Lunar-based Large Baseline Synthetic Aperture Radar Interferometry of the Earth

Kamal Sarabandi
The University of Michigan

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• **Objective:** Use of lunar outpost for radar remote sensing of earth
  – Create solid Earth, topography, altimetry, 3D tomography, and vegetation maps.

• **Form SAR images of the Earth from the Moon surface using the relative motion of the Earth with respect to Moon.**
  – Use multiple antennas to form a microwave interferometer with a long baseline and extreme stability.
  – This configuration also allows bistatic operation for remote sensing of complex terrestrial processes.
Important Features:

- Unique capability to observe the entire Earth's disk at any given time.
- Simultaneous multi-baseline interferometry
- Very large baseline for InSAR is easily accomplished (very accurate topography).
- Multi-static SAR, Tx on the Moon and many inexpensive receivers on LEO orbit
- All weather capability.
- Dual-band will allow global observation of the ionosphere.
Using Moon

➤ Planetary Synthetic Aperture Radar (SAR) imaging technique:
  ➤ Utilize the relative motion between Earth and Moon to get cross-range resolution
  ➤ Utilize the signal bandwidth to get range resolution
  ➤ Utilize a transmitter and multiple receiver antennas with large baseline to achieve INSAR images with large and stable baselines.

\[
h = H - R \cos \theta
\]

\[
\sin (\alpha - \theta) = \frac{(R + \Delta R)^2 - R^2 - B^2}{2RB} \approx -\frac{\Delta R}{B}
\]

\[
\Delta R = \frac{\lambda}{2\pi} \phi \quad \text{where} \quad \phi = \angle E_1^s - \angle E_2^s
\]

The larger is B, the more accurate is the estimate of h.

h: height of a particular pixel
H: height of the radar
Advantages of Multi-baseline SAR interferometry:

- Automatic phase unwrapping
  - Several interferograms with different baselines have different elevation ambiguity intervals and allow for automatic phase unwrapping
- High quality Digital Elevation Model (DEM) reconstruction.
  - Combined DEM from multi-baseline is more accurate than individual DEMs.
- Tomography to produce full 3D imaging (e.g. of forest layers.)
- Using multiple coherence images is possible to produce high resolution coherence maps (i.e. an ensemble averaging is used instead of a spatial averaging)
Tree Height Retrieval
Using SRTM-USGS Image

• Bumps are tree stands
• Vertical scale exaggerated

Min=-7m  Black-->Blue-->Green-->Yellow-->Red  Max=18m
Multi-Baseline Lunar SAR, INSAR Concept
Earth: spin $\omega_e$  

Moon: elliptical orbit around Earth $\omega_{m\text{ orb}}$ and spin $\omega_m$

The relative motion of the Earth in view of a lunar radar on the Moon can be represented by a line-of-sight velocity and a tangential velocity plus a rotation around the apparent rotation axis.
The apparent rotations of the Moon can be considered in terms of the motion of the sub-Radar point (the point on the lunar surface closest to the observer on the Earth) on the surface of the Moon.

Major components are:

1. Latitude libration: result of lunar spin axis inclined to lunar orbital plane
2. Longitude libration: result of the difference between $\omega_{m\text{ orb}}$ and $\omega_m$
3. Diurnal libration: due to spin of Earth.

Magnitude of such rotation is at $10^{-6}$ rad/s.
Doppler Spectrum

Axis of apparent rotation (the component perpendicular to the radar direction)

To Radar

Doppler Frequency (Hz)
removing bulk motion of Moon

Doppler Spectrum, ctd.

Axis of apparent rotation

To Radar
North-south ambiguity (pixel A and B) is avoided by pointing the antenna to either semi-sphere normal incidence, prohibited.

Two lunar antennas taking SAR images of same pixels on the ground forming interferometric SAR (InSAR).
The footprint size is large enough to cover a large portion of the earth’s surface.

![Graph showing footprint size on the Earth's surface vs. desired aperture size for different frequencies: 1.2 GHz, 7.2 GHz, and 35 GHz. The x-axis represents the desired aperture size, and the y-axis represents the footprints size on the Earth's surface in kilometers.](image)
AstroMesh Deployable Large reflector antennas

Diameter: 20-100m

Surface accuracy < 2.5Dx10^-5 rms

Stowed in a small volume
• Doppler calculation

\[ f_{\text{Doppler}} = \frac{2v_{\text{los}}}{\lambda} \approx \frac{2\omega_a r_y}{\lambda} \]

• Cross range resolution

\[ \Delta r_y = \frac{\lambda \delta f_{\text{Doppler}}}{(2\omega_a)} \]

\[ \delta f_{\text{Doppler}} = \frac{1}{t_{\text{coh}}} \]

• \( t_{\text{coh}} \) = Coherent observation of the surface

• Slant range resolution:

\[ \Delta R = \frac{c}{2B} \]

