EFFICIENT CALCULATION OF EFFECTIVE POTENTIAL AND GRAVITY ON SMALL BODIES. A. F. Cheng[1], O. S. Barnouin[1], C. M. Ernst[1], and E.G. Kahn[1], [1]The Johns Hopkins University Applied Physics Laboratory, Laurel, MD 20723, USA (andrew.cheng@jhuapl.edu).

Introduction: Small-scale topography is key to characterizing surface morphology and geological processes on asteroids and comets as well as on planets. Such small bodies are often irregular in shape, and numerical calculations of surface gravitational and centrifugal potential as well as surface effective gravity are essential for studying the sedimentation and mass motion of surface materials. Evidence of such processes is found on all small bodies observed to date at sufficient resolution. For the two asteroids Eros and Itokawa, studied from rendezvous, the measured gravity field is consistent with a body of constant density.

Werner and Scheeres [1] introduced an exact method for calculating the gravitational potential of constant-density, polyhedral bodies as a sum over surface polygon faces and another sum over polygon edges. While this potential calculation is exact for polyhedral bodies, the polyhedral shape model is itself an approximation, as is the constant density assumption. For application to Eros and Itokawa, we have used a simpler, approximate method for calculating the effective potential and effective gravity of a small body, summing over faces of shape models in which the surface is tessellated into triangular plates [2,3].. Here we use various high-resolution, polyhedral shape models of a spheroid, Eros, and Itokawa to make quantitative comparisons of the accuracy of the effective potential and gravity calculated from the exact polyhedral method [1], the approximate method [2], and a spherical harmonic method applied outside the body. We also compare the computational burdens for these methods.

Calculation Method: The plate models are polyhedral shape models with triangular faces whose vertices are control points on the surface. The vertex vectors are $\boldsymbol{v_m}$, with $\mathrm{m} = 1, \ldots,$ vertex number.

The plate centroids, where plate n is made from vertices i, j, k, are defined by the vectors

$$\boldsymbol{R_n} = \frac{\boldsymbol{v_i} + \boldsymbol{v_j} + \boldsymbol{v_k}}{3}, \qquad n = 1, \ldots, \text{plate number}$$

Outward normal vectors are defined for each plate, such that each has length equal to twice the plate area,

$$\boldsymbol{N_n} = (\boldsymbol{v_j} - \boldsymbol{v_i}) \times (\boldsymbol{v_k} - \boldsymbol{v_i})$$

where again n is the plate number index.

The gravitational potential is a sum over plates [2]

$$\mathrm{U}(\boldsymbol{x}) = \frac{\mathrm{G}\rho}{4} \sum_{\mathrm{n}} \frac{(\boldsymbol{x} - \boldsymbol{R_n}) \cdot \boldsymbol{N_n}}{|\boldsymbol{x} - \boldsymbol{R_n}|}$$

The gravity acceleration is also a sum over plates

$$\boldsymbol{g} = -\frac{\mathrm{G}\rho}{4} \sum_{n} \left[\frac{\boldsymbol{N_n}}{|\boldsymbol{x} - \boldsymbol{R_n}|} - \frac{(\boldsymbol{x} - \boldsymbol{R_n})(\boldsymbol{x} - \boldsymbol{R_n}) \cdot \boldsymbol{N_n}}{|\boldsymbol{x} - \boldsymbol{R_n}|^3} \right]$$

If the field point is at a plate centroid, $\boldsymbol{x} = \boldsymbol{R_n}$, then the contribution of that plate to the potential U is vanishing, but the contribution to the gravity is

$$-\frac{\mathrm{G}\rho}{4} \boldsymbol{N_n} \left[\frac{3}{|2\boldsymbol{v_j} - \boldsymbol{v_i} - \boldsymbol{v_k}|} + \frac{3}{|2\boldsymbol{v_i} - \boldsymbol{v_j} - \boldsymbol{v_k}|} + \frac{3}{|2\boldsymbol{v_k} - \boldsymbol{v_i} - \boldsymbol{v_j}|} \right]$$

Conclusion: The error of our plate model calculations, when compared with exact analytic solutions, decreases as the number of faces increases and also decreases rapidly above the surface. Similar behavior is found for the Werner and Scheeres polyhedral model, which is more complex and requires several times more computation time at the same resolution. The plate model calculation yields a comparably accurate approximation to the actual potential of a small body.

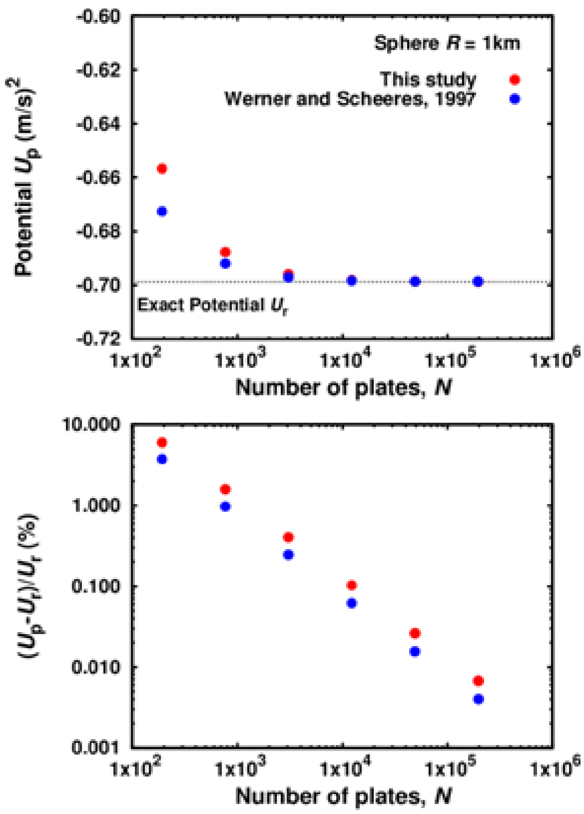

Figure 1 . Comparison of the gravitational potential U_p for a sphereical shape model to the exact potential U_r

References: [1] Werner R. A. and D. J. Scheeres (1997) CeMDA, 65, 313-344. [2] Cheng A.F. et al. (2002) Icarus 155, 51-74. [3] Barnouin O. S. et al. (2008) Icarus 198, 108-124