This memorandum summarizes the effort to develop a mathematical model of the power dissipation within the PCU and to adapt that model to assist in the central station power/thermal analytical studies that have been required during ALSEP system development.
1. Introduction and Summary

Throughout the ALSEP Program, ATM 449 (Power Budget) has provided a catalog of power availability and power demand associated with the various equipment items of the system. This type of power budgeting has been essential to system power management and to the forecasting of system performance in terms of:

- permissible experiment power demands
- thermal support power availability
- maintenance of voltage regulation under all operational modes
- thermal control of central station electronics.

This budget process was simplified by the fact that all units except the Power Conversion Unit (PCU) have, at most, two or three operational modes in which their power demands differ appreciably. Hence, although the number of operational modes is relatively large, the associated power demands at the output of the PCU are normally predictable. The power dissipation ($W$) associated with the PCU requires a knowledge of this system power demand as well as the power coming in to the system from the RTG, and is determined by summing the electrical dissipations associated with the two PCU functions of power conversion and voltage regulation, viz.:

$$ W = P_{rt} + P_c $$  \hspace{1cm} (1)

where

- $P_{rt}$ = Reserve power dissipated within PCU
- $P_c$ = Voltage conversion losses.

It is the purpose of this memorandum to discuss the history of development of the model used for the prediction of PCU power dissipation and to discuss the application of this model during ALSEP system test and lunar surface operation.
2. Mathematical Modelling of PCU Dissipation

The distribution of power in the ALSEP system can be described in terms of the subdivisions shown in Figure 1 to be

\[ P_{\text{in}} = P_o + P_c + P_r \]  

where

- \( P_{\text{in}} \) = Power into PCU (RTG - OUTPUT)
- \( P_o \) = Power out of PCU (ALSEP LOAD)
- \( P_c \) = Power used in voltage conversion (CONVERSION LOSS)
- \( P_r \) = Power dissipated in shunt regulator (RESERVE POWER)

The last two items are determined by the performance of the PCU. Hence, it is necessary to be able to establish the power usage of the PCU in order to evaluate the balance of power throughout the system. A first approximation of the PCU conversion loss was made during engineering model testing of that unit. At that time it was assumed that the power consumed in the PCU for voltage conversion and associated functions varied linearly with PCU power output in the following manner:

\[ P_c = C + K P_o \]  

where

- \( C = 5.0 \) watts
- \( K = 0.08 \)

The reserve power dissipated within the PCU is simply stated as:

\[ P_{\text{rt}} = P_r - P_{\text{rr}} \]  

where

- \( P_r \) = Total reserve power
- \( P_{\text{rr}} \) = Reserve power external to PCU
Figure 1: Simplified Diagram of Power Distribution
Since \( P_{rr} = I_r^2 \cdot R_{ext} \)

or \( P_{rr} = \left( \frac{P_r}{E_{in}} \right)^2 \cdot R_{ext} \)

then \( P_{rr} = \frac{P_r^2}{P_v} \)  \( (5) \)

where

- \( I_r \) = Current in shunt regulator
- \( R_{ext} \) = Shunt regulator circuit resistance external to PCU
- \( E_{in} \) = Voltage at PCU input

\[ P_v = \frac{E_{in}^2}{R_{ext}} \]

Substituting (5) into (4) yields

\[ P_{rt} = P_r \left( 1 - \frac{P_r}{P_v} \right) \]  \( (6) \)

By definition, the sum of equations (3) and (6), represent the total electrical dissipation within the PCU and can be written as

\[ W = P_r \left( 1 - \frac{P_r}{P_v} \right) + C + K P_o \]  \( (7) \)
When the system parameters other than reserve power \( P_r \) and output power \( P_o \) are available, this model can be expressed in alternate, but equivalent, forms such as,

\[
W = P_s - \left( \frac{P - P_c}{P_v} \right)^2
\]

where

\[
P_s = P_{in} - P_o
\]

and

\[
W = \frac{\left( P_{in} - C - (1+K) P_o \right) \left( P_v - P_{in} + C + (1+K) P_o \right)}{P_v} + C + K P_o
\]

This last equation was investigated for the effect of changes in \( P_{in} \), \( P_o \) and \( P_v \) by use of an analog computer. The results of this analog modelling effort were published in ATM 675, and are reproduced in Figure 2.

