

Section B

Lunar Surface Geometry

by

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## Section B

### LUNAR SURFACE GEOMETRY

#### Introduction--data

This section provides numerical descriptive information on lunar surface roughness contributed by the geometry of the terrain, without regard for specific features such as craters or blocks. The data were obtained through photoclinometry, the method of deriving slope information from the brightness distribution of monoscopic lunar images by calibrated techniques (Van Diggelen 1951, Watson 1968). Data sources are high resolution imagery (effective resolution one meter) from Lunar Orbiter spacecraft (Missions II, III, and V) and selected terrestrial telescopic photographs (effective resolution 0.75 km).

The photoclinometric method affords a means of gathering large quantities of data in a relatively short time, once the necessary instrumentation and computer software have been assembled. Thus, it has some advantage over the more laborious shadow measurements. As currently developed, the photoclinometric method for reducing spacecraft imagery is suited more to the smoother lunar terrains, the mare, than to the rougher, upland areas (Lambiotte and Taylor, 1967). One meter resolution data for the lunar uplands are comparatively meager and are less reliable than mare data. Hence, considerable reliance has been placed upon the 0.75 km data, originally reduced by Rowan and McCauley (1966). This information has been extrapolated into the resolution range most relevant to vehicle mobility problems.

Photogrammetric terrain data, best obtained from Apollo 8 and Lunar Orbiter V medium resolution images, are not yet available in sufficient quantity. Additionally, photogrammetric reduction of the Orbiter and Apollo 8 imagery does not yield the one meter resolution of which the photoclinometric method is capable, and which is the most useful scale for the present study. The available lunar slope data have been supplemented and enhanced by a slope distribution model of a type previously employed in studies of terrestrial surface roughness conditions.

The most critical question, "How reliable are the high resolution photoclinometric data," has yet to be answered satisfactorily, particularly for the rougher lunar areas. When available, such an evaluation will be made available to users of the information contained in this section. At most, the present terrain models and data should require but a simple recalibration.

#### Classification of Lunar Terrain

Heterogeneity of the Moon's surface character virtually precludes a complete numerical description of all types of terrain that might be encountered by a lunar roving vehicle. The only meaningful alternative is a sampling of representative lunar terrain types or classes. Such a sampling may be as generalized or as inclusive as time and the availability of large scale terrain data permit. These two constraints thus far have limited the number of lunar terrain classes to four: smooth and rough mare and hummocky and rough upland. These four terrain classes are the principle divisions used by Rowan and McCauley (1966). Table 1 shows the topographic features typically

Table 1

## FOUR-PART CLASSIFICATION OF LUNAR TERRAIN

MARE		UPLAND	
Smoother Mare	Rougher Mare	Hummocky Upland	Rough Upland
(1) many Eastern sites	(1) many Western sites	(1) older basin rim material (Fra Mauro fm)	(1) younger basin rim material (Oriental)
(2) dark mare material	(2) rille, dome, & ridge areas	(2) older large craters	(2) younger large craters
(3) older, subdued craters	(3) fresh craters	(3) blanketed craters	(3) scarps
(4) low crater densities	(4) high crater densities	(4) older, subdued crater hash	(4) fresh crater hash
(5) craters with few blocks	(5) blocky craters	(5) outer rim slopes of large craters	(5) inner rim slopes of large craters
	(6) secondary swarms especially on rays	(6) crater floors & basin fill	(6) trenches & rifts
	(7) large crater rims		

included in each category. As increasing data allow, this breakdown will be expanded to a six-part classification (Table 2). It is anticipated that each of these six classes eventually will be subdivided, yielding perhaps as many as two dozen different terrain types.

#### Selection of Descriptive Topographic Parameters

The three parameters chosen to describe the Moon's large scale surface geometry were selected on the following considerations (1) application to profile data, the only format in which photoclinometric data can be obtained reliably; (2) ability to express surface roughness rather than other, interesting, but less appropriate terrain characteristics; (3) ease of interpretation and application to problems of vehicle mobility; (4) ability to characterize surface roughness at any desired base length pertinent to vehicle trafficability problems. The three parameters are power spectral density, slope angle, and angle of slope curvature. The varied expression and presentation of each of these measures for the four gross lunar terrain types is discussed at length below.

#### Power Spectral Density

This parameter expresses the relief frequency content of a terrain profile as a time series, and is used principally to evaluate the dynamic response of a vehicle to different types of terrain (Jaeger and Schuring, 1966, Rozema, 1968). Like slope curvature, it is a measure of relative terrain roughness, and can be somewhat independent of absolute slope angle and regional slopes. The measure is expressed as a full logarithmic graph of power spectral density, in meters<sup>2</sup>/cycle/meter, against frequency, in cycles/meter.

Table 2

## SIX-PART CLASSIFICATION OF LUNAR TERRAIN

Large Craters (over 10-20 km diameter)	MARE		UPLANDS	
	Smoother mare	Rougher mare	Smooth Uplands	Hummocky Uplands
				Rough Uplands

Figures 1-6 present the most recent power spectral density curves available for the Moon. They are a substantial improvement over previous curves (Pike 1968). Two curves, representing roughest and smoothest terrain conditions within each of the four main lunar terrain types, are shown in each diagram. The upper curve in Figure 5 is an educated hypothesis only. Figure 6 shows the maximum roughness range of the entire Moon. While all curves presented here are subject to revision or replacement by still more representative curves, Figures 1-6 probably include most of the roughness conditions present on the Moon.

Power spectral density functions of terrestrial terrains easily bracket the entire range of lunar curves presented here. Figure 7 shows curves for three freshly formed rough terrains, a lava flow surface, a blocky nuclear crater excavated in basalt, and a volcanic fissure vent. These terrains would be trafficable only with the greatest difficulty, if at all. These curves are displaced well above the roughest lunar curve yet obtained. Thus, most lunar terrains are appreciably smoother than these exceedingly rough terrestrial samples and should be traversable with relative security by a roving vehicle.

The frequency range most applicable to vehicle dynamics lies between 0.05 and 0.5 cycles/meter. Expressed more simply, this means that topographic features having base lengths between two and twenty meters have the greatest effect upon vehicle dynamics. Table 3 gives the power spectral density values of these two critical frequencies for the four lunar terrain classes. Maximum and minimum values, as determined by the bounding power spectral density curves in Figures 1-6, are given for each terrain type.

Power Spectral Density (meters<sup>2</sup>/cycle/meter)

