# Lunar-based Large Baseline Synthetic Aperture Radar Interferometry of the Earth

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- Objective: Use of lunar outpost for radar remote sensing of earth
  - Create solid Earth, topography, altimetry, 3D tomography, and vegetation maps.
- Form SAR images of the Earth from the Moon surface using the relative motion of the Earth with respect to Moon.
  - Use multiple antennas to form a microwave interferometer with a long baseline and extreme stability.
  - This configuration also allows bistatic operation for remote sensing of complex terrestrial processes.

Important Features:

- Unique capability to observe the entire Earth's disk at any given time.
- Simultaneous multi-baseline interferometry
- Very large baseline for InSAR is easily accomplished (very accurate topography).
- Multi-static SAR, Tx on the Moon and many inexpensive receivers on LEO orbit
- All weather capability.
- Dual-band will allow global observation of the ionosphere.

# Using Moon

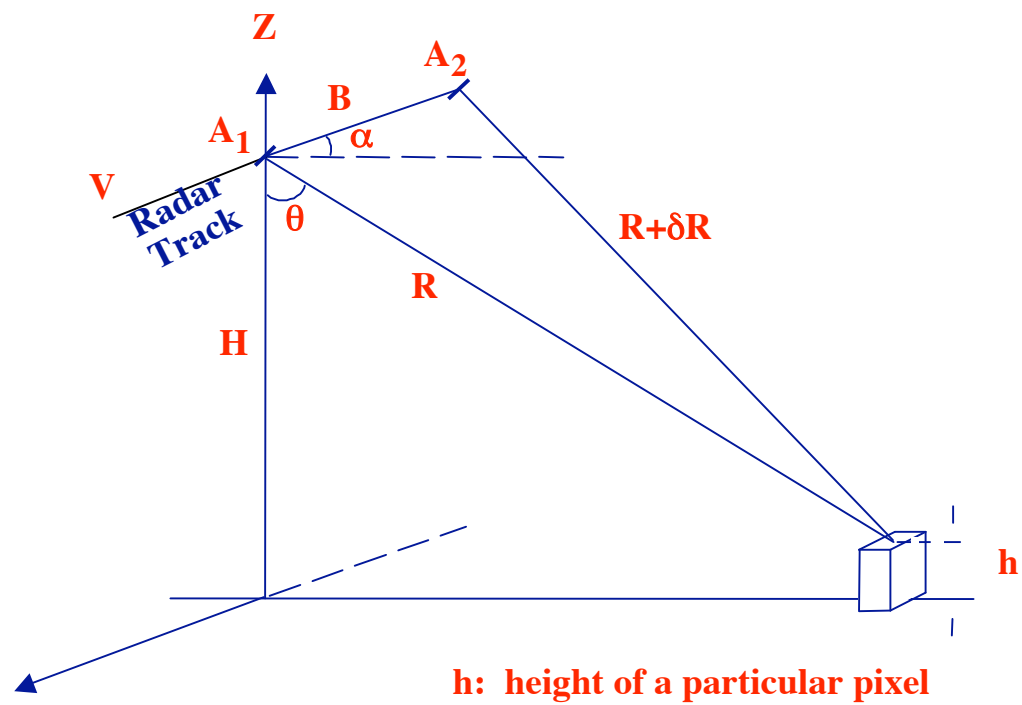
- Planetary Synthetic Aperture Radar (SAR) imaging technique:
  - Utilize the relative motion between Earth and Moon to get cross-range resolution
  - Utilize the signal bandwidth to get range resolution
- Utilize a transmitter and multiple receiver antennas with large baseline to achieve INSAR images with large and stable baselines.

$$h = H - R\cos\theta$$

$$\sin(\alpha - \theta) = \frac{(R+\Delta R)^2 - R^2 - B^2}{2RB} \approx -\frac{\Delta R}{B}$$

$$\Delta R = \frac{\lambda}{2\pi}\phi \qquad \text{where } \phi = \angle E_1^s - \angle E_2^s$$

The larger is B, the more accurate is the estimate of h.



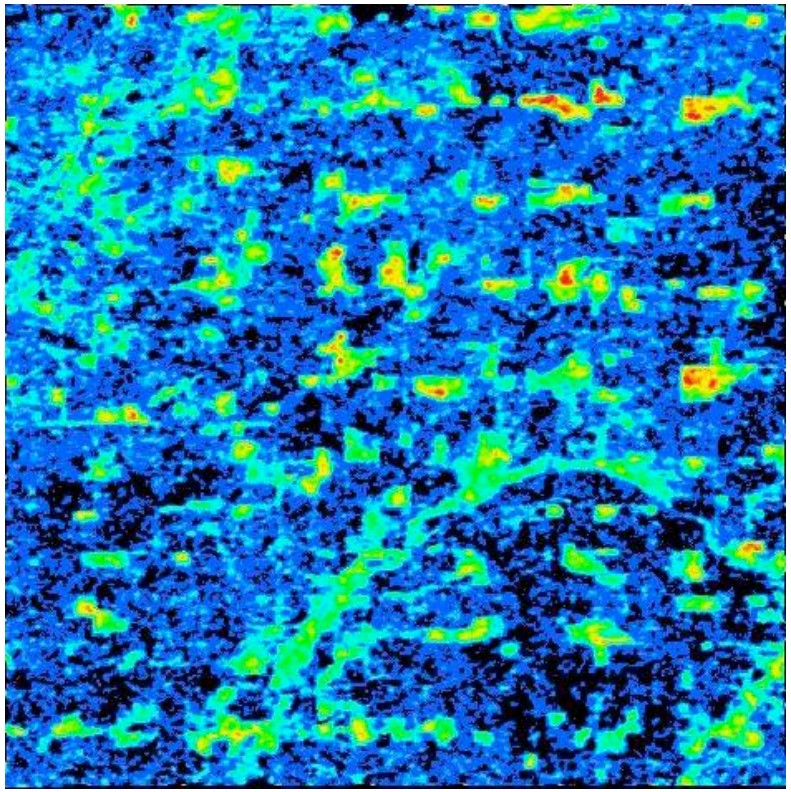

h: height of a particular pixel

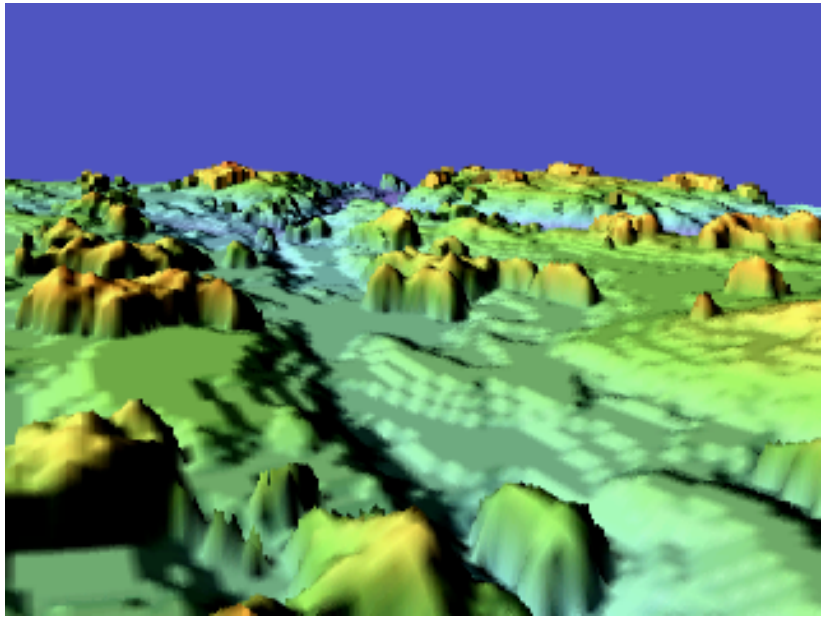
H: height of the radar

# Multi-baseline INSAR

- **Advantages of Multi-baseline SAR interferometry:**
  - **Automatic phase unwrapping**
    - **several interferograms with different baselines have different elevation ambiguity intervals and allow for automatic phase unwrapping**
  - **High quality Digital Elevation Model (DEM) reconstruction.**
    - **Combined DEM from multi-baseline is more accurate than individual DEMs.**
  - **Tomography to produce full 3D imaging (e.g. of forest layers.)**
  - **Using multiple coherence images is possible to produce high resolution coherence maps (i.e. an ensemble averaging is used instead of a spatial averaging)**

