MARKOV-CHAIN MONTE-CARLO METHODS FOR ASTEROID ORBIT COMPUTATION. D. A. Oszkiewicz[1], K. Muinonen[2], S. Mouret[3], M. Granvik[4], and J. Virtanen[5], [1]dagmara.oszkiewicz@helsinki.fi, Observatory, P.O. Box 14, FI-00014 University of Helsinki, Finland, [2]karri.muinonen@helsinki.fi, [3]serge.mouret@helsinki.fi, [4]mikael.granvik@iki.fi , [5]jenni.virtanen@fgi.fi.

Statistical asteroid-orbit-computation methods have proved to be important in problems such as computing the collision probability [1], performing dynamical classification, identifying asteroids [3], and aiding the recovery of lost objects. These methods will be of major importance during the Gaia mission (to be launched in 2012). Here we present novel variants of the methods. As earlier in [1, 2, 3], we examine the Bayesian a posteriori probability density of the orbital elements using Monte-Carlo methods that map an unbiased volume of solutions in the orbital-element phase space. In particular, we use the Markov-Chain Monte-Carlo (MCMC) method to map the phase space [4]. MCMC allows us to increase the performance of sampling over the phase space. The MCMC orbit-computation methods presented here have the benefit of performing a direct unbiased sampling of complicated distributions (see Fig. 1 for examples of marginal distributions of orbital elements).

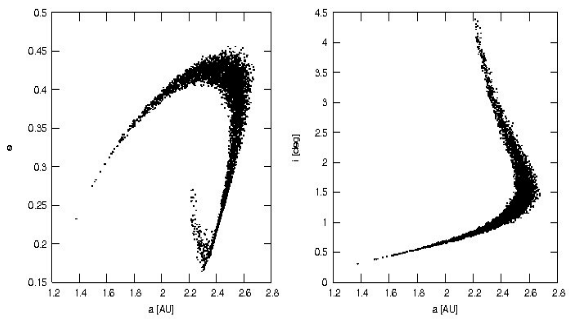

Fig. 1: The MCMC ranging method was used to generate 5000 orbits for main-belt object 1987 AN to map the 6D phase space of the orbital elements (epoch 1987 Jan 4.0 TDT). The distributions for the orbital elements were acquired using a standard deviation of 0.1 AU for the proposal distributions of ranges and 0.1 arcsec for the proposal distributions of angular coordinates. The observation set contained five observations (observational timespan 5.6 days) obtained in January 1987.

In general, our MCMC methods use the Metropolis-Hastings algorithm. We start by introducing Gaussian proposal densities for a set of orbital elements and use them to generate a chain of successive samples. If the probability-density-function (p.d.f.) value corresponding to the orbital elements is higher than the one from the previous iteration, we accept the proposed sample as the next value to originate from. If the p.d.f. value is lower than the one from the previous iteration, we accept the proposed sample as a new origin with a probability equal to the ratio of the two probabilities (previous and current one). The algorithm is run for many iterations until the initial state is "forgotten" (burn-in phase). The sample states, which are drawn during the burn-in phase, are then removed from the full sample. The remaining set of accepted trials represents the unbiased distribution of the orbital elements.

To assess the unbiased orbital-element distributions, one starts by generating a single sample orbit with any method (Gauss method, statistical ranging and so on). In the next step our new method, phase-space MCMC, is used to sample the orbital-element p.d.f. in Keplerian or Cartesian elements. The Metropolis-Hastings algorithm, discussed previously, is then applied to obtain distributions of orbital elements and O-C residuals.

One particular method for obtaining a single orbit, that we have been investigating, is MCMC ranging. This method can currently be used to estimate the extents of the orbital-element distributions (p.d.f. sampling under development). From the full set of observations, we select two and generate random topocentric distances (ranges). We assume Gaussian proposal densities for both ranges and angular coordinates and introduce random Gaussian deviates to them. We then compute a trial orbit and accept or reject it according to MCMC acceptance criteria described before.

The first series of tests on a number of objects with varying standard deviations have indicated that the proper choice of standard deviations for the proposal densities in MCMC methods is a critical factor for obtaining high efficiency and large coverage of the distribution. In particular, MCMC ranging is sensitive to changes in the standard deviations of the ranges. In addition to being method-dependent, the optimum standard deviations also depend on the dynamical classification of the object. We plan to optimize the MCMC methods by, for instance, introducing different types of proposal densities. Finally, we assess the development of MCMC methods for asteroid mass estimation based on mutual perturbations among asteroids.

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