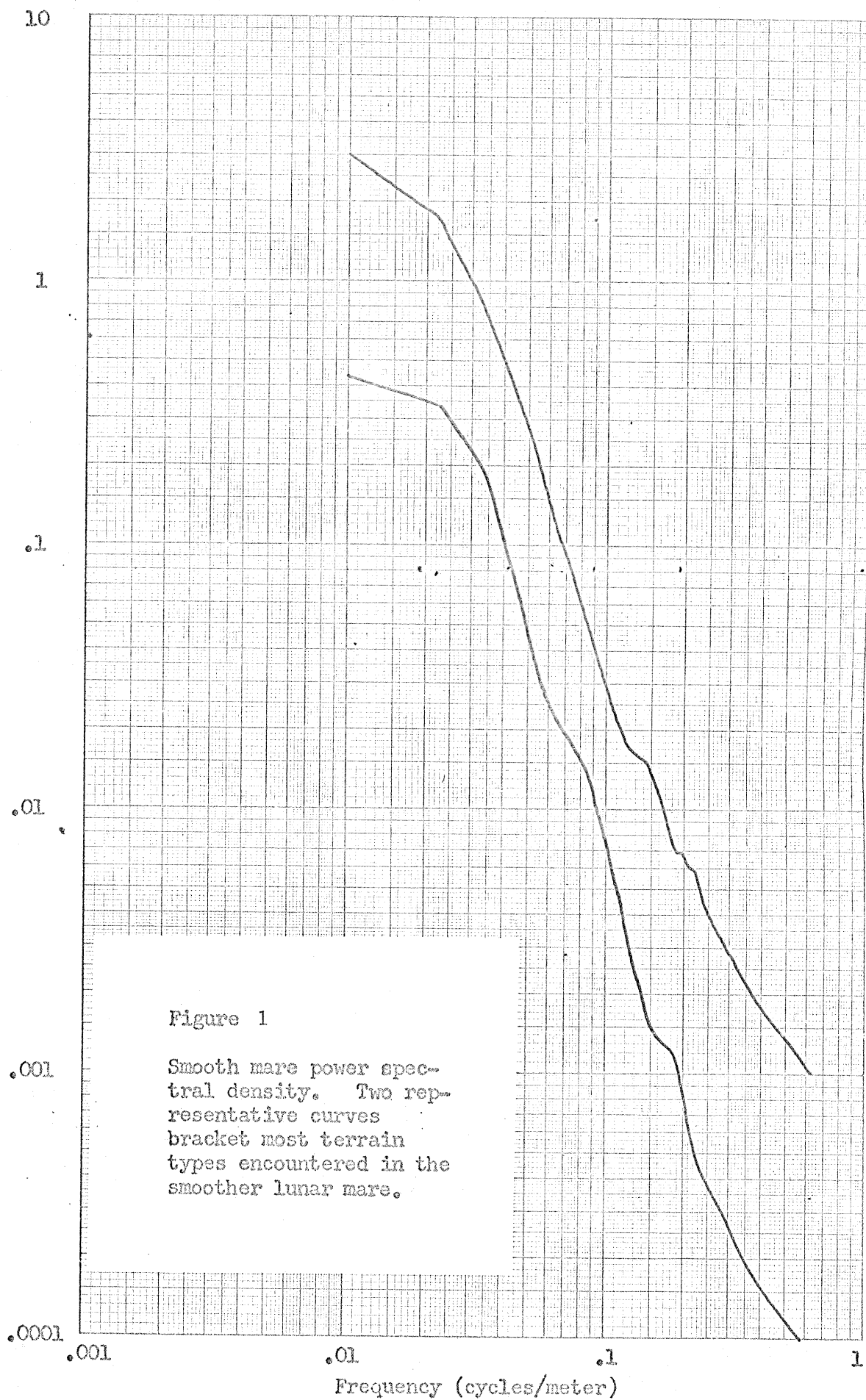
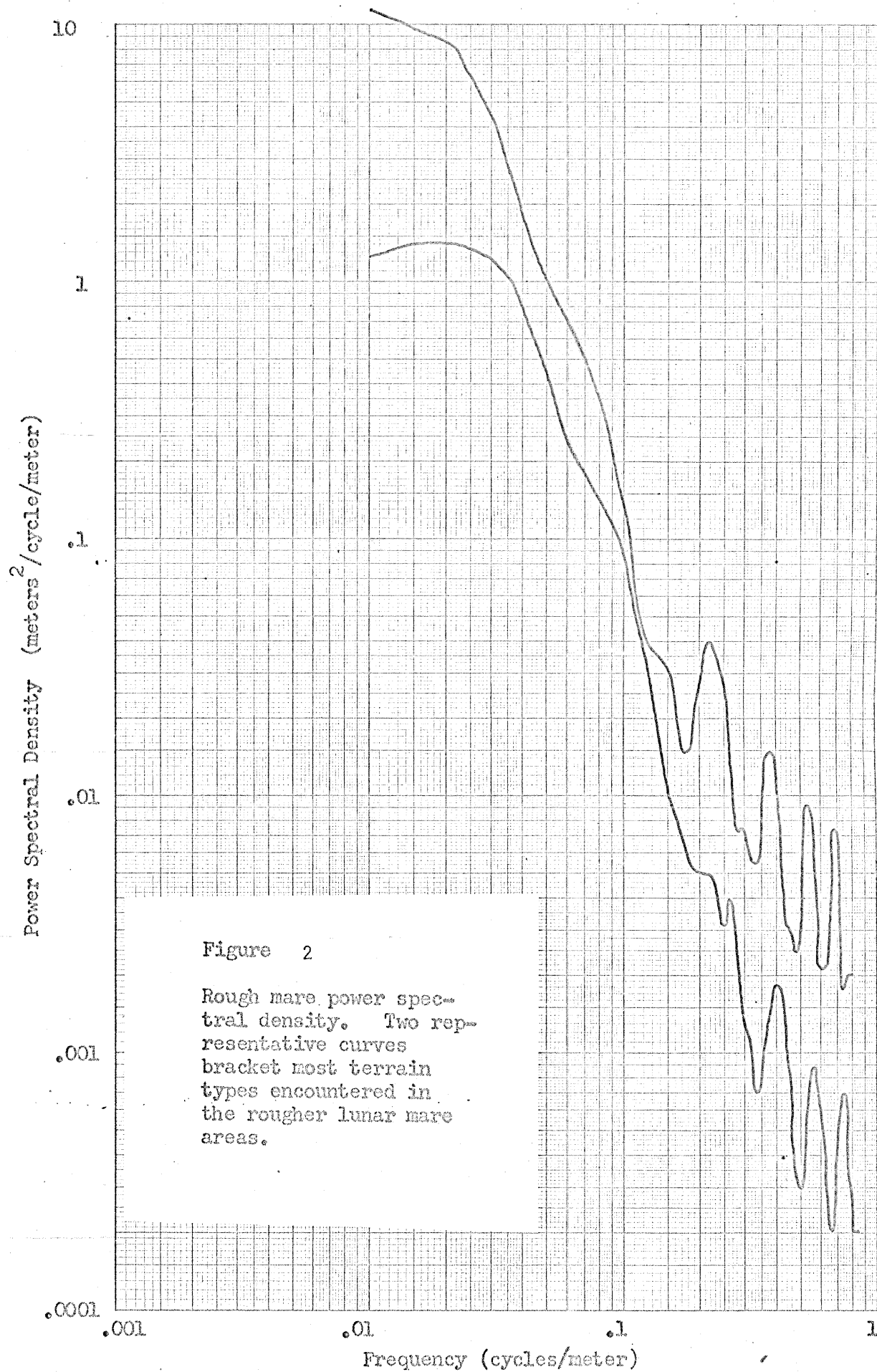
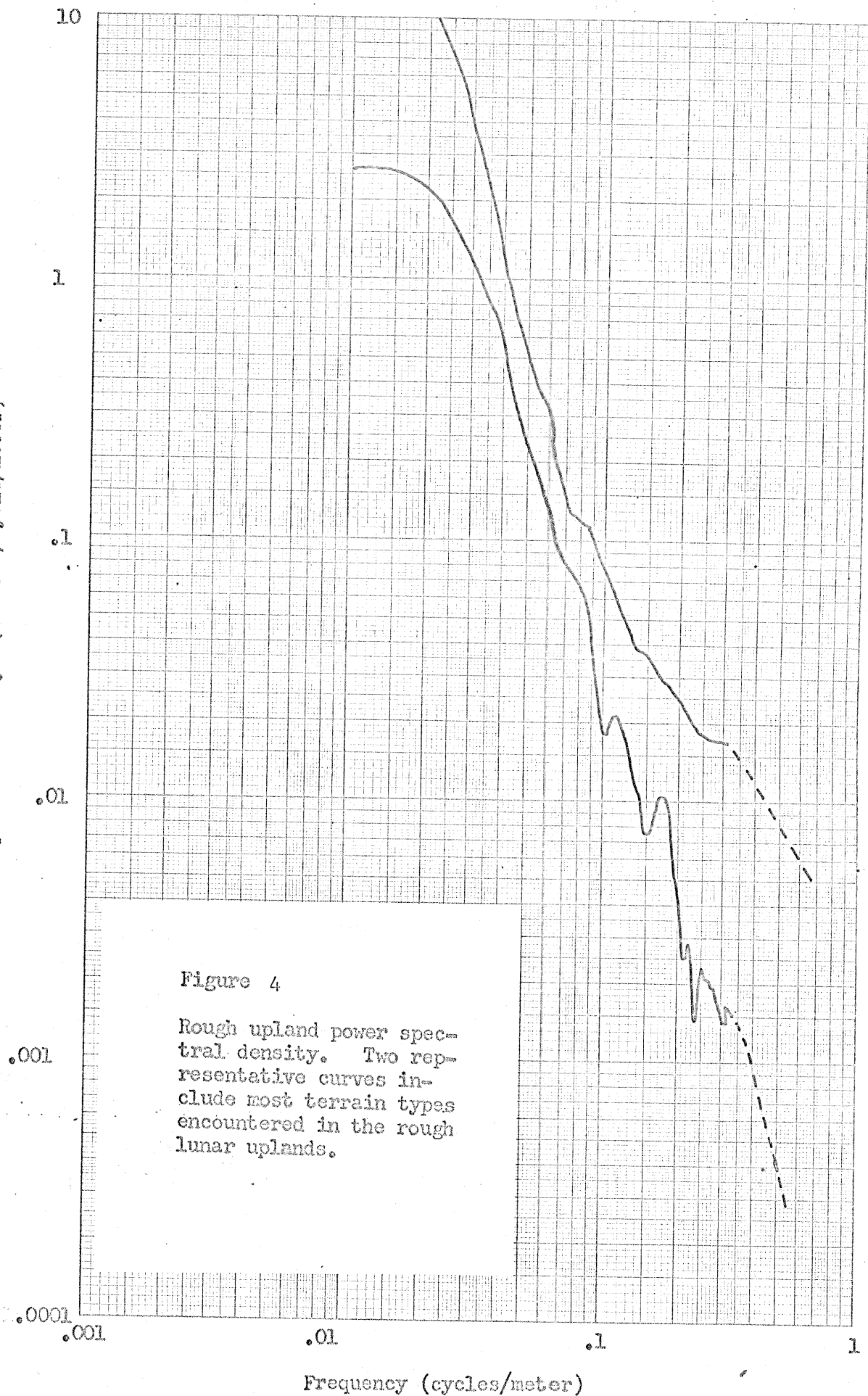