# Tree Height Retrieval Using SRTM-USGS Image

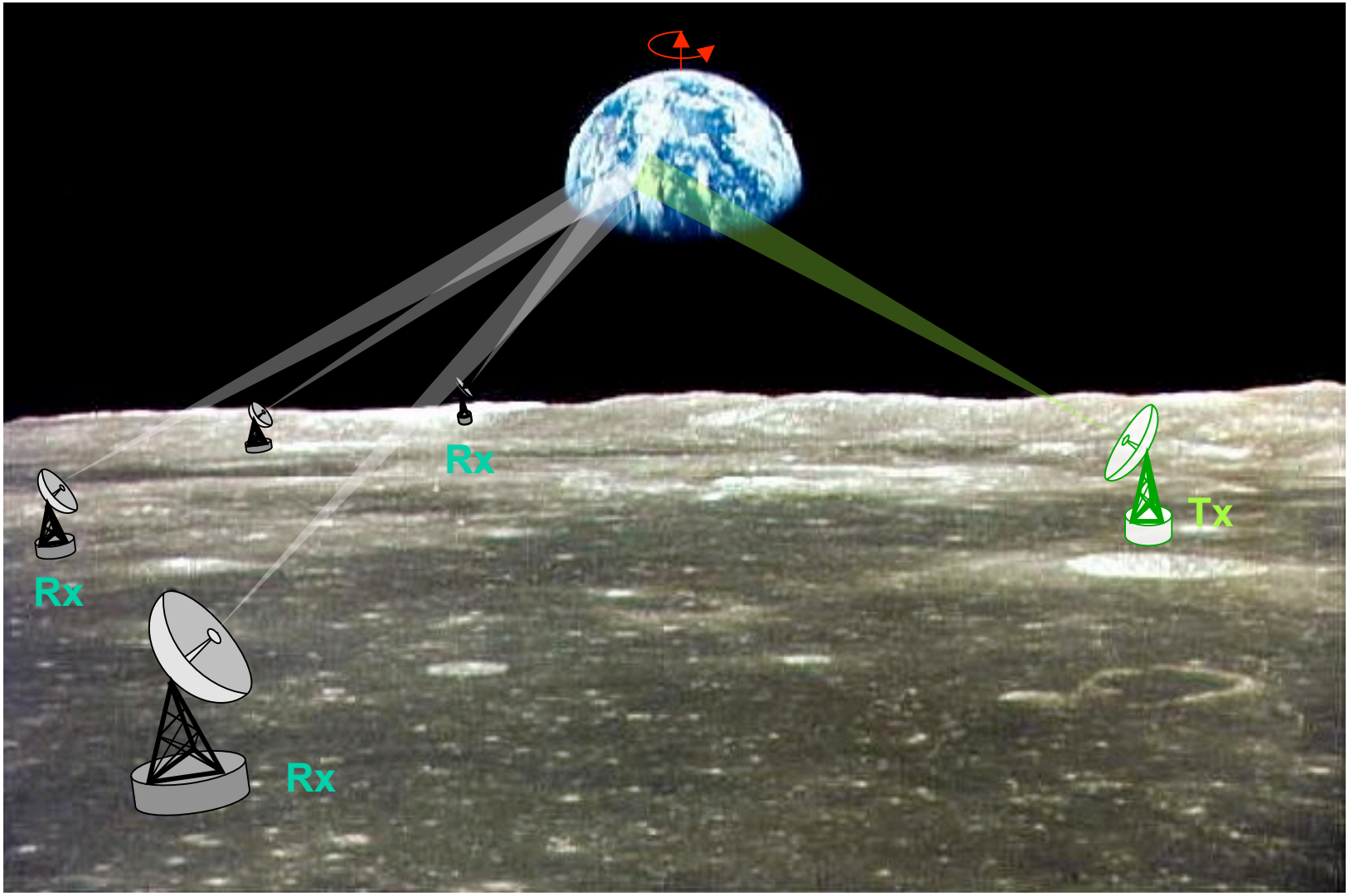




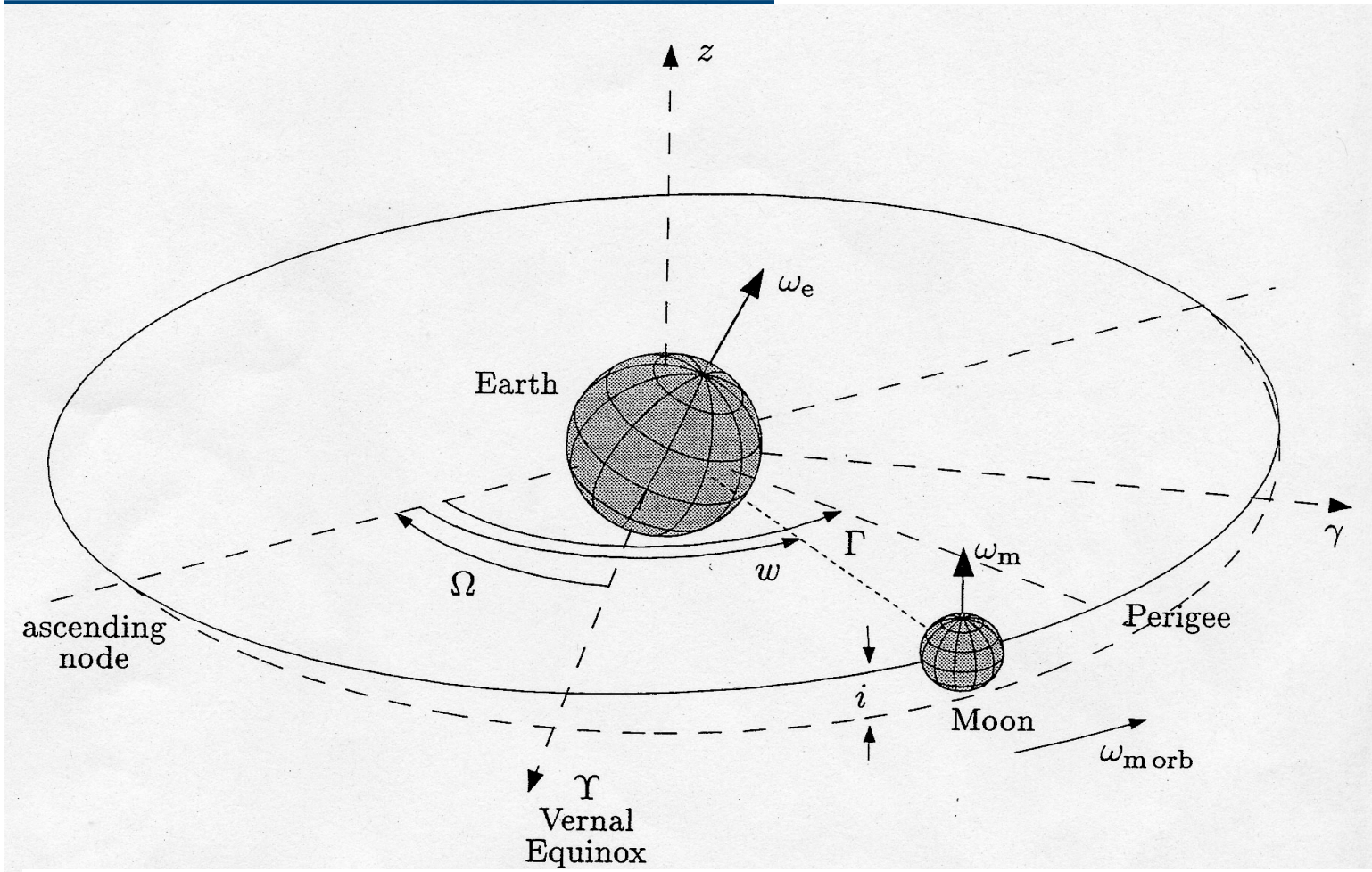
- Bumps are tree stands
- Vertical scale exaggerated

