LIGHTCURVE INVERSION USING MARKOV-CHAIN MONTE-CARLO METHODS K. Muinonen, Observatory, University of Helsinki, P.O. Box 14, FI-00014 U. Helsinki, Finland, Karri.Muinonen@helsinki.fi.

Introduction: In asteroid lightcurve inversion, the shape and spin of an asteroid as well as its scattering properties are solved for. We develop novel Markov-Chain Monte-Carlo methods (MCMC) for the statistical inversion of asteroid spins and shapes in the case of limited numbers of and/or sparsely distributed disk-integrated photometric observations.

In conventional lightcurve inversion, the shape model is a convex polyhedron, where the free parameters are either the individual polyhedron facet areas or the coefficients of the spherical-harmonics series describing the Gaussian curvature of the surface [1]. Spin and shape models can be derived using relative photometry by applying simplified scattering laws, such as a combination of the Lommel-Seeliger and Lambert laws. Extensive results of asteroid lightcurve inversion have been reported in, e.g., [2]. For more sophistigated scattering laws, the reader is refered to [3,4,5]. The scattering properties are usually assumed to be homogeneous over the surface.

Numerical methods: We have developed three MCMC inverse methods of gradually increasing complexity. First, we make use of single-parameter biaxial sphere-cylinder shapes consisting of finite cylinders with hemispherical caps. By introducing suitable additional plane elements, we obtain two-parameter triaxial sphere-cylinder-plane-element shape models [6]. Second, we utilize axisymmetric convex shapes composed of a number of interconnected conical frustums. We integrate analytically the disk-integrated brightnesses of the conical frustums for the combined Lommel-Seeliger and Lambert scattering law, thus efficiently reducing the computing times in MCMC inversion. Again, by introducing suitable plane elements, we introduce triaxial shape models that are no longer axisymmetric. Third, we make use of general convex shapes described using a finite number of triangles [7].

Whereas conventional lightcurve inversion consists of two parts, that is, the derivation of the normal-vector distribution and the subsequent derivation of the convex shape from the normal vectors, in MCMC inversion, the convex shape solutions are directly sampled (as in simplex inversion, [7]). There are four parameters for the spin characteristics: the rotational period, the ecliptic longitude and latitude of the rotational pole, and the rotational phase of the object at a given time. For the general convex shape model, the shape is specified using triangles with the Cartesian coordinates of the nodes as free parameters. Altogether, there are 3+3N free parameters where N is the number of nodes, the rotational phase becoming redundant because of the general shape model. For the sphere-cylinder and axisymmetric convex shape models, the rotational phase remains as one of the free parameters. The initialization of the sampling can be accomplished, e.g., by using prolate spheroids. For a detailed description of MCMC methods, the reader is refered to [8].

The MCMC approach allows for a flexible incorporation of conditions on the shapes to be sampled. For the general convex model, at each sampling step, the convexity of the shape is verified, returning a rejection for concave solutions. Solutions are constrained into a realistic regime in radial distances, that is, only radial distances within [0.3, 1.0] are presently accepted. Solutions are further constrained by the requirement that the triangle mesh be mathematically well defined.

Results: We have applied the MCMC inversion methods to the limited lightcurve observations of the near-Earth objects 2003 MS_2 and (1981) Midas, and to the extensive observations of (1580) Betulia. In all cases, we have succeeded in obtaining realistic spin and convex shape solutions within reasonable computing times (using often more than 100 free parameters). In order to validate the MCMC methods, we will apply the MCMC methods to the extensive lightcurve observations of the main-belt asteroid (951) Gaspra.

Conclusion: We have developed MCMC inversion methods for deriving asteroid spins, shapes, and scattering properties from photometric lightcurve observations using simplified and general convex shapes. With the help of the novel methods, we have successfully assessed both limited and extensive lightcurve observations of three near-Earth objects. In the future, we plan to study the MCMC inversion for concave shapes.

References: [1] Kaasalainen M. et al. (2001) *Icarus, 153*, 37-51. [2] Torppa J. et al. (2003) *Icarus, 164*, 346-383. [3] Muinonen K. et al. (2002) *Asteroids III*, 123-138. [4] Parviainen H. and Muinonen K. (2007) *JQSRT, 106*, 398-416. [5] Parviainen H. and Muinonen K. (2008) this meeting. [6] Muinonen K. et al. (2007) *Proceedings of IAU Symposium No 236, Near-Earth Objects, our Celestial Neighbors: Opportunity and Risk*, 309-320. [7] Muinonen K. and Torppa J. (2007) *Tenth Conference on Electromagnetic & Light Scattering*, 129-132. [8] Gilks W. R. et al. (1996) *Markov Chain Monte Carlo in Practice*, Chapman & Hall/CRC.