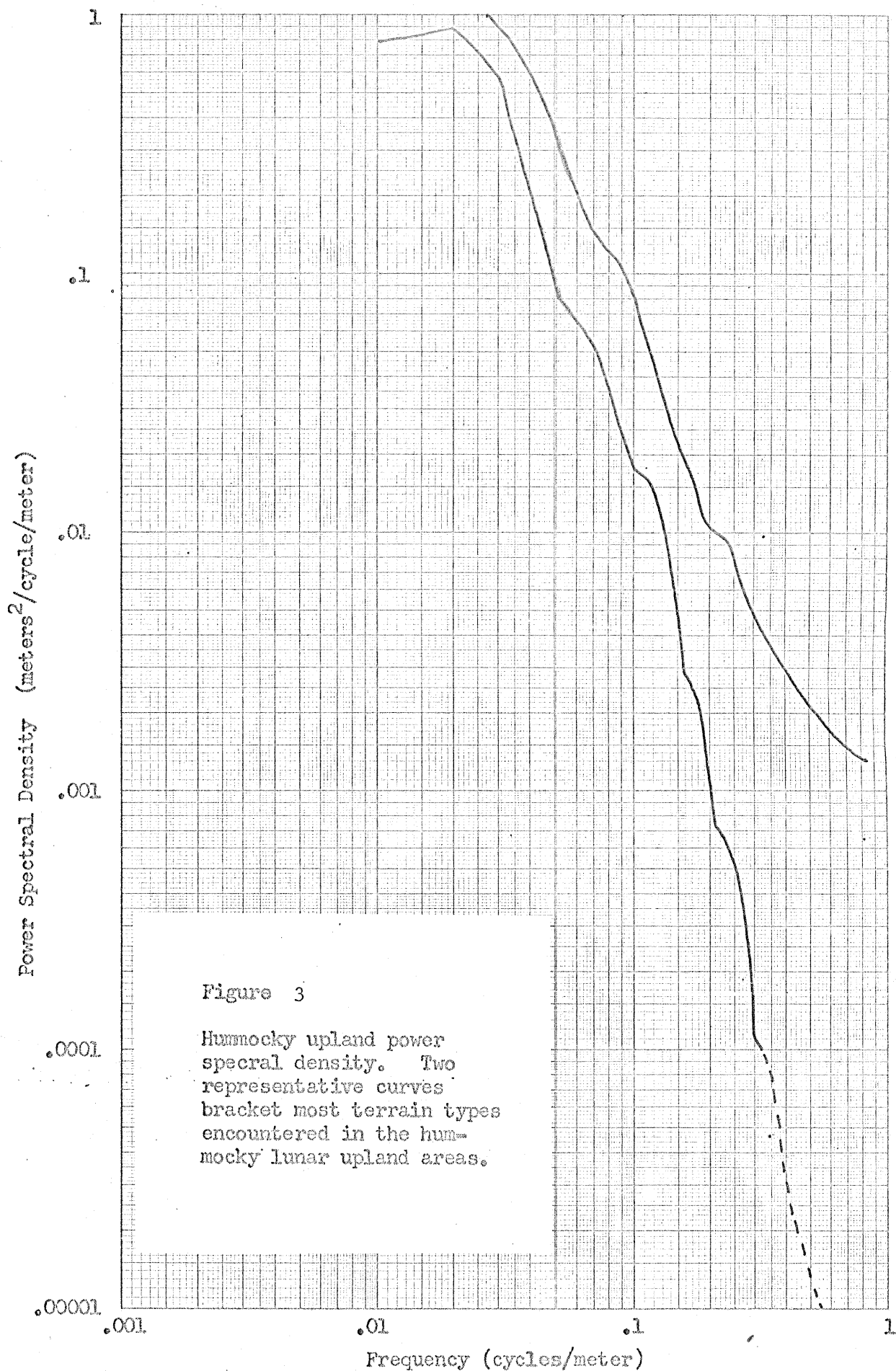
Figure 1

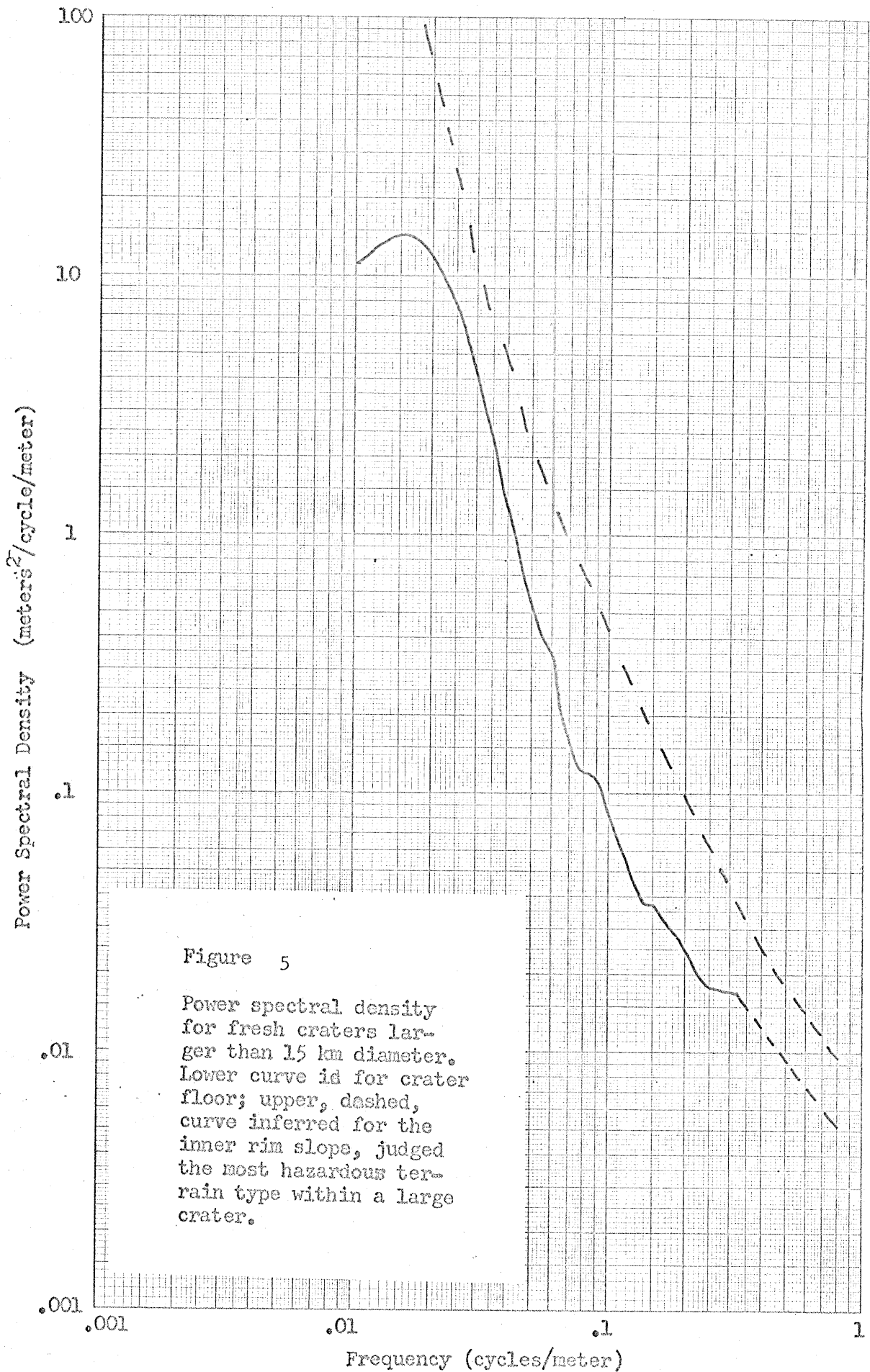
Smooth mare power spectral density. Two representative curves bracket most terrain types encountered in the smoother lunar mare.

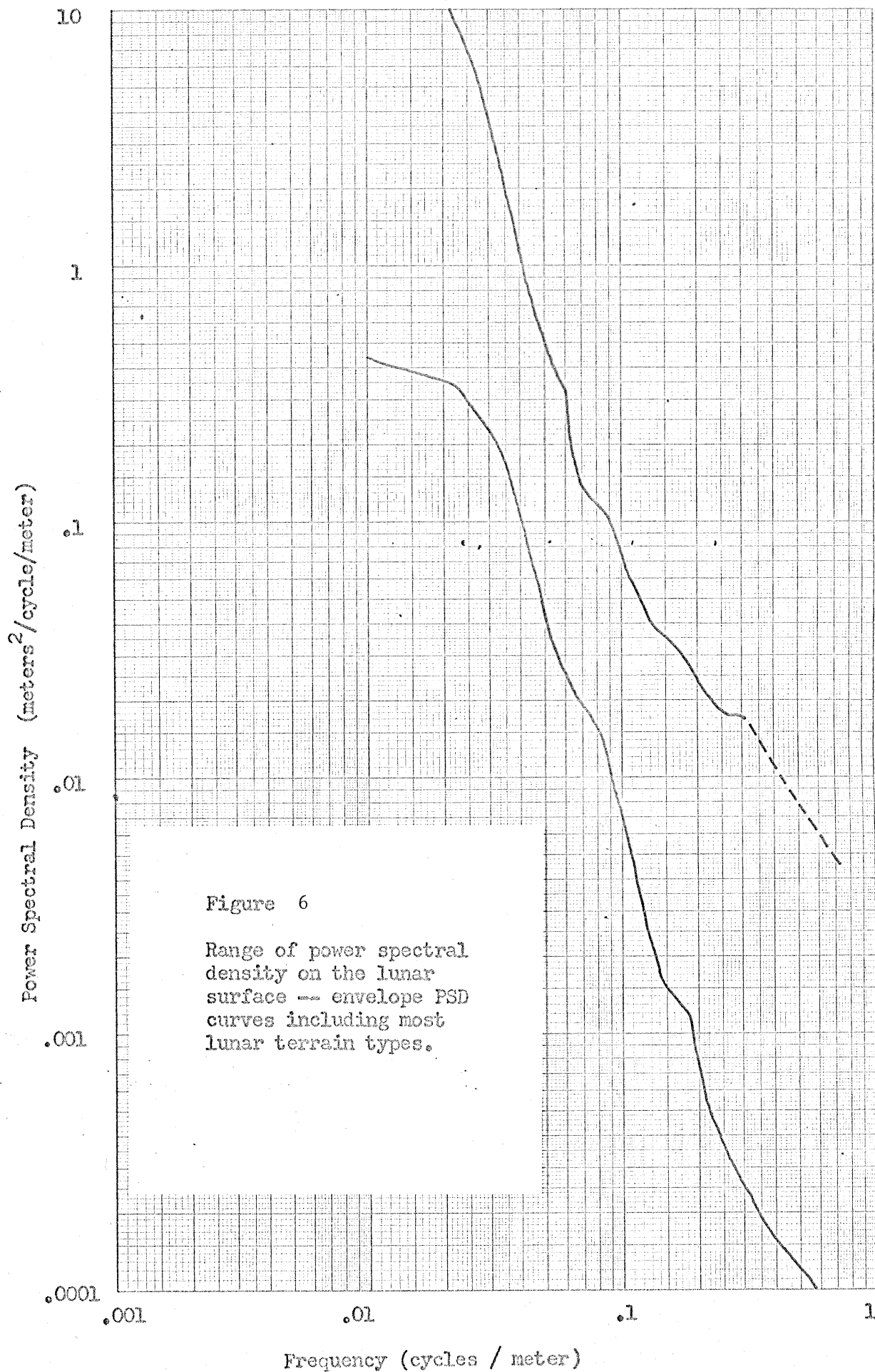


Power Spectral Density (meters<sup>2</sup>/cycle/meter)









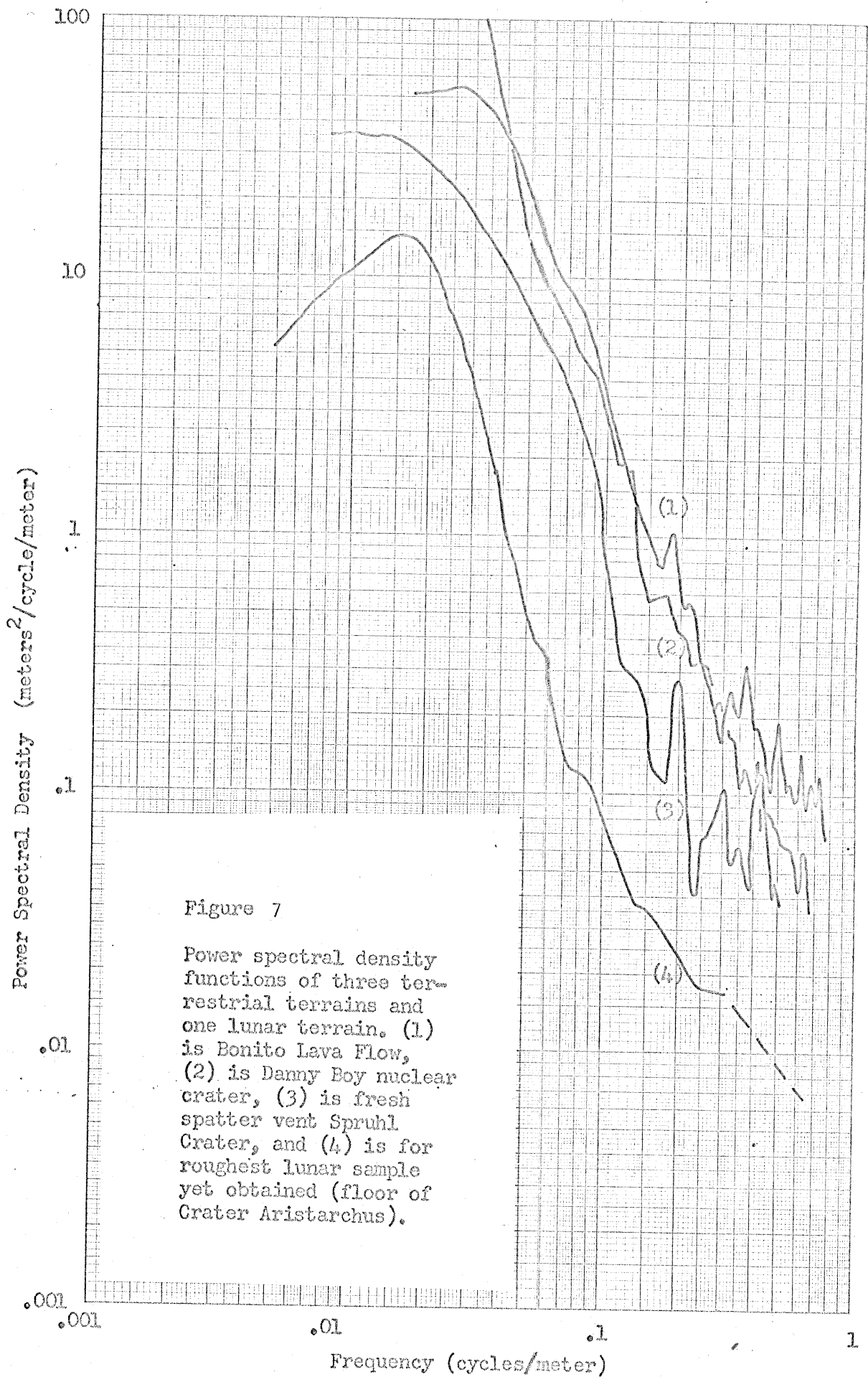


Table 3

Ranges of Power Spectral Density (PSD) Values for  
Four Gross Lunar Terrain Types at Two Frequencies

Terrain Type	PSD in meters <sup>2</sup> /cycle/meter	
	0.05 cycles/meter	0.5 cycles/meter
Smooth mare	0.045 - 0.25	0.00012 - 0.0013
Rough mare	0.420 - 1.00	0.0003 - 0.0035
Hummocky upland	0.075 - 0.34	0.000013 - 0.0021
Rough upland	0.200 - 0.50	0.0004 - 0.0080
All terrains	0.075 - 1.00	0.000013 - 0.008

### Slope Angle

Measuring the departure of topography from the horizontal, slope angle is an absolute index of terrain roughness. Slope angles measured along a profile may be expressed either as absolute values or as algebraic values, where slopes facing, for example, east, are designated positive, and the west-facing slopes negative.

On the Moon, algebraic slope-frequency distributions typically approach, but never quite achieve, the gaussian configuration. Thus, the usual central tendency and dispersion statistics (mean, mode, median, standard deviation, variance, skewness, and kurtosis) can be applied, albeit with caution, to algebraically expressed lunar slope samples. However, the applicability of all of these statistics to vehicle mobility problems has not been established, so lunar slopes customarily are expressed as absolute values.