Min=-7m Black-->Blue-->Green-->Yellow-->Red Max=18m

# Multi-Baseline Lunar SAR, INSAR Concept



# Earth-Moon Dynamics

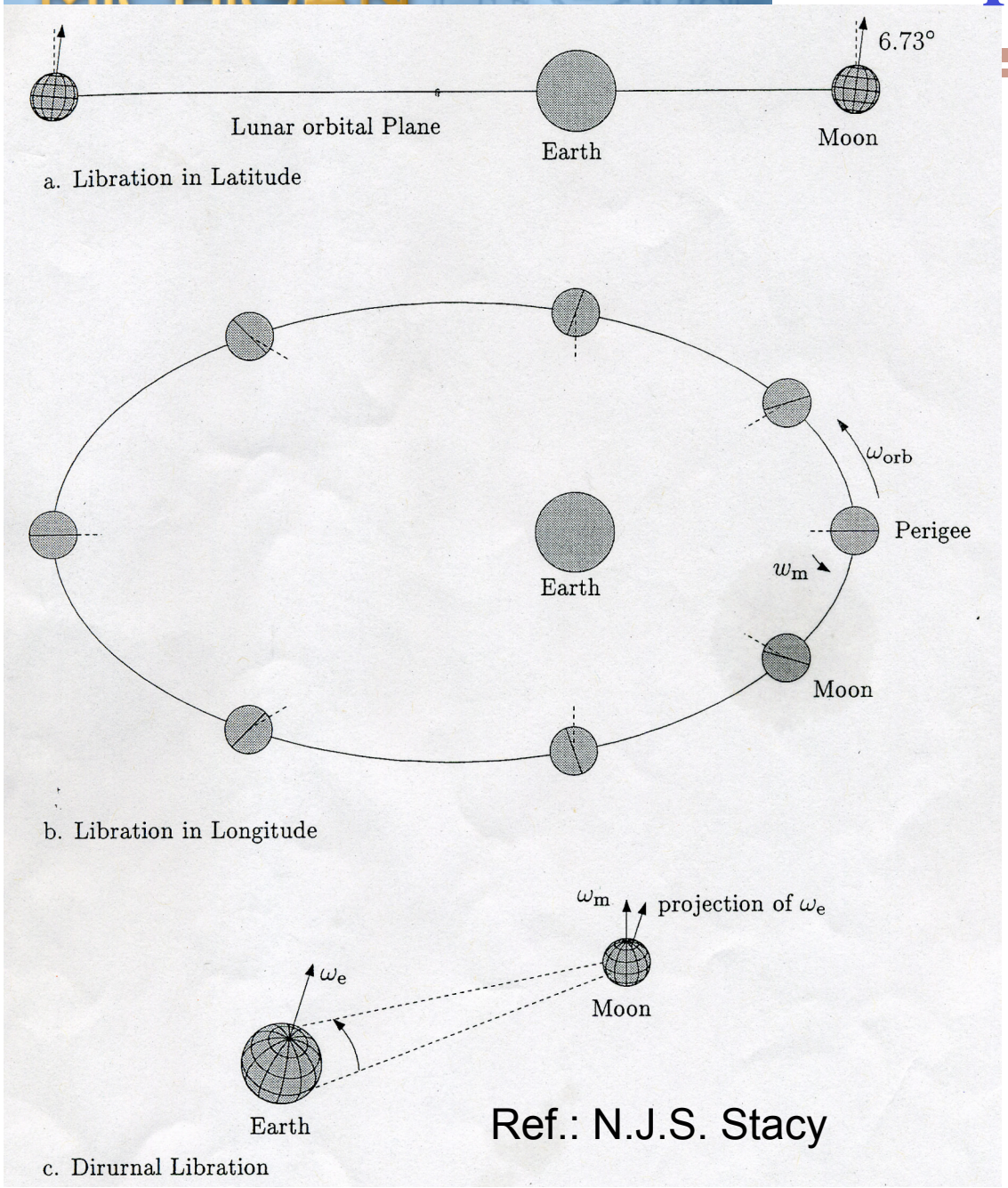



Ref.: N.J.S. Stacy

Earth: spin $\omega_e$ Moon: elliptical orbit around Earth $\omega_{m\,orb}$ and spin $\omega_m$

The relative motion of the Earth in view of a lunar radar on the Moon can be represented by a line-of-sight velocity and a tangential velocity plus a rotation around the apparent rotation axis.

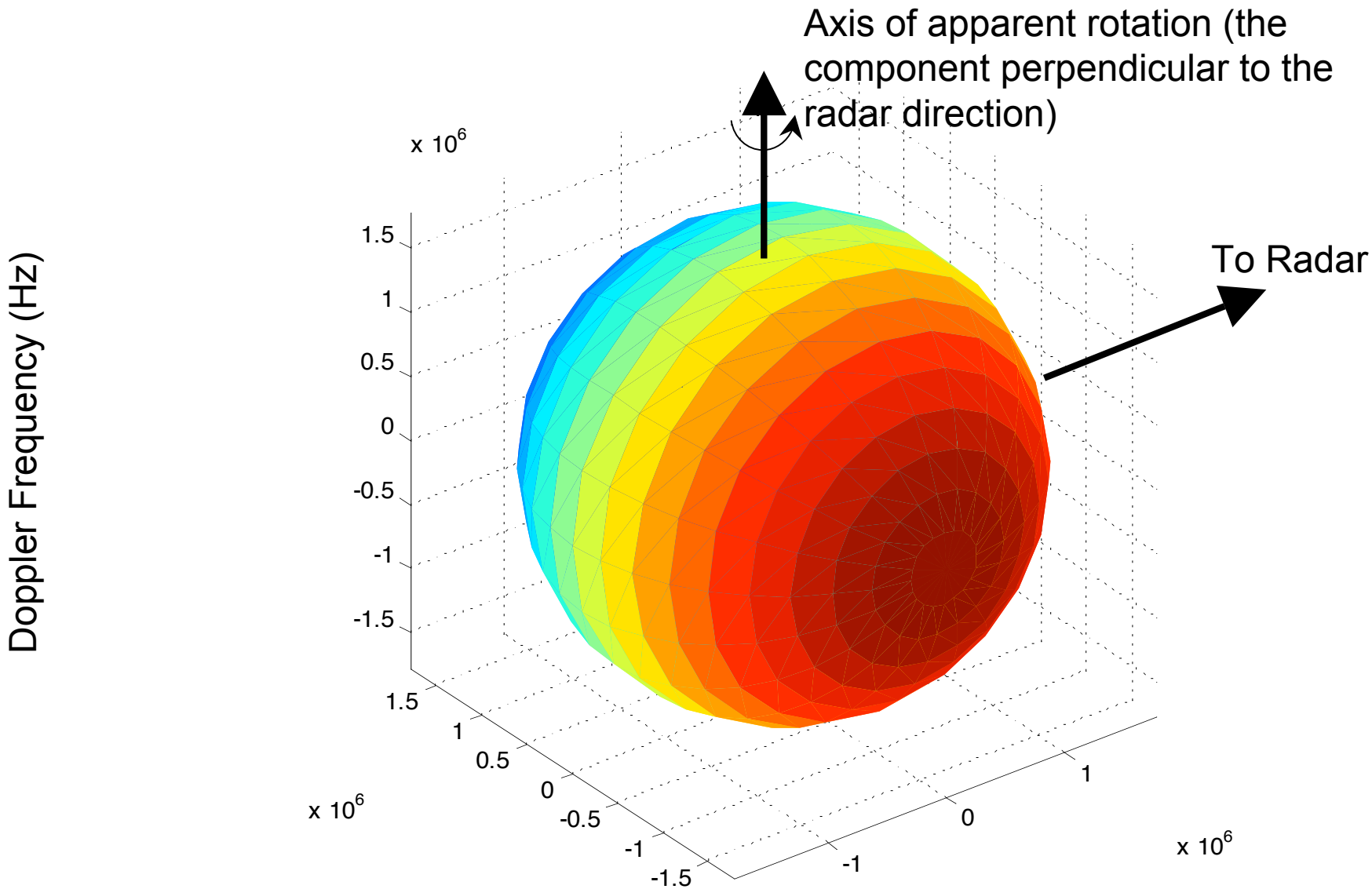
# Apparent Rotation




Ref.: N.J.S. Stacy

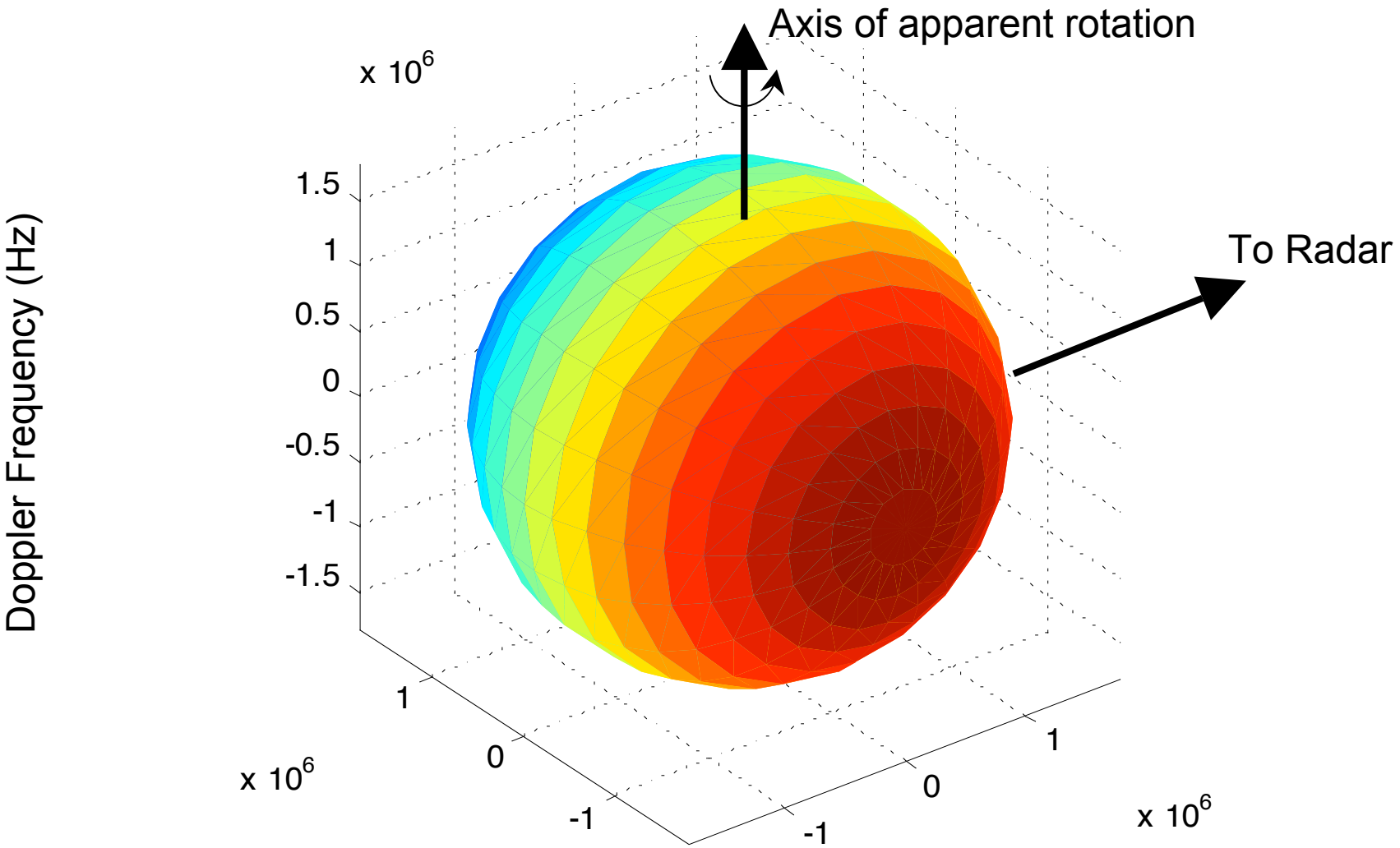
- The apparent rotations of the Moon can be considered in terms of the motion of the sub-Radar point (the point on the lunar surface closest to the observer on the Earth) on the surface of the Moon.
- Major components are:
  - Latitude libration: result of lunar spin axis inclined to lunar orbital plane
  - Longitude libration: result of the difference between $\omega_{m\ orb}$ and $\omega_m$
  - Diurnal libration: due to spin of Earth.
- Magnitude of such rotation is at $10^{-6}$ rad/s.