\( B \) = chirp bandwidth
Power budget:

\[
SNR = \frac{P_t G_t}{4\pi R^2} \sigma^0 A_{\text{pixel}} \frac{A_r}{4\pi R^2} \frac{1}{kTB} \frac{\tau_p}{\delta\tau} f_p t_{\text{coh}}
\]

- \(P_t\): peak transmitter power;
- \(G_t\): transmitter antenna gain;
- \(R\): distance to the Moon;
- \(\sigma^0\): Surface backscatter cross section per unit area;
- \(A_{\text{pixel}}\): Pixel area
- \(A_r\): effective area of receiver antenna;
- \(k\): Boltzmann’s constant;
- \(T\): Receiver system temperature
- \(B\): bandwidth of the receiving system;
- \(\tau_p\): uncompressed pulse width;
- \(\delta\tau\): compressed pulse width;
- \(f_p\): pulse repetition frequency;
- \(t_{\text{coh}}\): coherent processing interval.

Range processing gain

Azimuth processing gain
Estimation of $\sigma^0$ for dry soil with effective $\varepsilon_r \approx 3.0$.

3D rough surface generator using power-law spectrum:

$$P(k) = 4\pi \nu h^2 k_c^{-2} \left[ \left( \frac{2\pi k}{k_c} \right)^2 + 1 \right]^{-(\nu+1)}$$

(Ref.: Smith et al., 1997)

$\nu = 0.7$, $k_c = 1/\text{corr. Length}$, $h = \text{rms height of large scale undulation}$
- The surface pixels are divided into triangular sub-pixels with small scale roughness which generate backscatter signals.
- Plane wave assumed for local incidence on each sub-pixel.
- Average RCS $\sigma_0$ from each sub-pixel is calculated using GO according to the local incidence angle:
  \[ \theta_i^L = \pi - \cos^{-1}(\hat{k}_i \cdot \hat{n}) \in [0, \pi/2] \], shadowing considered, i.e. if $> \pi/2$ then $\sigma_0 = 0$. 
- Histogram of backscatter coefficient for 640mx640m pixels generated from 8192 pixels at 7.2 GHz and nominal incidence angle of 40 deg.
- The average value is -12dB, and the 1-sigma confidence level is [-19dB, -10dB].

\[ P(k) = 4\pi v h^2 k_c^{-2} \left[ \left( \frac{2\pi k}{k_c} \right)^2 + 1 \right]^{-(v+1)} \]

- \( \delta_{rms} = 10\text{cm} \) and \( l_{cor} = 30\text{cm} \).
System Parameters:

- Antennas Diameter: 35m, Aperture Efficiency: 60%,
- Noise Temperature: $T_{\text{Moon}} + T_{\text{sys}} = 300K$,
- Wavelength: 4.17cm (7.2GHz).
- Observation time $t_{\text{coh}} = 2 \omega_a / (\lambda r_{az}) = 18750/r_{az}$ (s)
  - where $r_{az}$ is cross-range resolution in
  - $\omega_a$ is the apparent angular velocity $\approx 10^{-6}$ rad/s.
  - $r_{az} = 100m$  $t_{\text{coh}} = 187.50s \sim 3min$
- $\sigma^0 = -12$ dB
- Duty cycle $f_p \tau_p = 0.64$
- Power Radiated: 36 kW
- SNR $\sim 15$ dB
- Limitation: $t_{\text{coh}} \sim 3min$ Due to relative motion, the antennas have to track the illuminated spot (spotlight SAR).
  - The integration time of 4 minutes will require beam steering of about $1^0$
Alternative Approach: A multi-Static SAR INSAR System

- A high resolution multi-static imaging radar system with transmitter on the Moon with continuous illumination
- Receiver constellation on LEO satellite, airplanes, etc.

- Range to receiver is much reduced.
- Same amount of transmit power is needed (lower integration time)
- Much finer resolution can be achieved
A LEO Satellite 65° inclination at an altitude of 1214.5 km

1-day repeating ground-track

Sensor coverage for a sensor with a swath width of 660 km

Bistatic range resolution

Receiver direction

Transmitter direction

\[ r_{rg} = \frac{c}{2B \sin \alpha / 2} \]
All LEO orbits are circular and have 55° inclination angle.

LEOs or RAANs are equally spaced.
Bistatic SAR Interferometry

- Brute force cluster motion requires enormous propulsion

- Stable low-propulsion cluster configurations exist

Smart Formation Flying Works With Mother Nature

Two Orders of Magnitude Re

NRL tethered spacecraft mission called TiPS

(a) inclination difference, (b) node difference [Vadali, 2003]
Lunar-based planetary SAR technique seems feasible

- Multi-baseline SAR interferometry from the Moon surface
- Multistatic Lunar/LEO SAR and INSAR can be considered

- Technology to setup very large antennas on the Moon exists but need be advanced for the required tolerances
- Need infrastructure to produce about 50Kw of microwave power