Absolute value slope frequency distributions on the Moon are skewed strongly toward low slope angles and only the mean, median, and modal slopes should be extracted from these distributions. If all lunar slope data were available along any desired azimuth, then algebraic slopes would furnish useful information on slope symmetry. However, lunar photoclinometric data can only be obtained along the phase plane, which does not change sufficiently among the five Lunar Orbiter missions to provide the required variation in azimuth.

With comparatively little to be gained from using algebraic slopes, the slope angle data gathered for the present study are expressed mostly in absolute values. Three aspects of slope angle are presented here: (1) mean absolute slope, (2) algebraic standard deviation, and (3) absolute slope frequency distribution. Each of

these three parameters is given for slope base lengths of one, ten, and fifty meters for each of the four classes of lunar terrains listed in Table 1.

Scarcity of representative large-scale slope data for all four lunar terrain classes has necessitated extrapolation of small scale photoclinometric data and the derivation of statistical models of slope distributions. The photoclinometric method, designed expressly for the relatively smooth and level mare, is not readily applicable to steeper and rougher upland terrain (Taylor and Lambiotte, 1967). Few slope distributions have been successfully obtained for the upland terrains at the base length of one meter. However, predictive models have been formulated for the uplands using one meter resolution spacecraft data from mare sites together with the 0.75 km resolution terrestrial data for both uplands and mare. Derivation of these models is treated in Pike (1968) and in the appendix to this section, and need not be discussed further here.

Tables 4-8 and Figures 8-11 present slope information for the four lunar terrain types at one, ten, and fifty meter base lengths. Most of these data are straightforward; their calculation is discussed in the appendix. Tables 6-8 show only tentative 100th percentile slope values, which are not shown by the curves in Figures 8-11. Maximum slope values, which commonly are not accurately measured by the photoclinometric method in any but the smoothest areas, vary also according to the number of slope angles inspected in a particular sample, and are not readily predicted.

Table 4

Mean Values of Absolute Slope at Three Base Lengths  
for Four Gross Lunar Terrain Types

Terrain Type	Average Mean	Lowest Mean Slope	Highest Mean Slope	Range of Mean Slopes
Base Length is one meter				
Smooth mare	2.9°	1.2°	4.0°	2.8°
Rough mare	5.3	3.5	7.8	4.3
Hummocky upland	8.2	4.7	10.0	5.3
Rough upland	11.0	8.7	15.0	6.3
Base Length is ten meters				
Smooth mare	2.0	1.0	2.8	1.8
Rough mare	3.8	2.5	5.6	3.1
Hummocky upland	5.8	3.6	7.0	3.4
Rough upland	7.7	6.2	11.0	4.8
Base Length is fifty meters				
Smooth mare	1.4	0.7	1.9	1.2
Rough mare	2.5	1.7	3.7	2.0
Hummocky upland	3.9	2.4	4.7	2.3
Rough upland	5.2	4.1	7.3	3.2

Table 5

Mean Values of Algebraic Standard Deviation of Slope  
Angle for Four Gross Lunar Terrain Types

Terrain Type	Average Standard Deviation	Lowest Standard Deviation	Highest Standard Deviation	Range of Standard Deviations
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Base Length is One Meter

Smooth mare	3.6°	1.5°	5.0°	3.5°
Rough mare	6.6	4.4	9.7	5.3
Hummocky upland	10.2	5.8	12.4	6.6
Rough upland	13.7	10.0	18.6	8.6

Base Length is Ten Meters

Smooth mare	2.5	1.3	3.5	2.2
Rough mare	4.7	3.1	7.0	3.9
Hummocky upland	7.2	4.4	8.7	4.3
Rough upland	9.6	7.7	13.7	6.0

Base Length is Fifty Meters

Smooth mare	1.7	0.9	2.4	1.5
Rough mare	3.1	2.1	4.6	2.5
Hummocky upland	4.8	3.0	5.8	2.8
Rough upland	6.5	5.1	9.1	4.0

Table 6

Predicted Distributions of One Meter Slopes for Four Lunar  
Terrain Types Whose Mean Slope Values are Known or Estimated

Mean Slope Values		2.9°	5.3°	8.2°	11.0°
%N	Model % of Mean Slope	Smooth mare	Rough mare	Hummocky upland	Rough upland
100	(450)	(13°)	(24°)	(37°)	(50°)
98	346	10	18	28	38
95	273	8	14	22	30
90	216	6.2	11	18	24
80	152	4.4	8	12	17
70	116	3.4	6.1	10	13
60	96	2.8	5.1	8	10
50	76	2.2	4.0	6.2	8
40	58	1.7	3.1	4.8	6.4
30	44	1.3	2.3	3.6	4.8
20	28	0.8	1.5	2.3	3.1
10	15	0.4	0.8	1.2	1.7

Table 7

Predicted Distributions of Ten Meter Slopes for Four Lunar  
Terrain Types Whose Mean Slope Values are Known or Estimated

Mean Slope Values		2.0°	3.8°	5.8°	7.7°
%N	Model % of Mean Slope	Smooth mare	Rough mare	Hummocky upland	Rough mare
100	(450)	(9°)	(17°)	(26°)	(35°)
98	346	6.9	13	20	27
95	273	5.5	10	16	21
90	216	4.3	8	13	17
80	152	3.0	5.8	9	12
70	116	2.3	4.4	6.7	9
60	96	1.9	3.6	5.6	7.4
50	76	1.5	2.9	4.4	5.8
40	58	1.2	2.2	3.4	4.5
30	44	0.9	1.7	2.6	3.4
20	28	0.6	1.1	1.6	2.2
10	15	0.3	0.6	0.9	1.2

Table 8

Predicted Distributions of Fifty Meter Slopes for Four Lunar  
Terrain Types Whose Mean Slope Values are Known or Estimated

Mean Slope Values		1.4°	2.5°	3.9°	5.2°
%N	Model % of Mean Slope	Smooth mare	Rough mare	Hummocky upland	Rough upland
100	(450)	(6.3°)	(11°)	(18°)	(23°)
98	346	4.8	9	13	18
95	273	3.8	6.8	11	14
90	216	3.0	5.4	8.4	11
80	152	2.1	3.8	5.9	8
70	116	1.6	2.9	4.5	6.0
60	96	1.3	2.4	3.7	5.0
50	76	1.1	1.9	3.0	4.0
40	58	0.8	1.5	2.3	3.0
30	44	0.6	1.1	1.7	2.3
20	28	0.4	0.7	1.1	1.5
10	15	0.2	0.4	0.6	0.8

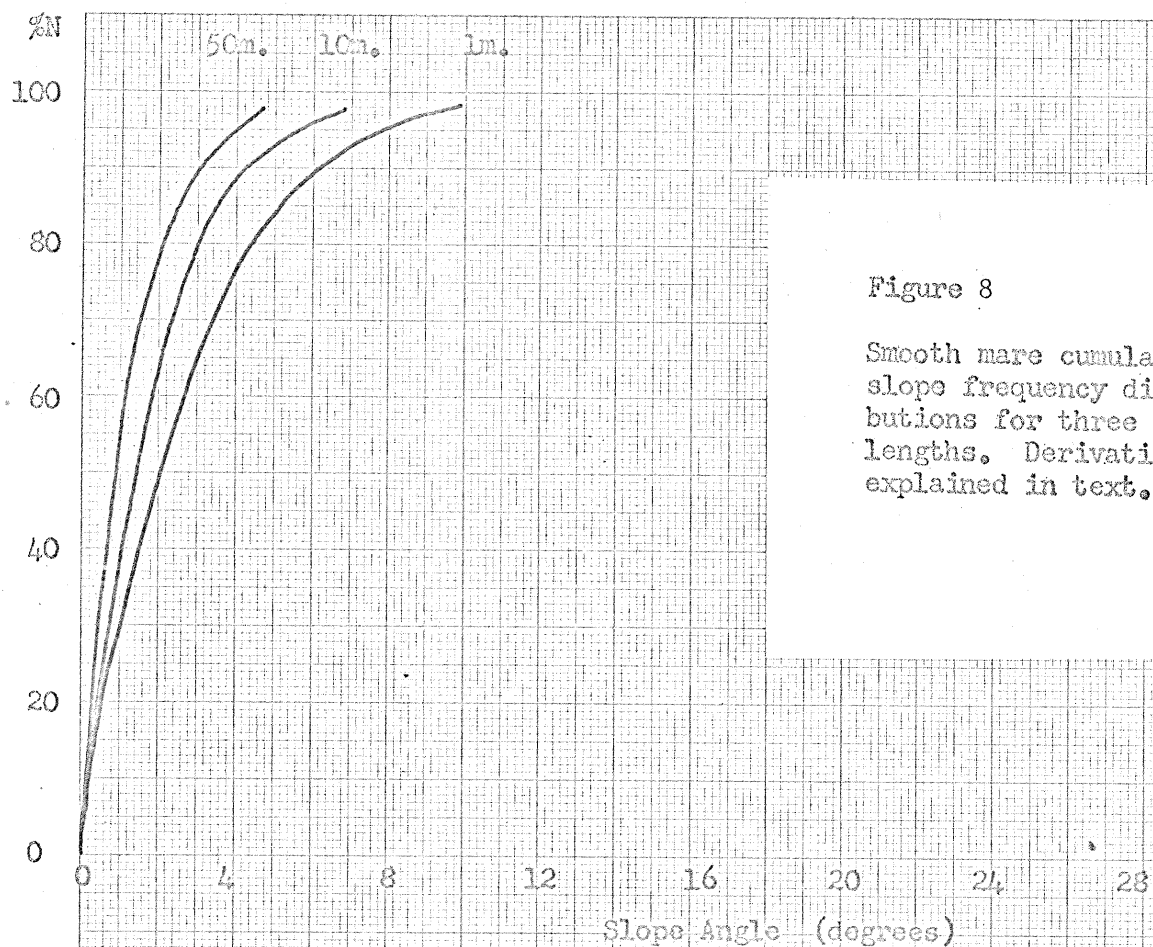


Figure 8

Smooth mare cumulative slope frequency distributions for three base lengths. Derivation explained in text.