# Doppler Spectrum



# Doppler Spectrum, ctd.

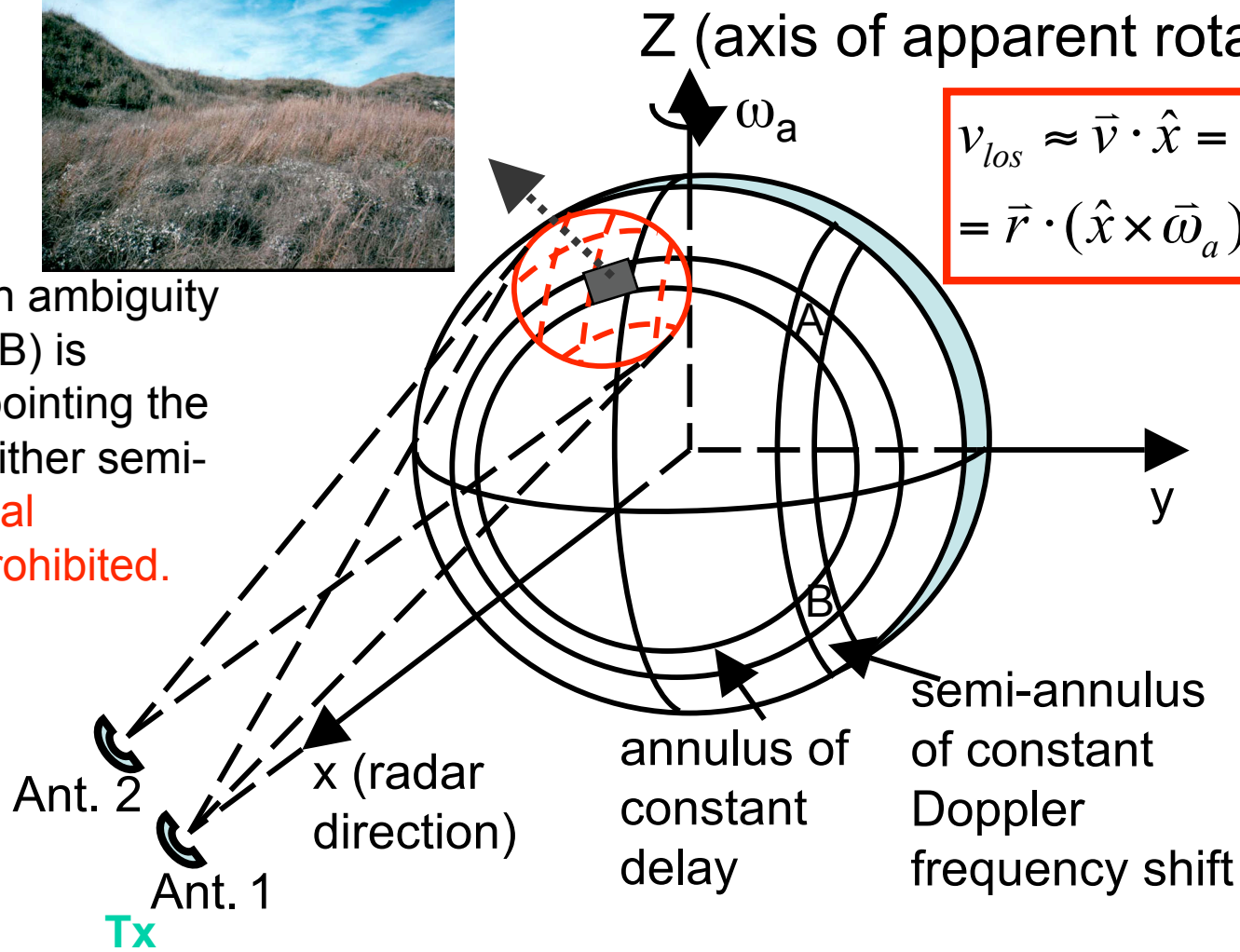
removing bulk motion of Moon



# Lunar InSAR Imagery



- North-south ambiguity (pixel A and B) is avoided by pointing the antenna to either semi-sphere normal incidence, prohibited.

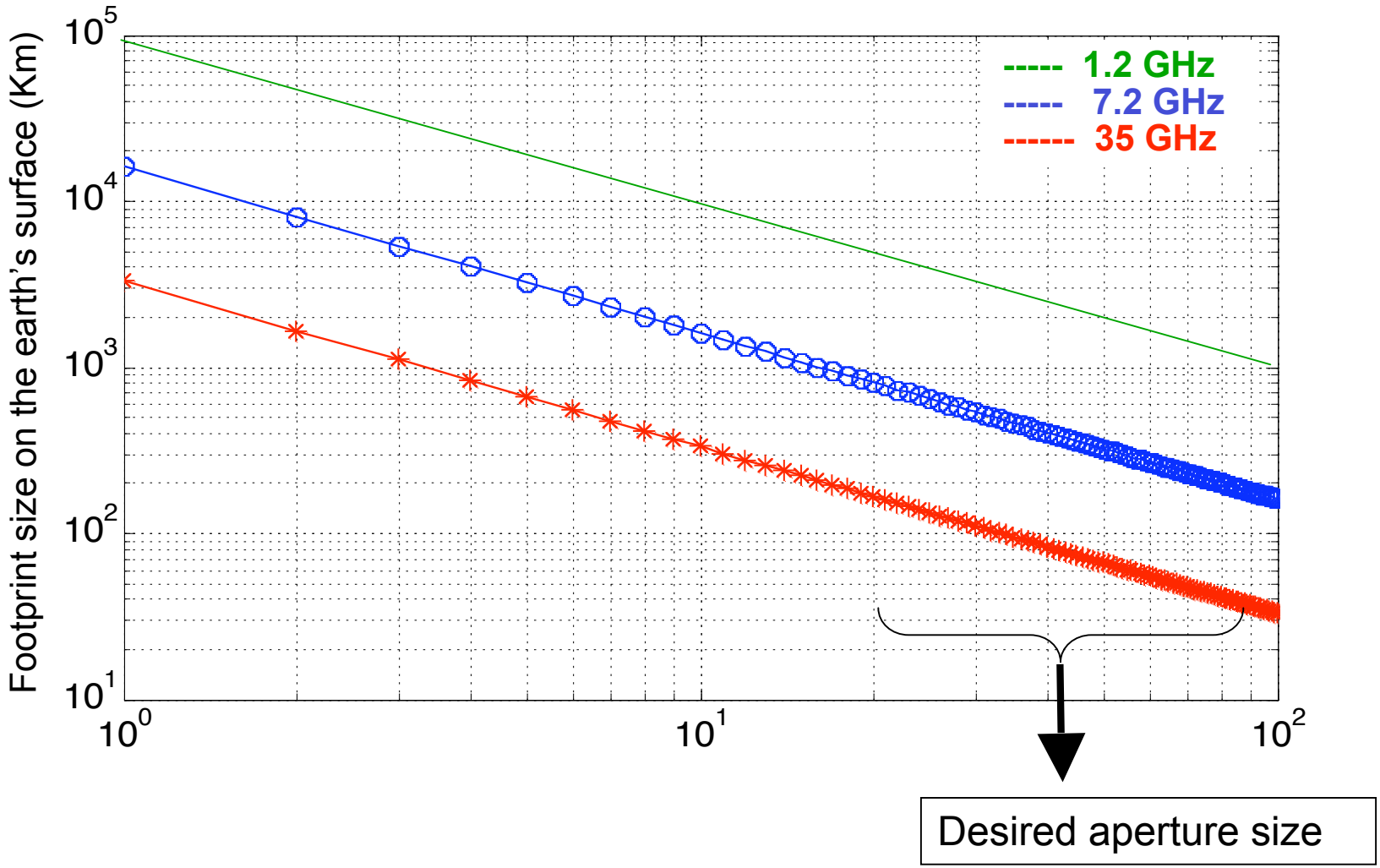



$$v_{los} \approx \vec{v} \cdot \hat{x} = (\vec{\omega}_a \times \vec{r}) \cdot \hat{x}$$
$$= \vec{r} \cdot (\hat{x} \times \vec{\omega}_a) \propto \omega_a r_y$$

- Two lunar antennas taking SAR images of same pixels on the ground forming interferometric SAR (InSAR).