To improve the flexibility and accuracy of these calculations, a digital computer program was written which provided a tabulated, printed output of the calculation of \( W \) for selected values of the primary parameters. This program was extended to provide plots of \( W \) vs \( P_o \) in several forms (see Figures 3 and 4) to aid in the analysis of various power distribution problems.

As a result of data obtained from tests performed on both the qualification, back-up and prototype Power Conversion Units, the need for modifying this preliminary model of PCU dissipation became evident. These tests were reported in ATM 753 and revealed, among other things, that \( P_c \) is actually a nonlinear function of \( P_o \). Figures 5 and 6 are reproduced from that report. Since a change in the dissipation model was necessary to reflect this new information, it was decided to also change the parameters used in the model so that it would be useful in the analysis of test and operational data. The only power parameters telemetered in the housekeeping data are:
Figure 2

Central Station
Dissipation (watts)

PCU Output $P_o$

Regulator Range (watts)

$P_V = \frac{(AE)^2}{4.2a} = 60 \text{ watt regulator}$
FIGURE 3
PCU DISIPATION VS PCU OUTPUT (IN WATTS)

Input Power (watts)

55 Watt Regulator
Figure 4

PCU Dissipation vs PCU Output (in Watts)

PCU Input = 65W

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Figure 5
DISSIPATION WITHIN PCU Vs PCU OUTPUT POWER
33 WATT REGULATOR

Total Load on All PCU Outputs (Watts)
Figure 6
DISSIPATION WITHIN PCU vs PCU OUTPUT POWER
55 WATT REGULATOR

Input Power = 74 watts

TOTAL LOAD ON ALL PCU OUTPUTS (Watts)

PCU INTERNAL DISSIPATION (Watts)
Development of a Power Dissipation Model for the Power Conditioning Unit (PCU)

This model was thus modified to express the test-derived values of PCU dissipation in terms of these input parameters.

The amount of reserve power dissipated inside the PCU, (i.e., $P_{rev}$) was already derivable from these parameters using equation (4). The relation between PCU conversion loss and the input power parameters was derived by empirical curve-fitting techniques. The details of this effort are given in Appendix A. The revised PCU dissipation model now has the form

$$W = I_r (E_{in} - I_r R_{ext}) + A + B (P_{in}) + c (P_{in})^2$$

where $I_r$ = shunt current, $R_{ext} = 4.2 \Omega$ (PCU), $P_{in} = RTG$ input power, $P_{rev} = RESERVE$ power

$$E_{in} = 15.90 \, \text{V}$$
$$R_{ext} = 4.2 \, \Omega$$
$$I_{in} = 3.77 \, \text{A}$$

A digital computer program has been written for this relation which provides tabulated and/or analog plotted presentations of the PCU dissipation as a function of $P_{in}$. Typical plots are presented in Figure 7.
PIN PRIME VS. W

$P_{in} = 75^\circ$

$R_{ext} = 4.27^\circ$

FIGURE 7
3. Application of the Revised Model

When substituting specific values for the parameters in this model it is, of course, important to properly interpret the definition of each parameter. Figure 1 illustrates the assumed distribution of power and emphasizes the assignment of all PCU power to "conversion" losses or "reserve power" dissipation. When considering the real PCU (Figure 8) it is necessary to be more specific in the definition of the $P_c$ and $P_{rt}$ terms in the mathematical model. As stated above, the $P_c$ term in the revised model was molded to suit the results of a specific test designed to measure the PCU conversion losses. To make these measurements under a range of load conditions it was necessary to establish the meaning of "conversion loss" so that the test data would be consistent. For purposes of this test the operating point of the regulator servo was set to "cut-off". This point was set by varying the voltage of the power source in such a manner as to yield a maximum voltage drop across the regulator transistor. This establishes a unique condition of power system operation where the regulator is imposing minimum load on the power source and the supply voltages are on the verge of dropping due to excessive load. Hence the measurements of power consumed within the PCU under this condition (and herein designated $P_c$) represent the total PCU dissipation when reserve current is at a minimum.