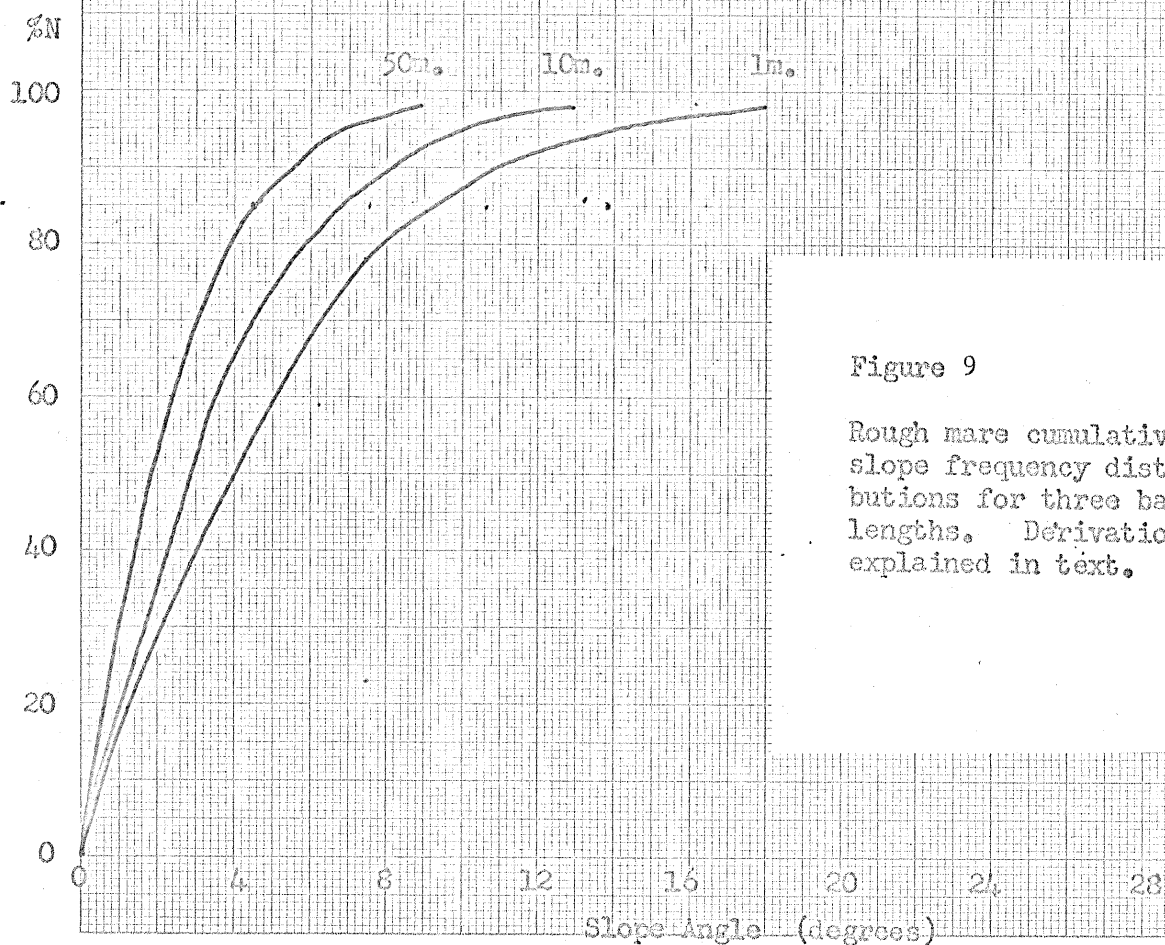


Figure 9

Rough mare cumulative slope frequency distributions for three base lengths. Derivation explained in text.

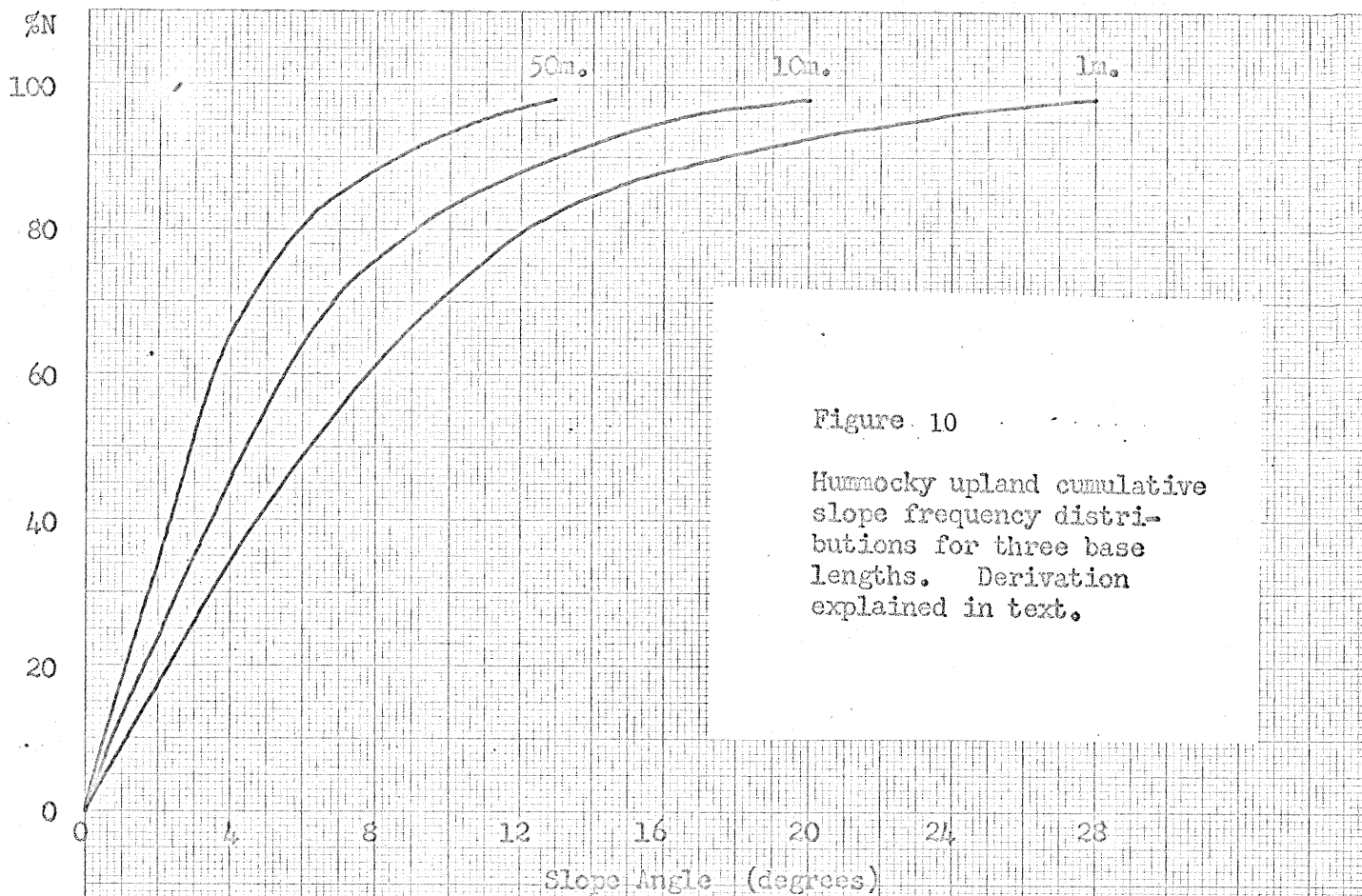


Figure 10

Hummocky upland cumulative slope frequency distributions for three base lengths. Derivation explained in text.

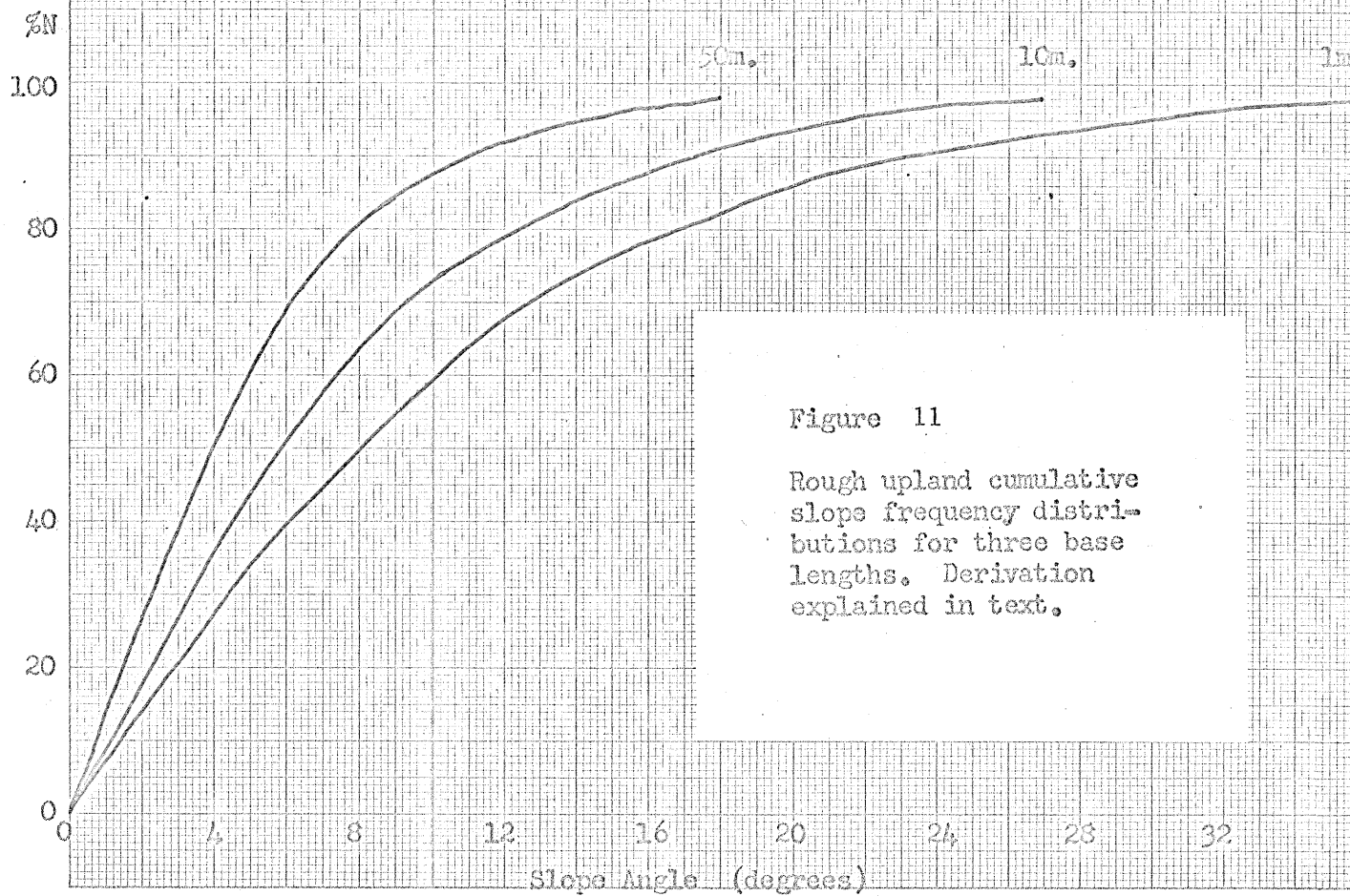


Figure 11

Rough upland cumulative slope frequency distributions for three base lengths. Derivation explained in text.