# Using Moon

- The footprint size is large enough cover a large portion of the earth's surface.



# Deployable Reflector Antennas

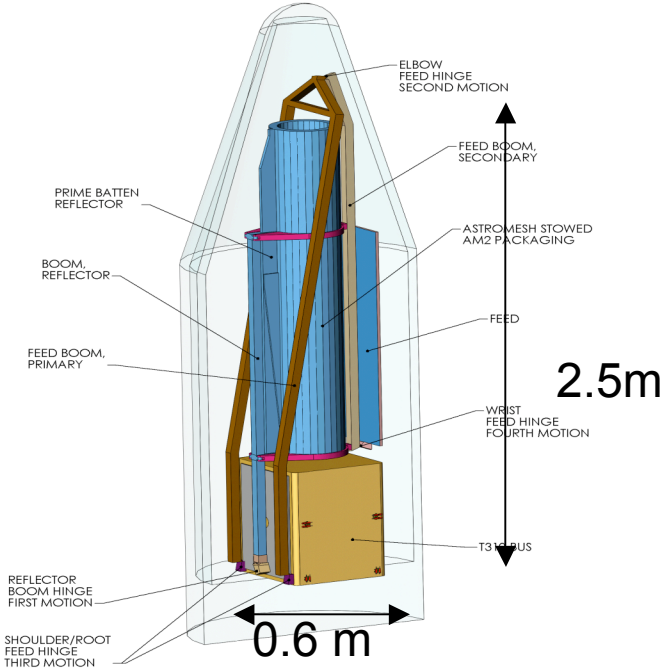
AstroMesh Deployable Large reflector antennas

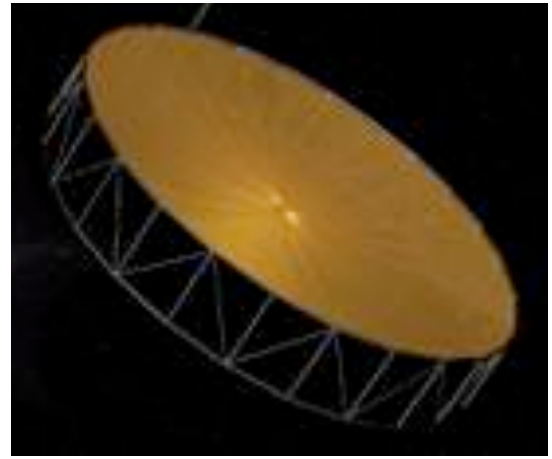
Diameter: 20-100m

Surface accuracy< 2.5DX10$^{-5}$ rms

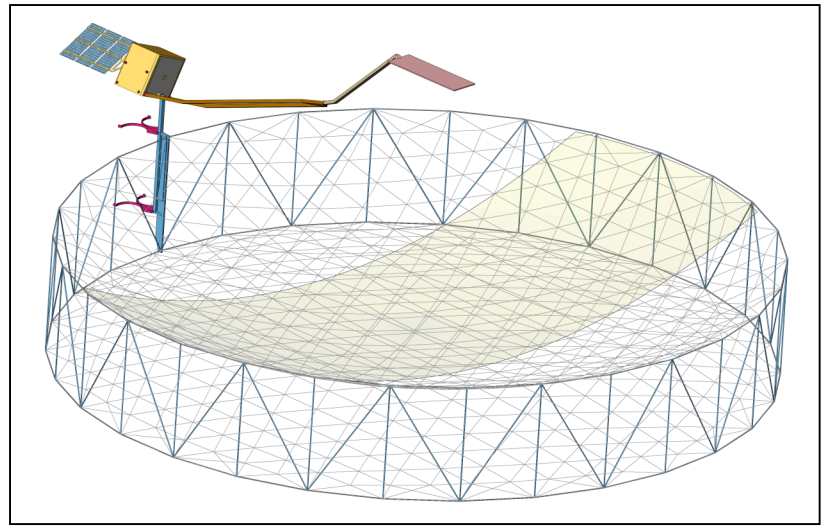
Stowed in a small volume

Stowed



Deployed



- Doppler calculation

$$f_{Doppler} = 2v_{los} / \lambda \approx 2\omega_a r_y / \lambda$$

- Cross range resolution

$$\Delta r_y = \lambda \delta f_{Doppler} / (2\omega_a)$$

$$\delta f_{Doppler} = 1 / t_{coh}$$

- $t_{coh}$ = Coherent observation of the surface
- Slant range resolution:

$$\Delta R = c / 2B$$

B= chirp bandwidth

# Power budget:

Range processing gain

$$SNR = \frac{P_t G_t}{4\pi R^2} \sigma^0 A_{pixel} \frac{A_r}{4\pi R^2} \frac{1}{kTB} \frac{\tau_p}{\delta\tau} f_p t_{coh}$$

Azimuth processing gain

| | |
|---|---|
| $P_t$ | peak transmitter power; |
| $G_t$ | transmitter antenna gain; |
| R | distance to the Moon; |
| $\sigma^0$ | Surface backscatter cross section per unit area; |
| $A_{pixel}$ | Pixel area |
| $A_r$ | effective area of receiver antenna; |
| k | Boltzmann's constant; |
| T | Receiver system temperature |
| B | bandwidth of the receiving system; |
| $\tau_p$ | uncompressed pulse width; |
| $\delta\tau$ | compressed pulse width; |
| $f_p$ | pulse repetition frequency; |
| $t_{coh}$ | coherent processing interval. |

# Bare Surface $\sigma^0$

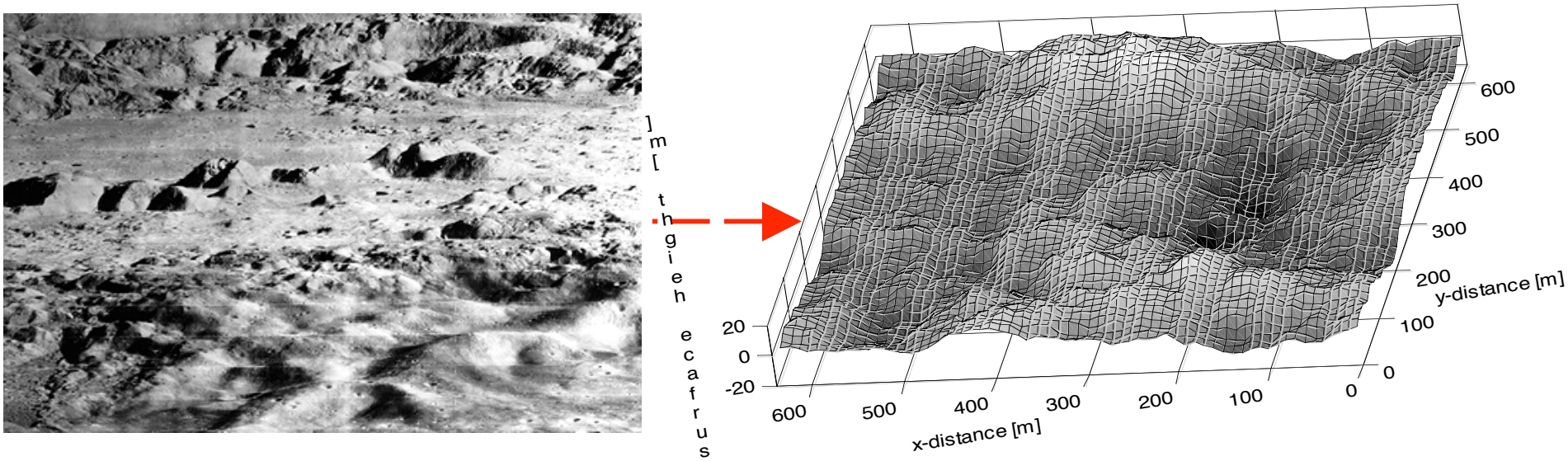
Estimation of $\sigma^0$ for dry soil with effective $\varepsilon_r \approx 3.0$.