When the above test measurements were made the data was correlated in terms of input power using a measurement of $E_{in}$ made at the PCU connector. When values of $E_{in}$ obtained during system test or operation (from the System Test Set or from the telemetered data) are substituted into this mathematical model, it must be recognized that these voltage readings may require some correction to represent the voltage at the input connector. The resolution of the telemetered input power parameters introduces an additional uncertainty into the results obtained by this model. To illustrate, the resolution of these parameters at a typical power operating point, in terms of the possible change in the parameter without a change in octal representation, is as follows:

Input Voltage Resolution: $0.08^V@16^V$

Input Current Resolution: $0.0275^A@4.25^A$

Reserve Current Resolution: $0.0172^A$
Figure 2 - PCU Electrical Schematic, Complete
This uncertainty is provided solely by the digitizing process and does not include any of the measurement errors imposed by voltage and temperature changes. These measurement uncertainties, if multiplied together in the most adverse combinations to determine power, yield the following typical resolution products:

\[ \Delta \text{Input Power} = + (16.08 \times 4.2775) - (15.92 \times 4.2225) \]
\[ = 68.7822 - 67.2221 \]
\[ = 1.56 \text{ W} (\approx 1.6 \text{ W}) \]

\[ \Delta \text{Reserve Power} = + (16.08 \times 0.3275) - (15.92 \times 0.2725) \]
\[ = 5.2662 - 4.3382 \]
\[ = 0.928 (\approx 1 \text{ W}) \]
APPENDIX A

Curve-Fitting to PCU Conversion Loss Test Results

On 9 May 1968, a test was performed on the Qualification Back-up Model of the PCU (Part No. 2330000-3, Serial No. 2) to determine the electrical power consumed by the PCU when the regulator is inactive. The procedure for this test was the first 62 pages of the Thermal Mapping Test Procedure for the PCU. The results of this test are plotted in Figure 9 against input power, $P_{in}$. Using the measurement data shown in Table I, rather than these hand-fitted curves, a series of equations of the form

$$P_{cn} = A_n + B_n (P_{in}) + C_n (P_{in})^2$$

where $P_{in} = P_{in} - P_r$

$= P_{in}$ (when $P_r = 0$)

$n$: relevant power converter (1 or 2)

were fitted numerically using simple curvilinear regression techniques.* This curve-fitting process yielded the following relations which are the ones being used to compute the PCU conversion loss, $P_c$, in the revised model:

$$P_{c1} = 3.893 + 0.0205 P_{in} + 0.000759 (P_{in})^2$$

$$P_{c2} = 3.97 + 0.0234 P_{in} + 0.000605 (P_{in})^2$$

The following paragraphs detail the method for determining the coefficients of these relations.

*REFERENCE:
Input Power (with reserve power = 0) --- watts

Figure 9: PCU Conversion Loss Test Results.
TABLE I:
PCU CONVERSION LOSS TEST DATA

<table>
<thead>
<tr>
<th>Power Converter 1</th>
<th>Power Converter 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pin</td>
<td>Pc</td>
</tr>
<tr>
<td>27.044</td>
<td>4.955</td>
</tr>
<tr>
<td>33.201</td>
<td>5.348</td>
</tr>
<tr>
<td>39.921</td>
<td>5.975</td>
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<tr>
<td>45.939</td>
<td>6.430</td>
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<td>52.437</td>
<td>7.197</td>
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<tr>
<td>58.637</td>
<td>7.766</td>
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<tr>
<td>64.719</td>
<td>8.408</td>
</tr>
<tr>
<td>71.964</td>
<td>9.184</td>
</tr>
</tbody>
</table>
Assuming a quadratic curve fit for the data and using a simple curvilinear regression technique, then

\[ P_1 = a + b (P_{in}) + (P_{in})^2 \]

In the following table \( Y = P_1, X = P_{in}, U = X^2 \)

Tabulation for \( P_1 \) Equation

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y )</th>
<th>( X^2 )</th>
<th>( UX )</th>
<th>( U^2 )</th>
<th>( XY )</th>
<th>( UY )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.009</td>
<td>4.336</td>
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<td>5.07 (10^4)</td>
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<td>27.044</td>
<td>4.955</td>
<td>731.38</td>
<td>19,779.4</td>
<td>53.5 (10^4)</td>
<td>134.00</td>
<td>3,624.00</td>
</tr>
<tr>
<td>33.201</td>
<td>5.348</td>
<td>1102.31</td>
<td>36,597.8</td>
<td>1.22 (10^6)</td>
<td>177.56</td>
<td>5,895.52</td>
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<tr>
<td>39.921</td>
<td>5.975</td>
<td>1593.69</td>
<td>63,621.7</td>
<td>2.54 (10^6)</td>
<td>238.53</td>
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<td>6.430</td>
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<td>9.184</td>
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<td>372,688.6</td>
<td>26.82 (10^6)</td>
<td>660.92</td>
<td>47,562.28</td>
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</tbody>
</table>