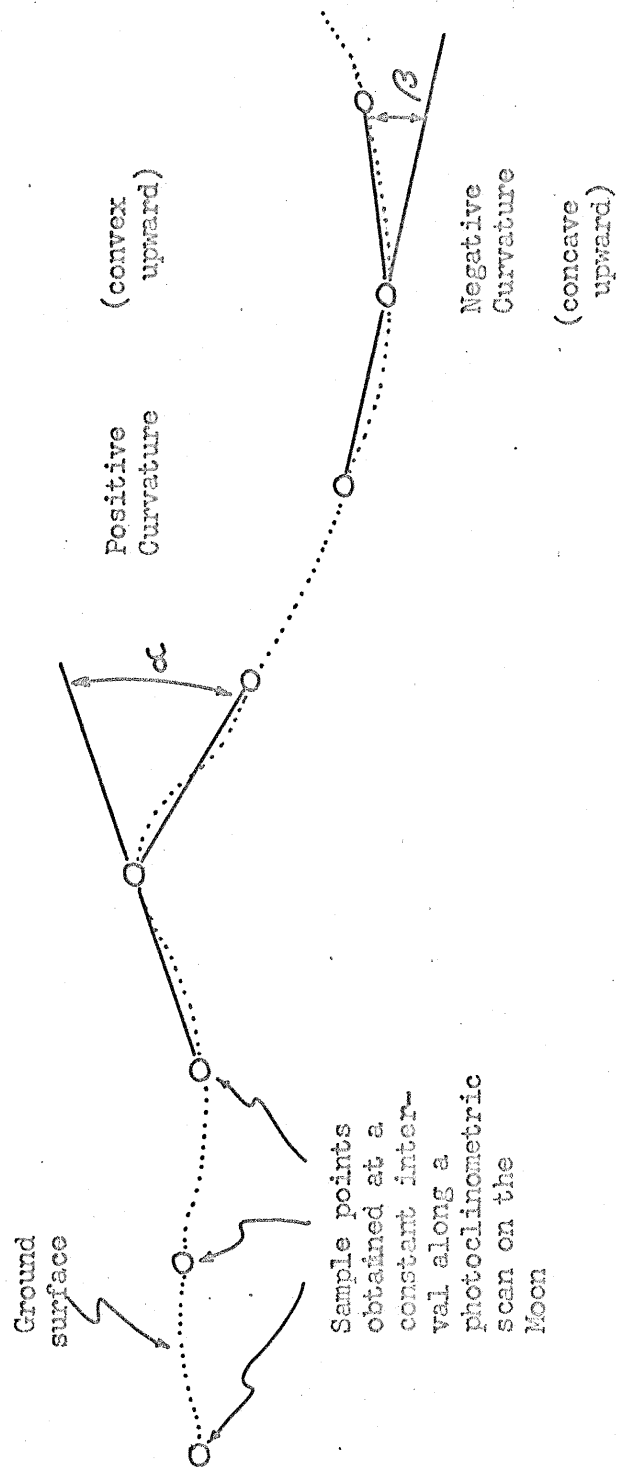
The slope information presented here suggests that the general terrain geometry will not pose a serious trafficability hazard in average smooth and rough mare areas. Special mare features, including rilles, domes, and fresh craters will have to be considered as specific hazards where encountered. The upland areas, however, can be expected to be totally impassable in places, due to the general terrain geometry alone. Remembering that the upland slope frequency curves in Figures 10 and 11 are only averages, it is a virtual certainty that some of the rougher upland areas will be exceedingly hazardous to a lunar roving vehicle. Of course, such areas still might prove to be trafficable if the vehicle was carefully routed around the most dangerous slopes.

An additional terrain type, large fresh craters (over 15 km in rim diameter), including desirable scientific sites such as Aristarchus, Tycho, and Copernicus, may well be even more hazardous than any of the four basic terrains discussed previously. Preliminary photogrammetric profiles (Wu 1969, and unpublished data) of fresh, large craters both in the mare and in the uplands, indicate that long and steep slopes at, and even exceeding, 30 to 40 degrees may be common in these areas. Data will be made available pending evaluation of the precision.

#### Angle of Slope Curvature

Like power spectral density, slope curvature is a measure of relative, rather than absolute, surface roughness. Curvature angle as used in the present study and defined in Figure 12 is considerably less complicated than that of Schloss (1965). The statistical properties

Figure 12  
DEFINITION OF SLOPE CURVATURE



of slope curvature as defined here are not yet well known. Algebraic curvature frequency distributions on the Moon are not strongly skewed, and, like slope angle, approach normality. While the mean absolute value of slope curvature usually correlates positively with mean absolute value of the slope angle, the degree of correspondence varies for different terrains. The parameter, calculated at several base lengths, is complementary to slope angle and power spectral density, and perhaps somewhat repetitive of the latter, although this redundancy has yet to be demonstrated.

Insufficient information on the properties of slope curvature angle precludes use of generalized predictive models similar to those computed for slope angle, above. Instead, four specific lunar samples were selected to represent each of the four major terrain types. Tables 9 and 10 present curvature absolute means and algebraic standard deviations at base lengths of one, five, ten, and fifty meters. Figures 13 and 14 are curvature angle frequency curves for the four terrain types at a base length of five meters.

Again, the lack of additional lunar and terrestrial curvature information precludes much comparison of the present lunar sites with other areas. However, the data do point up some interesting variations among the four lunar samples. At a base length of five meters, mean curvature values and the frequency curves display a systematic increase in curvature from smooth mare to rough upland. At base lengths of one and fifty meters, however, the progression is not as regular. The variation of curvature with base length likewise shows both systematic

Table 9

Mean Slope Curvature of Four Gross Lunar Terrain Types

At Four Base Lengths

Terrain Type	Values of Mean Slope Curvature in Degrees of Arc			
	One meter	Five meters	Ten meters	Fifty meters
Smooth mare	0.6	0.9	1.0	0.8
Rough mare	0.9	1.2	1.3	4.1
Hummocky Upland	0.8	1.7	2.4	1.5
Rough upland	2.0	2.5	2.7	3.8

Table 10

Algebraic Standard Deviation of Angle of Curvature  
Of Four Gross Lunar Terrain Types at Four Base Lengths

Terrain Type	Value of Algebraic Standard Deviation in degrees of arc			
	One meter	Five meters	Ten meters	Fifty meters
Smooth mare	0.8	1.3	1.3	0.9
Rough mare	1.3	1.9	1.8	5.0
Hummocky upland	1.2	2.7	2.9	1.9
Rough upland	3.1	3.3	3.4	3.4

Figure 13 Frequency distributions of slope curvature for the lunar maria. Curves shown represent smooth maria (left), rough maria (right). Base length is 5 meters

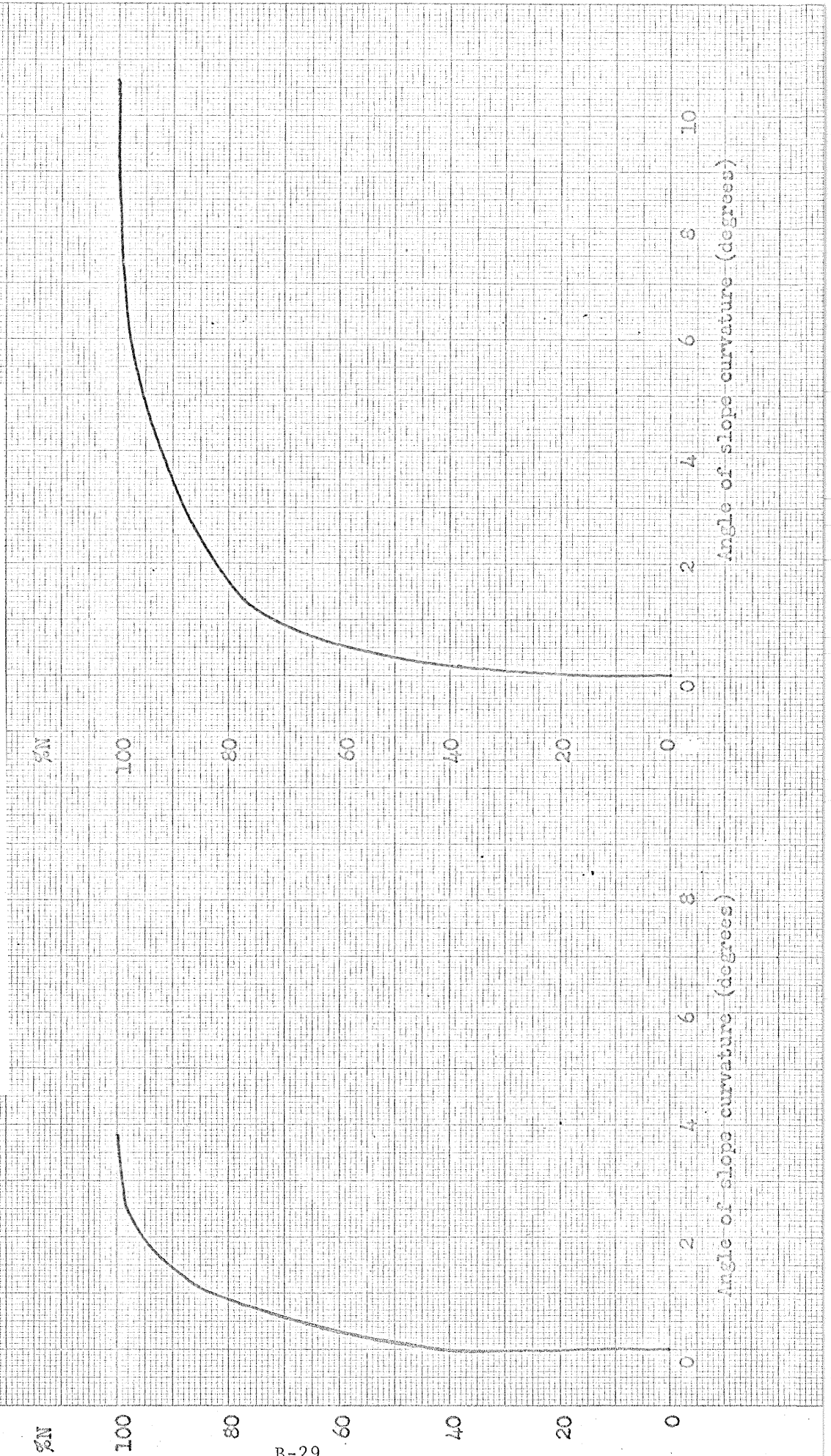
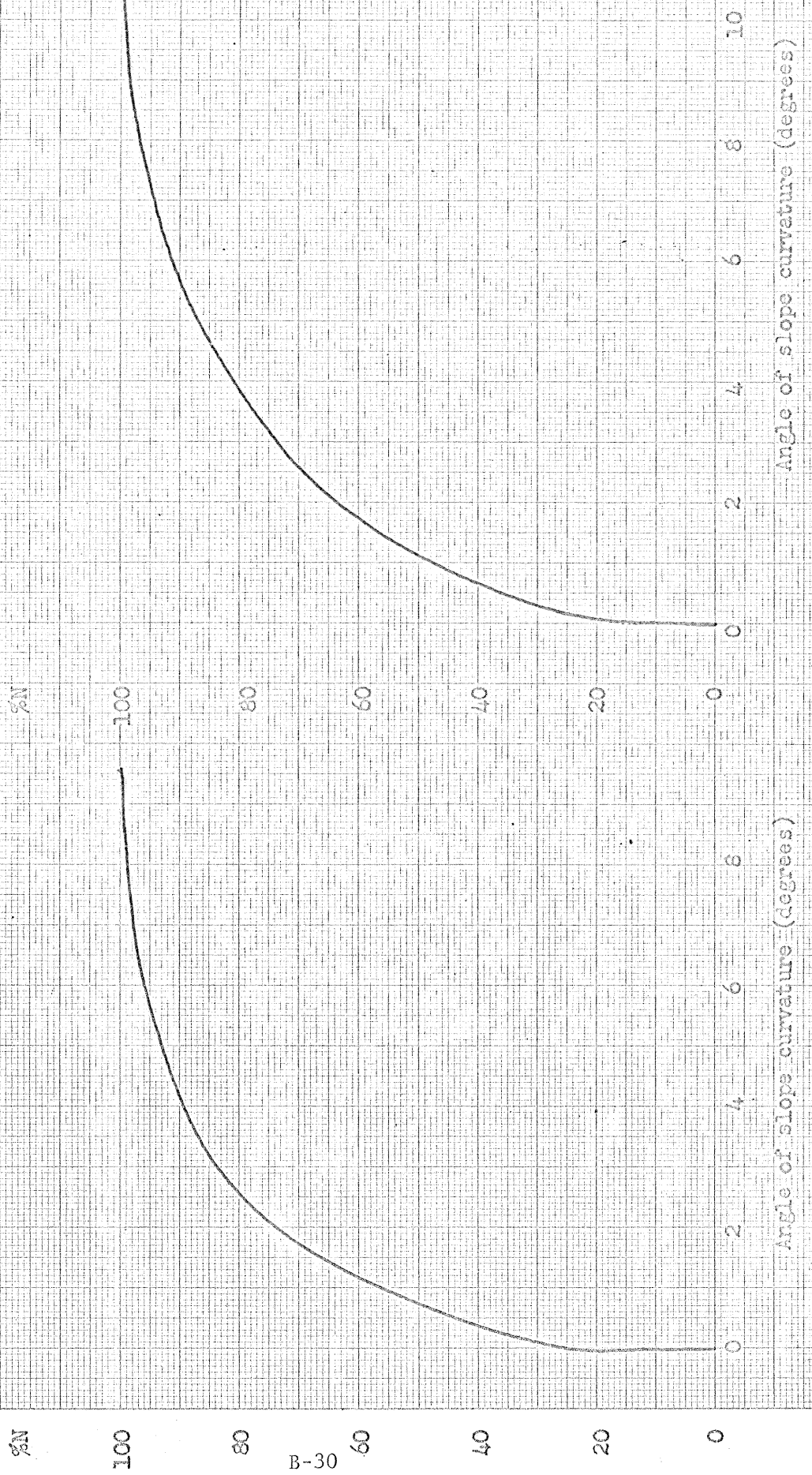