3D rough surface generator using power-law spectrum:

$$P(k) = 4\pi\nu h^2 k_c^{-2}\left[\left(\frac{2\pi\, k}{k_c}\right)^2 + 1\right]^{-(\nu+1)}$$

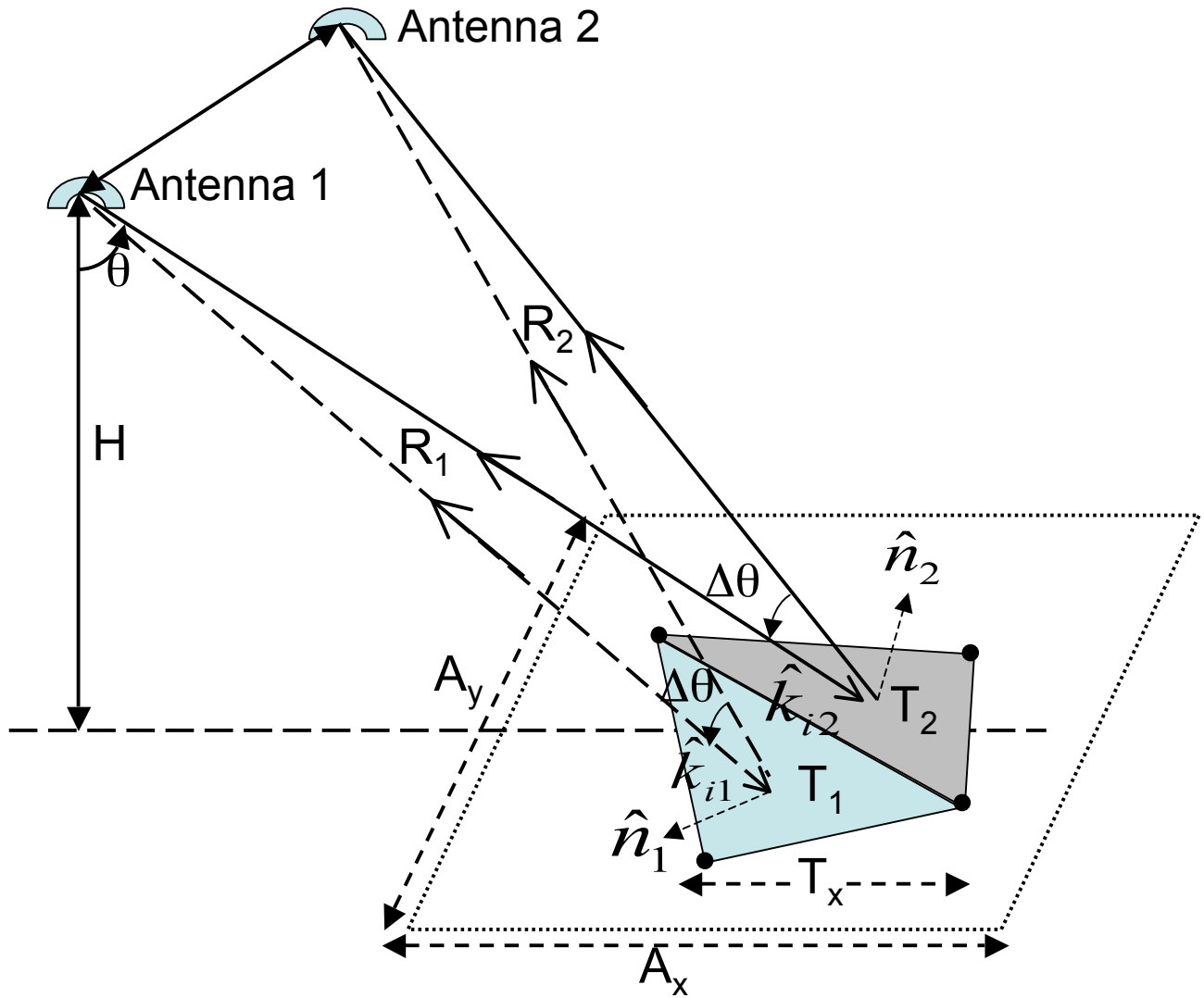
(Ref.: Smith *et al., 1997)*

*$\nu$ = 0.7, $k_c$=1/(corr. Length), h = rms height of large scale undulation*



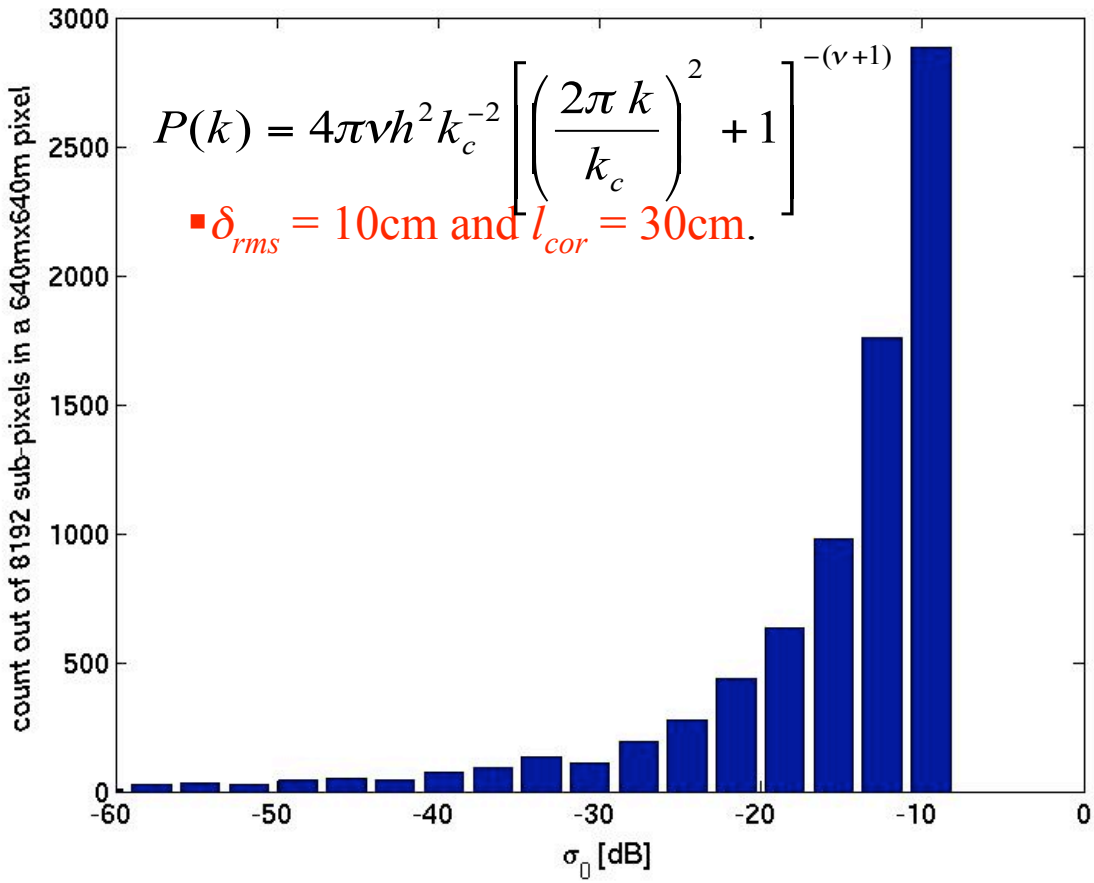
# Rough Surface Scattering: Two-scale model

- The surface pixels are divided into triangular sub-pixels with small scale roughness which generate backscatter signals.
- Plane wave assumed for local incidence on each sub-pixel.
- Average RCS $\sigma_0$ from each sub-pixel is calculated using GO according to the local incidence angle:




$$\theta_i^L = \pi - \cos^{-1}(\hat{k}_i \cdot \hat{n}) \in [0, \pi/2]$$, shadowing considered, i.e. if > $\pi/2$ then $\sigma_0$=0.

•Histogram of backscatter coefficient for 640mx640m pixels generated from 8192 pixels at 7.2 GHz and nominal incidence angle of 40 deg.
•The average value is -12dB, and the 1-sigma confidence level is [-19dB,-10dB].



