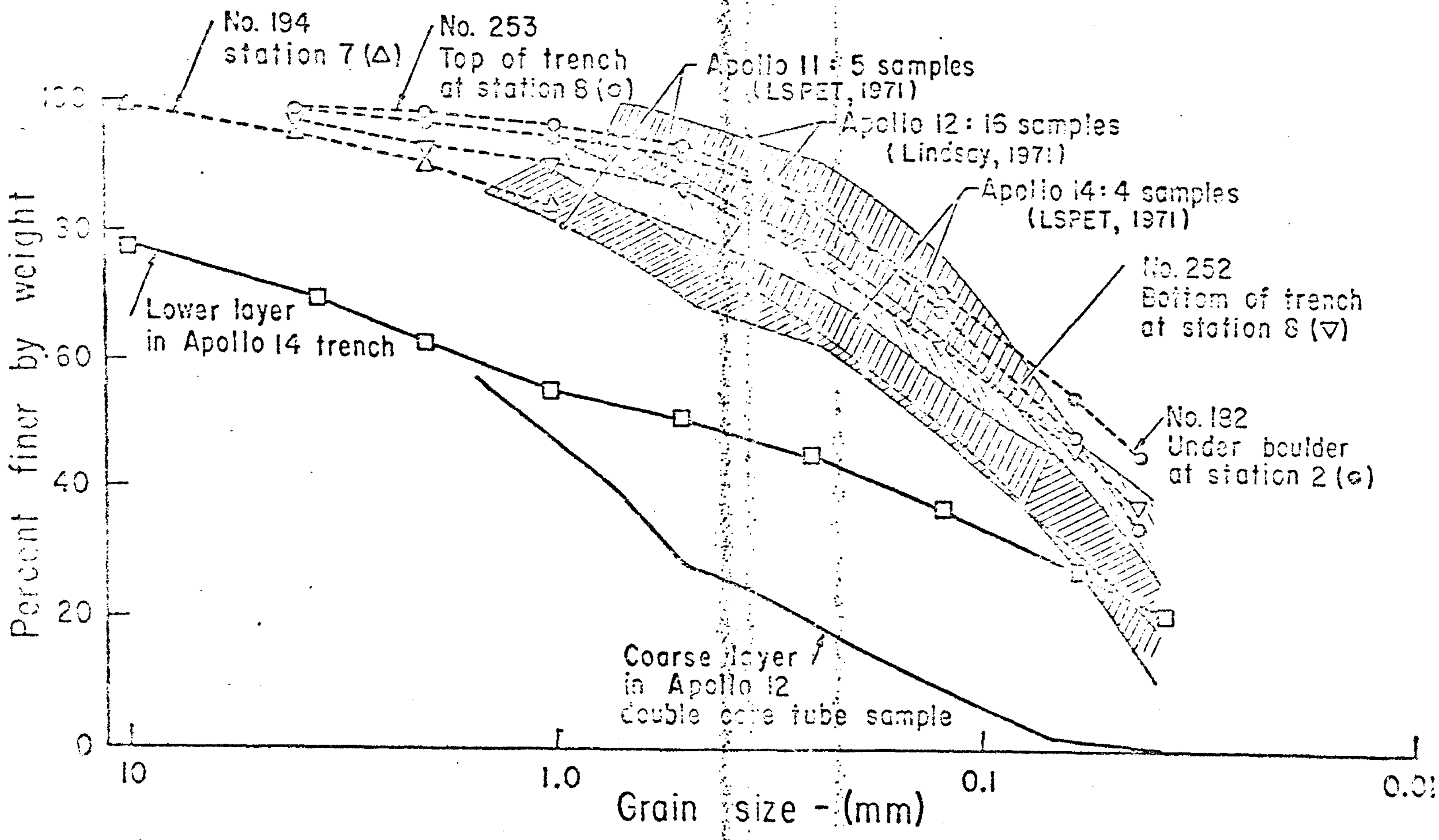

No. 194
station 7 (Δ)
No. 253
Top of trench
at station 8 (o)
Apollo 11 : 5 samples
(LSPET, 1971)
Apollo 12 : 16 samples
(Lindsay, 1971)
Apollo 14 : 4 samples
(LSPET, 1971)
No. 252
Bottom of trench
at station 8 (∇)
No. 192
Under boulder
at station 2 (o)
Lower layer
in Apollo 14 trench
Coarse layer
in Apollo 12
double core tube sample
Percent finer by weight
100
80
60
40
20
0
10
1.0
0.1
0.01
Grain size - (mm)

Fig 6

(1)	(2)	(3)	(4)	(5)	(6)*	(7)*	(8)*	(9)*	(10)*
CHARGE WEIGHT (LBS)	CRATER VOLUME (FT^3)	WEIGHT OF DEBRIS (LBS)	PACKAGE DIST x (METERS)	V_o EQ (15) (M/SEC)	x_m BASED ON V_o (METERS)	Θ	N_s (10^6 PARTICLES)	α	N_i (PARTICLES/FT^2)
1/8	.0384	2.8766	125	79.1	1919.3	.0001588	2.863	.000388	.17641
1/4	.082	6.143	250	91.0	2540.2	.00012	6.1143	.000194	.14237
1/2	.1805	13.522	500	107.2	3525.1	.0000865	13.4586	.00009702	.112904
1	.40	29.965	1000	114.1	3993.5	.00007632	29.8244	.00004852	.11045
3	1.404	105.177	3500	134.8	5995.1	.00005084	104.684	.00001386	.07377
6	3.1	232.978	3500	156.5	7512.96	.00004057	231.8859	.00001386	.13039

*NOTE

COL. (6) = $V_o^2/2g$ $\quad g = 1.63$ m/SEC

COL. (7) = $\Theta = 1 \times .3048 / x_m$

COL. (8) = N_s = COL. (3)/(1.00472×10^{-6})

COL. (9) = $1 \times .3048/(2\pi x)$

COL. (10) = $N_s \times \Theta \times \alpha$

$$N = \sum_{i=1}^{8} N_i = 2(.17641 + .14237) + .112904 + .11045 + .07377 + .13039 = 1.065 \text{ PARTICLES/FT}^2$$

TABLE 5 DISTRIBUTION OF GRAIN SIZE SOIL PARTICLES

VI CONCLUSIONS

The main task of this study concerns the probability of hitting the Command Service Module under the contingent situation that the eight explosive packages detonate on the Moon's surface before the CSM leaves the lunar orbit. The CSM has a circular orbit with a period of two hours and altitude of 60 NM. Based on two different approaches, one using empirical data and the other using theoretical energy considerations, this probability is as follows:

1. If the test data by SRI Report(1) is adopted, the maximum velocity given (600 m/sec) is not sufficient to reach the 60 NM altitude. Therefore the hit probability is zero.
2. With a very conservative approach, converting the entire explosive energy into projectile kinetic energy, the CSM hit probability is calculated to be less than 7.04×10^{-9}.

Since the probability of the CSM being hit by fragments is zero or negligible, no damage criterion concerning the penetration of the space vehicle is needed.

By an extension of the previous work, Reference 5, the probability of striking the ALSEP Central Station is modified to include the debris from cratering as well as the fragments from the case. The data shows that the ALSEP Central Station should have a negligible amount of debris impact.

The total striking probability, by either soil debris of EP case fragment is calculated to be less than 5.837% for the assumptions specified. Note that the major part of this comes from the soil debris 5.66%. The latter probability is for the impact of only one soil particle. The probability that more than one soil particle will strike the ALSEP Central Station is, of course, still less.

For an assumed grain particle size of 800 micron, the eight explosive packages give a maximum value of 1.065 particles per square feet. Any lunar surface protuberance between the explosive charges and the Central Station can greatly reduce the amount of particles reaching the Central Station.

VII REFERENCES

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