Figure 14 Frequency distributions of slope curvature for the lunar uplands. Curves shown represent hummocky (left) and rough upland (right). Base length is 5 meters



progressions and much less orderly changes, depending upon the terrain type. Such irregularities can readily be interpreted in terms of the varying geomorphic development of the four sample areas. All-in-all, the curvature angles shown in Figures 13 and 14 do not appear to be large enough to severely impede the mobility of a lunar roving vehicle. When available, curvature data for analagous terrestrial terrains will be furnished for comparison with the lunar samples given here.

#### Surface Geometry and Traverse Distance

Irregularities of the lunar terrain will materially, if but slightly, increase the map distance of any projected vehicular traverse. While avoidance of small steep craters, block fields, steep-walled rilles, and other hazards will contribute most to such an increased distance, the overall surface geometry has some affect as well. The figures in Table 11 were obtained by computing secants of median slope values of seven slope angle classes for each of the one-meter slope frequency curves in Figures 8-11. Percentage increase figures for large, fresh craters were estimated from photogrammetric profiles of the craters Copernicus and Tycho (unpublished data).

Table 11

Probable Contribution of Topographic Slope to Increase  
of Actual LRV Travel Distance over Planned Map Distance  
As a Function of Lunar Type

Terrain Type	Average percentage of distance added to map distance by topo- graphic slope (data at one meter base length)
Smooth mare	0.21%
Rough mare	0.40%
Hummocky upland	1.00%
Rough upland	1.86%
Fresh Craters over 15 km diameter	2.3% to 4.6 %

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(Appendix to Section B)

PRELIMINARY MODELS FOR THE PREDICTION  
OF SLOPE ANGLE ON THE LUNAR SURFACE

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March 17, 1969

U.S. Geological Survey  
Astrogeologic Studies

Mean slope angle values for four lunar terrain types

McCauley (1964) and Rowan and McCauley (1966) in previous photo-clinometric studies of lunar surface geometry, obtained abundant slope data at a resolution of 0.75 km and greater. One result of their work (the curve WXYZ on Figure 1), depicts median slope against slope lengths for an "average lunar mare," comprising both smoother and rougher mare types. The slope of this curve, almost exactly  $-0.25$ , is very well determined by the four points. Rowan and McCauley also calculated median lunar slope for 53 different lunar localities, and divided the samples into four gross terrain units, Smooth Mare, Rougher Mare, Hummocky Upland, and Rough Upland. By averaging medians of samples in each category, we obtain points B, C, D, and E. Point A, the average for all mare samples, demonstrates the validity of the curve WXYZ, derived earlier by McCauley.

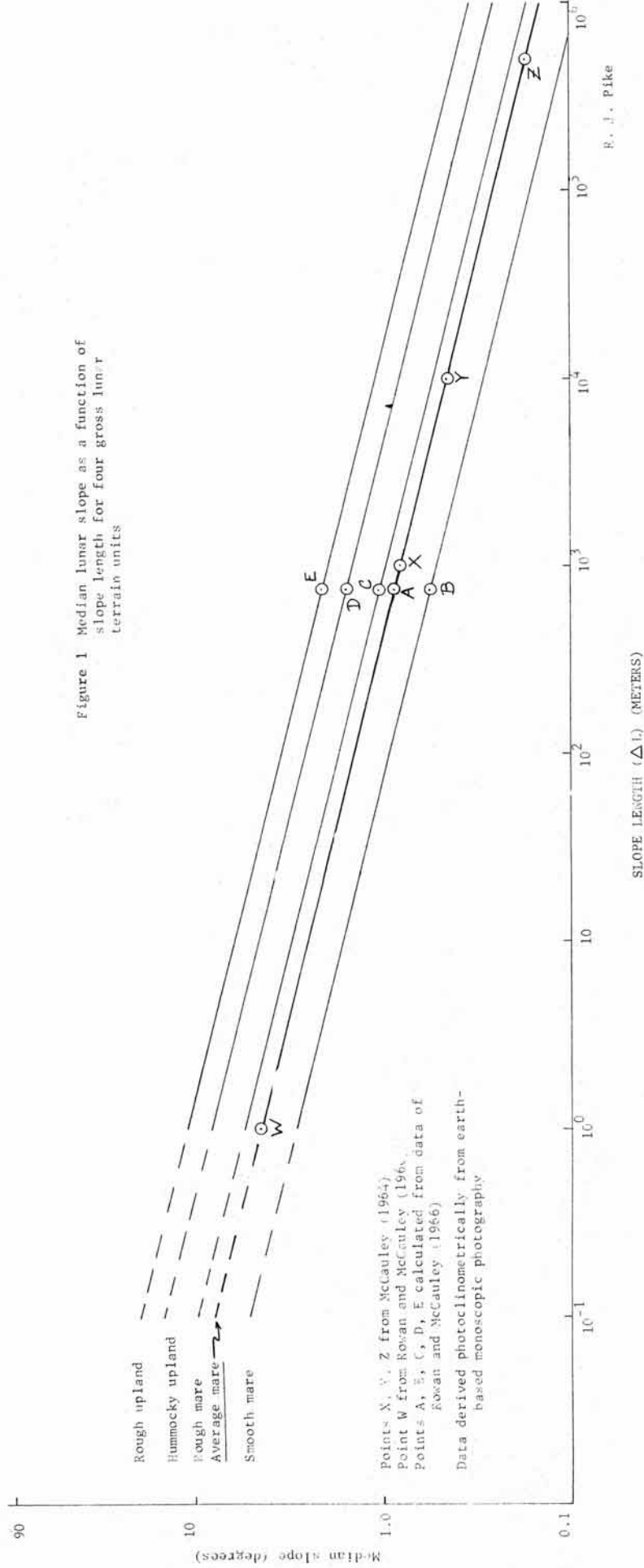
TERRAIN TYPE	CURVE IN FIG. 1	AVERAGE MEDIAN SLOPE at 0.75 km $\Delta$ L
Smooth Mare	(B)	$0.55^\circ$
Rough Mare	(C)	$1.03^\circ$
Hummocky Upland	(D)	$1.56^\circ$
Rough Upland	(E)	$2.07^\circ$

We let the four points in the above table define four curves describing the relationship between median slope and slope length. While the slope of these curves may differ from that of the WXYZ curve, there are no data to show what this difference, if any, might be. Hence, we set the four curves parallel to that for the average mare, assuming it is likely that the median slope:slope length relationship undergoes a similar rate of change with varying slope length, regardless of the terrain type. Since these curves are constructed from so few data, we have omitted confidence intervals, which would be meaningless here.

The four curves are described by the following simple power expressions:

LUNAR TERRAIN TYPE	EQUATION
Smooth Mare	$S_{med} = 2.9 DL^{0.25}$
Rough Mare	$S_{med} = 5.4 DL^{0.25}$
Hummocky Upland	$S_{med} = 8.1 DL^{0.25}$
Rough Upland	$S_{med} = 10.8 DL^{0.25}$

Where  $S_{med}$  is median slope in degrees, and  
DL is slope length in meters.



Although Figure 1 gives useful information concerning the lunar surface, the median is not an especially powerful statistical parameter. Rowan and McCauley (1966) also computed mean slope values for their 53 lunar terrain samples at 0.75 km resolution. Using techniques similar to those by which the previous set of curves was produced, we derive four curves depicting the relationship between mean lunar slope and slope length, Figure 2. The mean slopes at 0.75 km resolution, F, G, H, and I, are listed below for the four gross terrain types:

TERRAIN TYPE	CURVE IN FIG. 2	AVERAGE MEAN SLOPE (at 0.75 km DL)
Smooth mare	(F)	0.70°
Rough mare	(G)	1.29°
Hummocky Upland	(H)	1.96°
Rough Upland	(I)	2.61°

Again, these data were averaged from Rowan and McCauley (1966). It is assumed that the slopes of the curves are similar, and for most of their length, do not depart significantly from the value of -0.25 used for the median slope curves of Figure 1. Confidence in this choice increases somewhat upon calculating the mean slope of samples at base lengths smaller than 0.75 km. Point J is located on a hummocky material near the crater Aristarchus. The slope length is 12.0 meters, and the terrain appears to be older, pre-mare topography best classified as

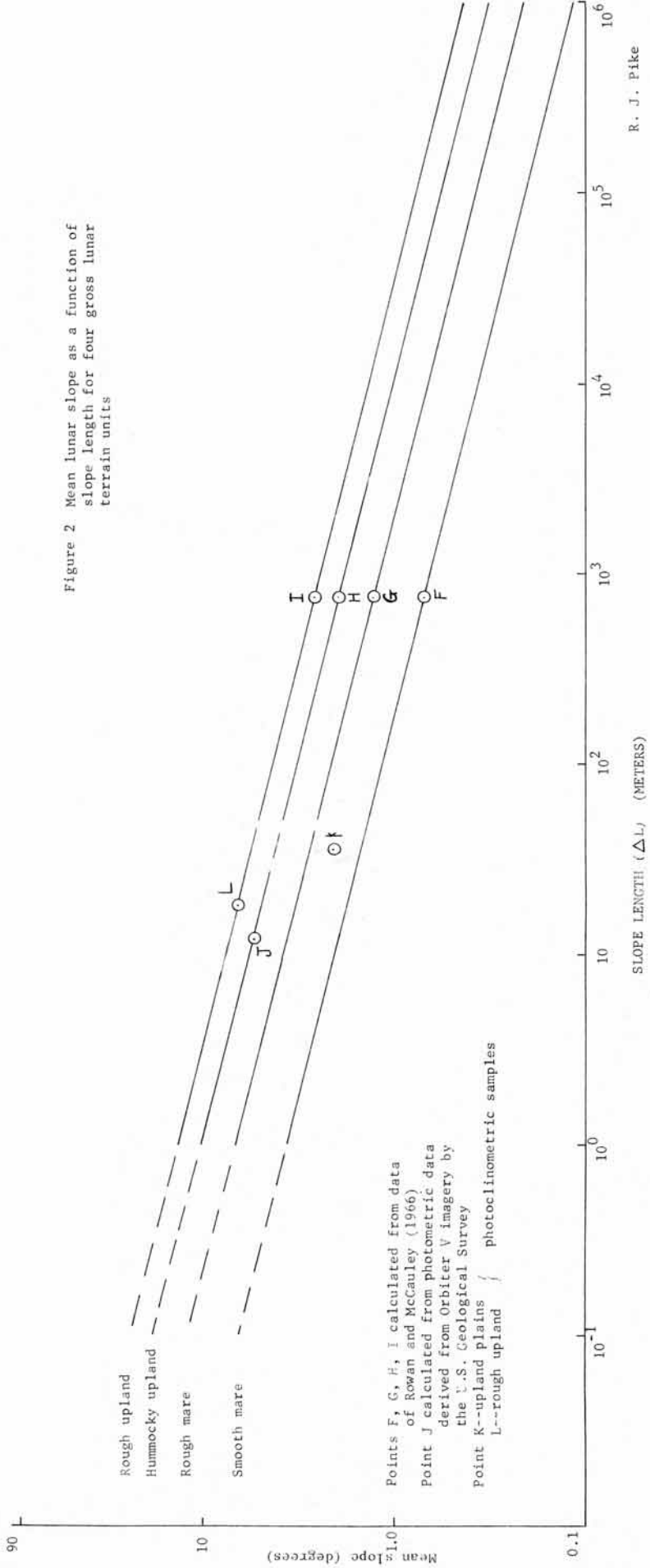


Figure 2 Mean lunar slope as a function of slope length for four gross lunar terrain units

Hummocky Upland. Point K is the floor of the crater Ptolemaeus, an upland plains unit rather like an average mare surface. Point L is a sample from the rougher uplands.