# Simulation Results

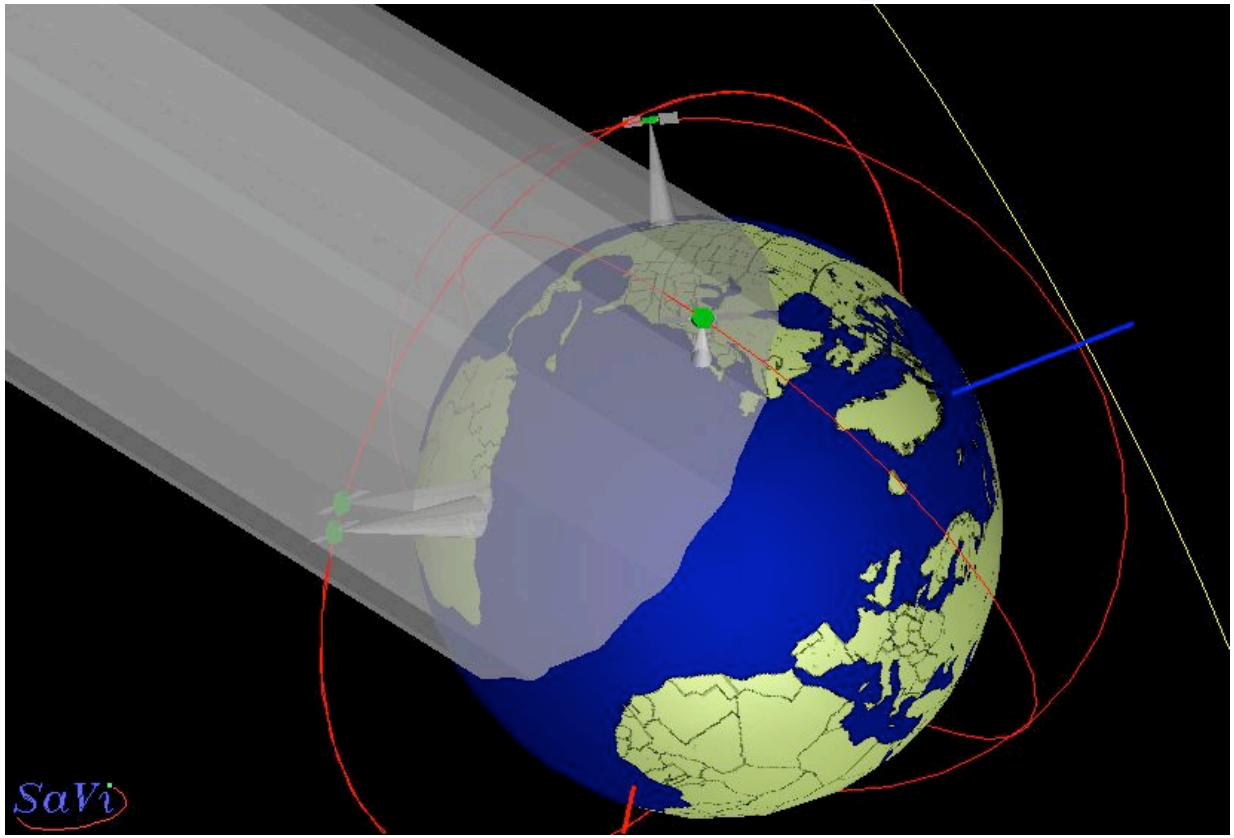
## System Parameters:

- Antennas Diameter: 35m, Aperture Efficiency: 60%,
- Noise Temperature: $T_{Moon}+T_{sys}$=300K,
- Wavelength: 4.17cm (7.2GHz).
- Observation time $t_{coh} = 2\omega_a/(\lambda\, r_{az}) = 18750/r_{az}\,(s)$
  - where $r_{az}$ is cross-range resolution in
  - $\omega_a$ is the apparent angular velocity $\approx 10^{-6}$ rad/s.
  - $r_{az}=100m$ $t_{coh} = 187.50s\sim3min$
- $\sigma^0=-12\ dB$
- Duty cycle $f_p\tau_p=0.64$
- Power Radiated: 36 kW
- SNR~15 dB
- Limitation: $t_{coh}\sim3min$ Due to relative motion, the antennas have to track the illuminated spot (spotlight SAR).
  - The integration time of 4 minutes will require beam steering of about $1^0$

# Alternative Approach: A multi-Static SAR INSAR System

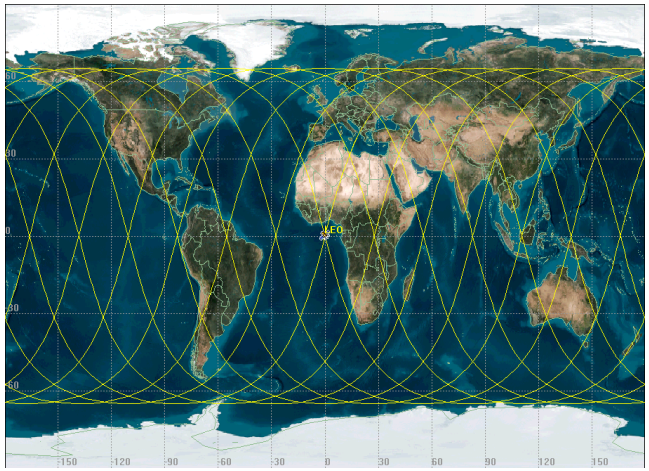
- A high resolution multi-static imaging radar system with transmitter on the Moon with continuous illumination
  - Receiver constellation on LEO satellite, airplanes, etc.

- Range to receiver is much reduced.
- Same amount of transmit power is needed (lower integration time)
- Much finer resolution can be achieved

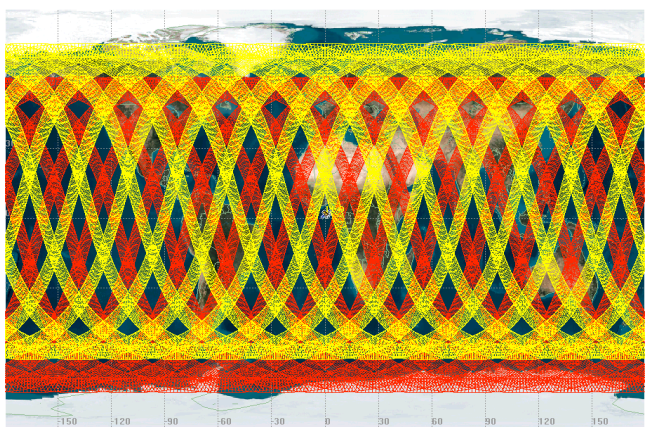


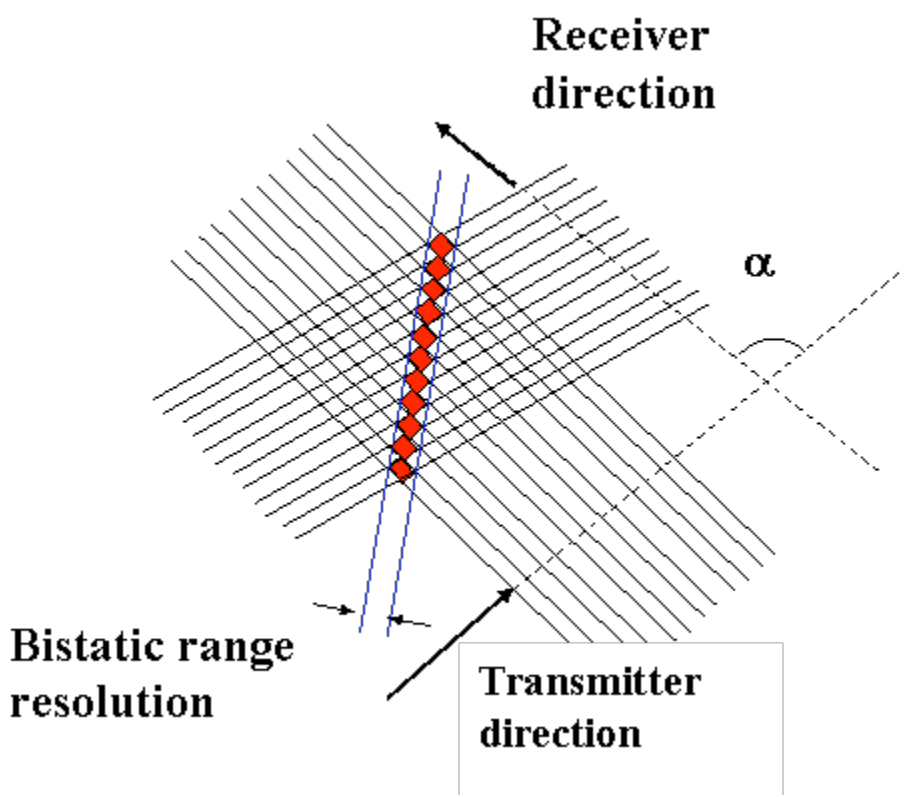
# A LEO Satellite 65° inclination at an altitude of 1214.5 km