\[ \Sigma X = 430.02 \quad \Sigma Y = 64.27 \quad \Sigma U = 21,765.60 \quad \Sigma UX = 1,219,349 \quad \Sigma U^2 = 72.73 \times 10^6 \quad \Sigma XY = 3,046.63 \quad \Sigma UY = 164,936 \]

Determine the following auxiliary relations in order to find the parameters \( a, b, \) & \( c \);

\[ M_x = \frac{\Sigma X}{n} = \frac{430.02}{10} = 43 \quad M_y = \frac{\Sigma Y}{n} = \frac{64.27}{10} = 6.427 \quad M_u = \frac{\Sigma U}{10} = 2,176 \]

\[ \Sigma X^2 = \Sigma X^2 - nM_x^2 = 3,275.6 \]

\[ \Sigma UX = \Sigma UX - nM_x M_u = 283,669 \]

\[ \Sigma U^2 = \Sigma U^2 - nM_u^2 = 25.43 \times 10^6 \]

\[ \Sigma XY = \Sigma XY - nM_x M_y = 282 \]

\[ \Sigma UY = \Sigma UY - nM_u M_y = 25,085 \]
b and c can now be found by solving the following equations simultaneously.

\[(\sum x^2)b + (\sum xu)c = \Sigma xy\]
\[(\sum xu)b + (\sum u^2)c = \Sigma uy\]

Then
\[c_i = 0.00759\]
\[b_i = 0.0205\]

and the following relationship is used to determine a;

\[a_i = My - bMx - cMu\]
\[a = 3.893\]

Therefore,

\[P_{ci} = 3.893 + 0.0205 P_{in}' + 0.000759 (P_{in}')^2\]
Using the same assumptions given in Appendix A-1, then

$$P_c2 = a + b(P_{in'}) + c(P_{in'})^2$$

Tabulation for $P_c2$ Equation

TABLE III

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>U</th>
<th>XU</th>
<th>U^2</th>
<th>XY</th>
<th>UY</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.997</td>
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<td>225</td>
<td>3,375.0</td>
<td>5.06(10^4)</td>
<td>66.09</td>
<td>991.35</td>
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<td>441</td>
<td>9,262.8</td>
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<td>4.984</td>
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<td>52.59(10^4)</td>
<td>134.22</td>
<td>3,614.50</td>
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<td>32.980</td>
<td>5.322</td>
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<td>70.519</td>
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</table>

$\sum X = 423.74$  $\sum Y = 61.73$  $\sum U = 21,067.7$  $\sum XU = 1,157,861.7$  $\sum U^2 = 67.74(10^6)$  $\sum XY = 2,849.26$  $\sum UY = 150,444$

$$M_X = \frac{\sum X}{n} = 42.374$$  $$M_Y = \frac{\sum Y}{n} = 6.173$$  $$M_U = \frac{\sum U}{n} = 2,106.8$$

$$\sum x^2 = \sum X^2 - nM_X^2 = 3,112.2$$
$$\sum xu = \sum XU - nM_XM_U = 265,842$$
$$\sum u^2 = \sum U^2 - nM_U = 23.35(10^6)$$
$$\sum xy = \sum XY - nM_XM_Y = 233.8$$
$$\sum uy = \sum UY - nM_M_Y = 20,391$$
Development of a Power Dissipation Model for the Power Conditioning Unit (PCU)

\[
(\sum x^2)b + (\sum xu)c = \sum xy = 233.8
\]

\[
(\sum xu)b + (\sum u^2)c = \sum uy = 20,391
\]

Solving Simultaneously,

\[
c_2 = 0.000605
\]

\[
b_2 = 0.0234
\]

and

\[
a_2 = My - bM_x - cM_u = 3.97
\]

Then

\[
P_{c2} = 3.97 + 0.0234 P_{in} + 0.000605 (P_{in})^2
\]