Figure 3 shows that the linear model is less satisfactory at base lengths under 10 meters. Point M is the average of 10 smooth mare slope means, and point N the average of 19 rough mare means. These data are at 0.6 m resolution. Since the vertical distance between points M and N is identical to that between G and F on Figure 2 it is likely that the two upland curves also follow below 10 m base lengths, the same trend as do the mare curves.

It is to be emphasized that the curves in Figure 3 are for average lunar terrains within each of the four gross morphologic categories. It is likely, therefore, that specific lunar terrain samples will be both rougher and smoother than the idealized cases depicted here.

Ranges of mean slope values were determined using the data of Rowan and McCauley (1966---their Table 4 and Figure 13) and the 0.6 m resolution spacecraft data generated by the Langley II photoclinometric program. Considerable overlap is present, and is to be expected in a classification as elementary as that available at this time.

The four average mean slope:slope length curves (Figure 3) for base lengths over 10 meters are described by the following simple power functions:

B-41-missing

TERRAIN TYPE	$\leq 10$ m EQUATION
Smooth mare	$\bar{S} = 3.65 \text{ DL}^{0.25}$
Rough mare	$\bar{S} = 6.75 \text{ DL}^{0.25}$
Hummocky upland	$\bar{S} = 10.40 \text{ DL}^{0.25}$
Rough upland	$\bar{S} = 13.80 \text{ DL}^{0.25}$

Where  $\bar{S}$  is mean slope in degrees, and

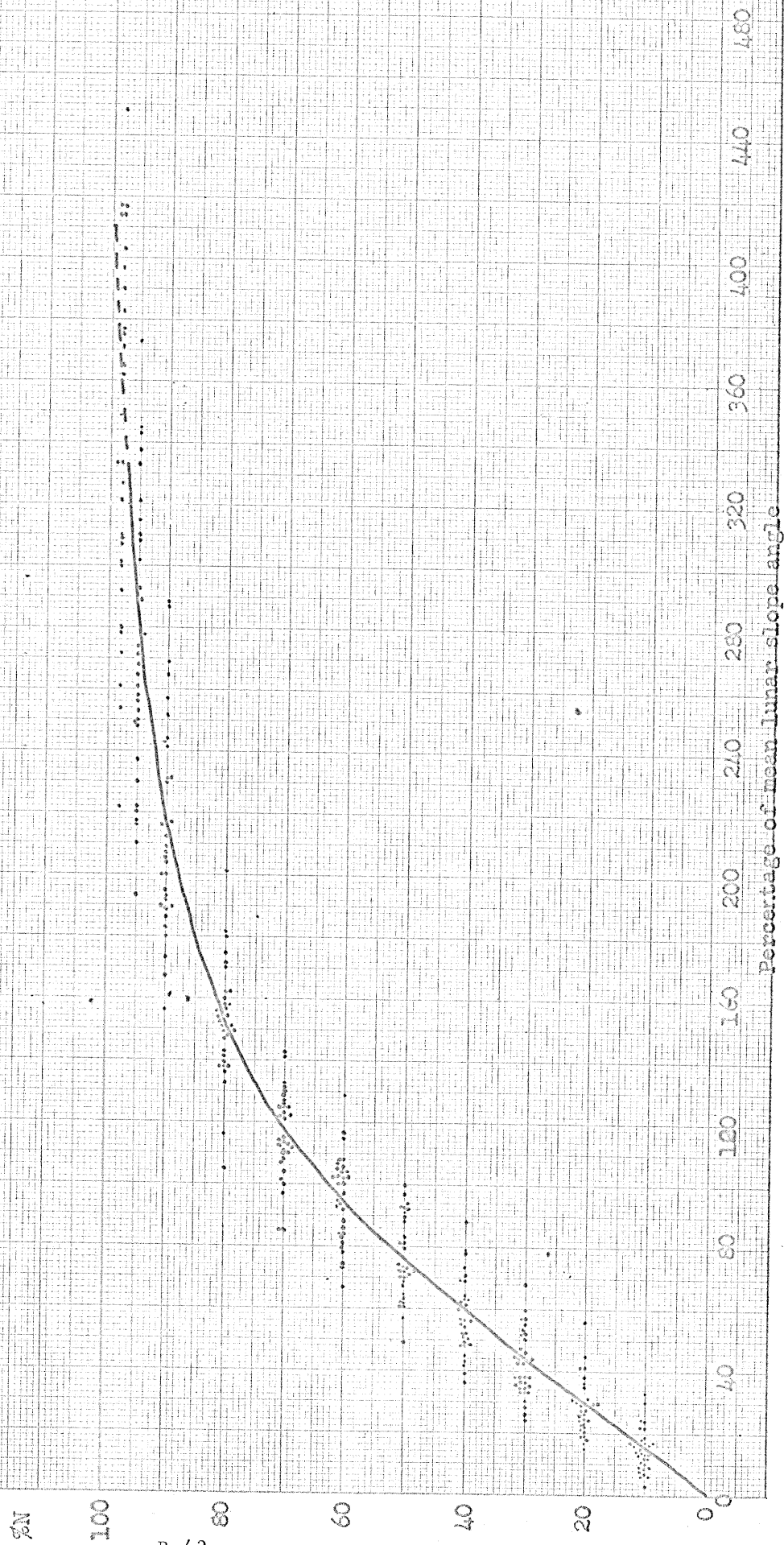
DL is slope length in meters

#### Lunar slope frequency distribution model

Previous experience with terrestrial slope distributions by W. F. Wood (1961, 1962) and the writer (1961) has shown that, given a sufficiently large number of cases ( $50 \leq N \leq 100$ ) all distributions can be "normalized" to yield nearly identical percentage-of-mean slope frequency curves, regardless of how steep or gentle the average slopes might be. The resulting model percentage-frequency curve can be used to predict slope-frequency distributions for any type of terrain for which the mean slope value can be derived.

This technique has been extended to lunar slope distributions in this preliminary investigation. Figure 4 is the model curve derived from existing photoclinometric slope-frequency distributions at 0.6 m resolution. Thirty-two slope-frequency distributions, aggregating 171.025 individual slope values, were plotted for different lunar localities (mostly mare). Each cumulative-frequency curve was then converted into a percentage-frequency curve: cumulative % of cases is plotted on vertical axis, and % of mean slope angle is plotted on the

Figure 4. Percentage-of-mean slope distribution for the lunar surface. Model curve determined from 171,025 slopes in 32 lunar samples. 100th percentile is indeterminate, but probably is about 450 %-of-mean slope angle.



horizontal axis (arithmetic coordinates). While all these curves differ slightly from one another, they are remarkably similar in overall configuration. Percentages of mean slope were read from each curve at 11 convenient intervals. These values were averaged at each of the 11 intervals of % N, and the averages plotted as Figure 4, the preliminary lunar slope distribution model. This curve is virtually identical ( $r = 0.997$ ) to a similar curve derived for terrestrial slope distributions (Table 1). At this stage, no attempt has been made to demonstrate the correspondence of the model lunar curve with the curves from which it was computed. Previous experience with terrestrial distributions suggests that correlation is exceedingly good (probably  $>0.95=r$ ) for the lunar data.

This likelihood of high correlation of model with individual curves suggests that the model curve can be used for prediction of real lunar slope-frequency distributions when the mean slope value is known (or can be inferred statistically). Figures 2 and 3 provides estimates of mean slope for four gross lunar terrain units, smooth mare, rougher mare, hummocky upland, and rough upland, at any desired level of generalization. These estimates can be used in conjunction with Figure 4 to compute probable slope-frequency distributions for any of the four terrain types at any slope length. Tables 6 through 8 in the text are nomographs by which this can be accomplished. Values of % mean slope have been converted back to real slope values simply by multiplying the model % mean slope values by all of the mean slope values listed across the top of the table. Slope distributions presented in Tables 6, 7, 8, and Figures 8, 9, 10, and 11 in the text were computed from

Table 1

Comparison of lunar and terrestrial slope distribution models

Cumulative percentage of cases (%N)	Model Percentages of Mean Slope Angle	
	Lunar Slope Angle (in degrees of arc at a constant base length of about 0.6m-1.0m from photoclinometry of Lunar Orbiter I, II, III, and V imagery)  Total N is 171,025	Terrestrial Slope Angle (tangent of valley side slopes comprising linear segments between major slope reversals; variable base length; data from aerial photos of scale 1:20,000)  Total N is 2876
(100)	(450)	(531)
98	346	369
95	273	269
90	216	210
80	152	150
70	116	115
60	96	91
50	76	71
40	58	57
30	44	45
20	28	34
10	15	22

Correlation coefficient,  $r$ , for above data (omitting 100th percentile) is 0.99774.

mean slope values at one, ten, and fifty meter slope lengths read from Figure 3 of the appendix (Table 4 in the text).

Figure 4 shows that departures of the constituent curves from the average, or model, percentage-of-mean slope curve increases markedly over the 90th percentile of N. Analysis of percentage-of-mean slope values for the 95th, 98th, and 100th percentiles confirms that the dispersion increases sharply with percentile, and that prediction of the 100th percentage-of-mean slope from the model is poor. Analysis shows also that the percentage of mean slope at the 100th percentile of N depends strongly upon the number of slope values included in the sample. Thus, the 100th percentile slope values in Figures 8-11 of the text are only test estimates, relying in part upon terrestrial slope data. Efforts will be made to provide better estimates of the 100th percentile values.

Like all theoretical models, the lunar slope distribution curve (Figure 4) and the predictive nomographs (Tables 6, 7, 8 in text) must be treated with some caution. Visual examination of high-resolution Lunar Orbiter imagery suggests that many terrains which appear exceedingly rough at lower resolutions are in fact quite smooth at slope lengths on the order of a meter. Conversely, other terrain types (crater floors, etc.) which appear smooth at low resolution are quite rough at higher resolution and smaller slope lengths. However, the model derived here should constitute a useful basis from which to proceed in the determination of lunar surface roughness from absolute slope angle data.