## 1-day repeating ground-track



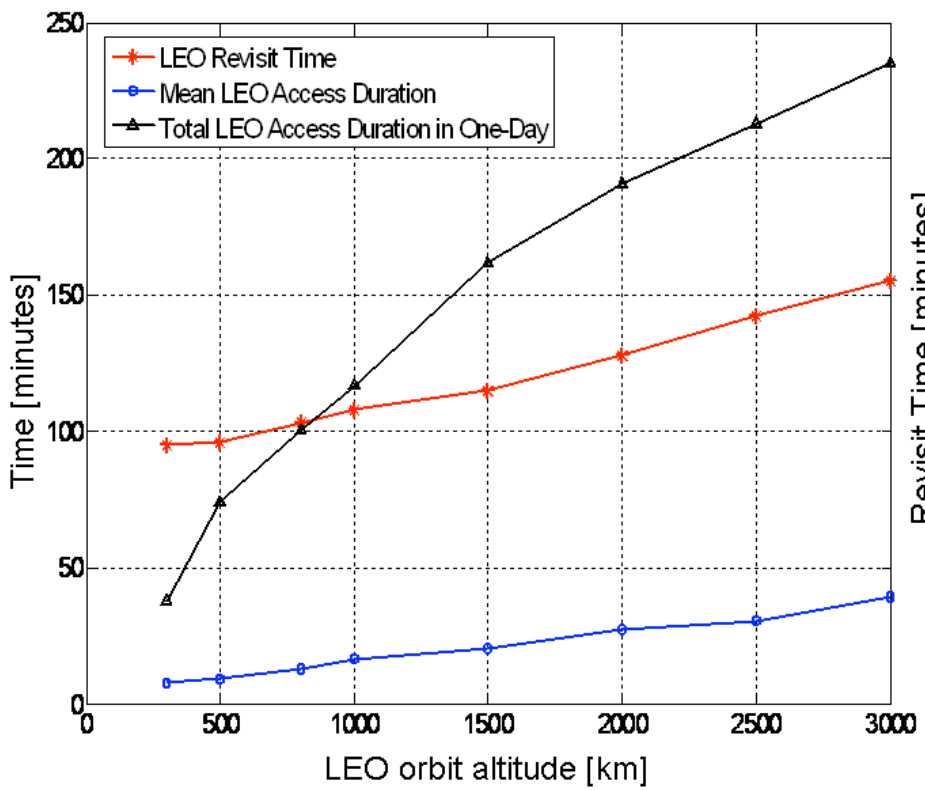
Sensor coverage for a sensor with a swath width of 660 km





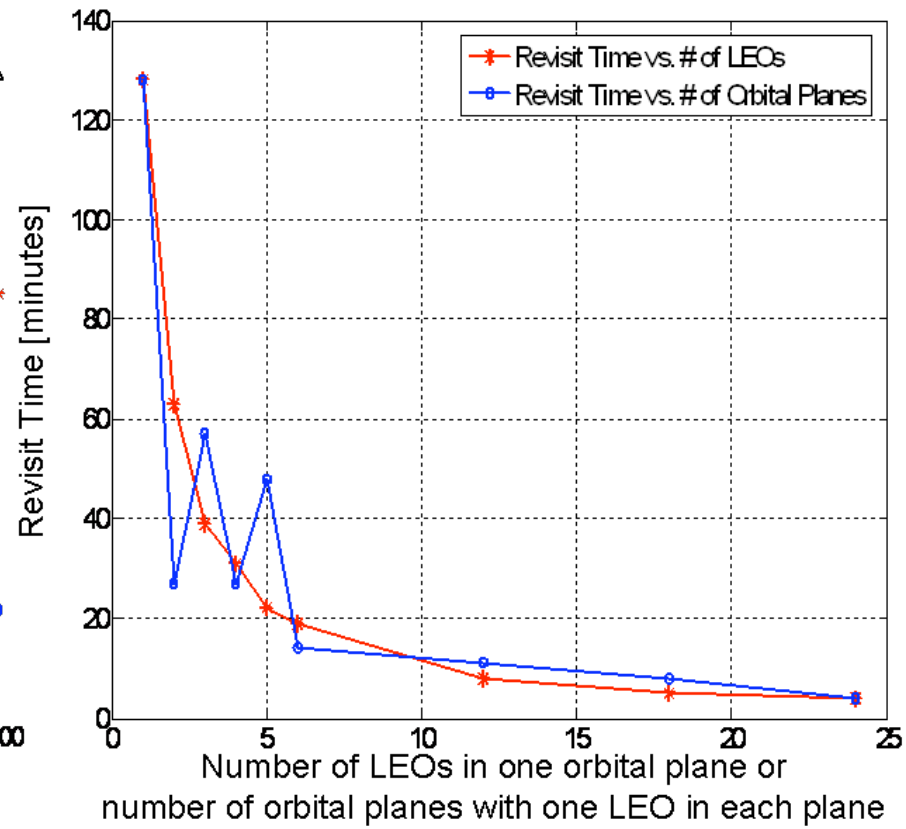

$$r_{rg} = \frac{c}{2B\sin\alpha/2}$$

All LEO orbits are circular and have 55° inclination angle.



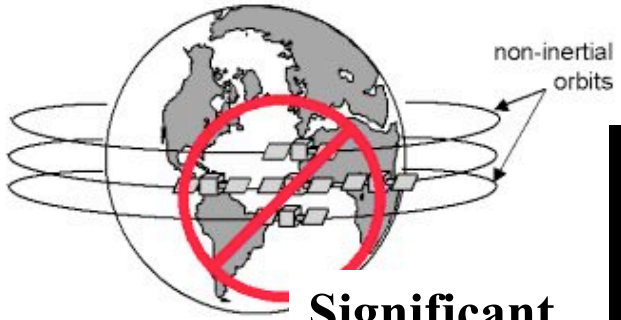

All LEO orbits are circular and have 55° inclination angle.

LEOs or RAANs are equally spaced.



# Bistatic SAR Interferometry

- Brute force cluster motion requires enormous propulsion




**Significant drift**

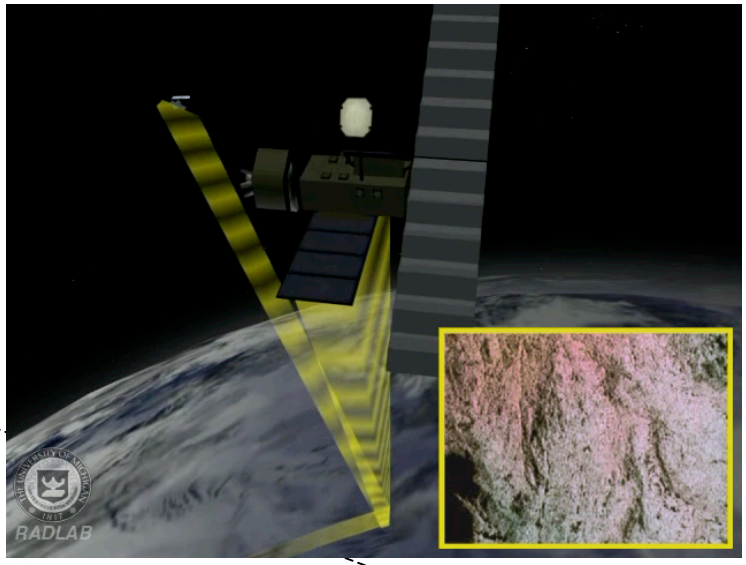
- Stable low-propulsion cluster configurations exist

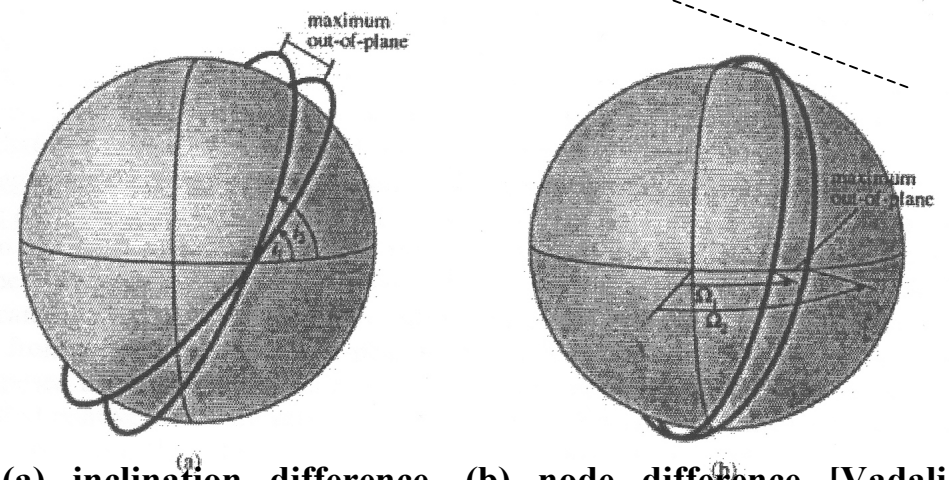
**Low drift**

Smart Formation Flying
Works With Mother Nature

Two Orders of Magnitude Re

**NRL tethered spacecraft mission called TiPS**




**(a) inclination difference, (b) node difference [Vadali, 2003]**

- Lunar-based planetary SAR technique seems feasible
  - Multi-baseline SAR interferometry from the Moon surface
  - Multistatic Lunar/LEO SAR and INSAR can be considered
- Technology to setup very large antennas on the Moon exists but need be advanced for the required tolerances
- Need infrastructure to produce about 50Kw of microwave power