THE MIMAS PARADOX REVISITED plus CRUSTAL SPREADING ON ENCELADUS?  William B. McKinnon¹ and Amy C. Barr², ¹Department of Earth and Planetary Sciences and McDonnell Center for the Space Sciences, Washington University, Saint Louis, MO 63130 (mckinnon@wustl.edu); ²Department of Space Studies, Southwest Research Institute, 1050 Walnut St., Suite 400, Boulder, CO 80302.

Introduction: Enceladus presents a number of paradoxes or at least mysteries. One is the so-called Mimas paradox, which asks why Mimas is not as active as Enceladus despite its larger orbital eccentricity and closer orbit to Saturn [1]. We argue below that Mimas is simply misunderstood, and that there is no paradox. This does not mean that Enceladus is understood, however. Both the magnitude of the tidal heat flow observed, and its localization at the south pole are puzzles [2,3]. We present a simple model for crustal spreading at the south polar terrain that accounts for the heat flow and its localization.

The Mimas-Enceladus Paradox: In [3] it was pointed out that Mimas should be much more tidally heated than Enceladus, for the same tidal dissipation factor Q, which has become known as the “Mimas paradox.” The tidal heating on Enceladus is maintained by its 2:1 mean motion resonance with Dione. It is an eccentricity-type resonance [4]:

\[ n_E - 2n_D = -\dot{\phi} \]  

(1)

where \( n_E \) and \( n_D \) are the mean motions of Enceladus and Dione, respectively, and \( \dot{\phi} \) is the rate of advance of their conjunctions. In this resonance, \( \dot{\phi} = \omega_E (= 0.0044^{\circ} \text{day}^{-1}) \), where \( \omega_E \) is Enceladus’ longitude of periapsis, which results in the forcing of Enceladus’ eccentricity (0.0047 at present). The dissipation associated with mechanical distortion of the body of Enceladus as it orbits Saturn must be the ultimate source of Enceladus’ heat flow anomaly (and associated tectonics). The 2:1 resonance allows Enceladus to tap an effectively infinite reservoir of energy to drive its geological activity — the rotation of Saturn.

Mimas is also in a 2:1 mean motion resonance, with Tethys, but it is an inclination-type resonance [5]:

\[ n_M - 2n_T = -\dot{\phi} \]  

(2)

where \( n_M \) and \( n_T \) are the mean motions of Mimas and Tethys, respectively, but in this case \( \dot{\phi} = (\Omega_M + \Omega_T)/2 \), where \( \Omega_M \) and \( \Omega_T \) are the longitudes of the ascending nodes of Mimas and Tethys, respectively. The periapsis of Mimas’ orbit precesses through its ascending node (i.e., is not locked to it, or to the mean value for the two satellites) and so the eccentricity of Mimas’ orbit is not forced by the resonance. Mimas’ eccentricity (0.02) must be a primordial remnant, possibly enhanced by previous, now disrupted, eccentricity resonances or resonances [6], or due to some other dynamic event, such as a large impact (e.g., Herschel) or the formation of Saturn’s rings (and the opening of the Cassini division).

If Mimas were as dissipative as Enceladus, its \( e \) would rapidly decay toward zero, releasing a total specific energy

\[ \Delta E = \frac{GM_S e_M^2}{2a_M} \]  

(3)

where \( e_M \) and \( a_M \) are Mimas’ eccentricity and semi-major axis, respectively, \( M_S \) is the mass of Saturn, and \( G \) is the gravitational constant. For \( e_M = 0.02 \), \( \Delta E = 41 \text{kJ g}^{-1} \), which for a composition dominated by cold ice (80% ice by mass, and 100 K) implies \( \Delta T \approx 60 \text{K} \). A one-time temperature pulse of this magnitude would be of little lasting consequence to Mimas, and the energy reservoir available to Mimas would not be refilled. Its free eccentricity would stay damped.

The persistence of Mimas’ present, finite free eccentricity is logically due to the cold and geologically inert nature of this small, icy moon (as is obvious to casual inspection). The tidal \( Q \) of Mimas must be large (>100) for this eccentricity to survive for billions of years [5], but this does not seem implausible. There is no Mimas paradox.

Puzzles: The amount of tidal heating in Enceladus is ultimately limited by the torque Saturn can apply to Enceladus’ orbit [e.g., 7]. Based on the present understanding of Saturn’s \( Q \), Enceladus’ measured heat flow of \( 5.8 \pm 1.9 \text{GW} \) cannot be supplied in steady state. This is one puzzle. The other is the concentration of heat flow and tectonics at the south pole, although given Enceladus’ small size, perhaps a single, volcanically active province (compared with, say, Io) is not so unreasonable. Below we present a new mechanism — active crustal spreading — that may explain the heat flow and tectonics of the south polar terrain.

Active Spreading: Several sites have been proposed for tidal energy deposition at Enceladus’ southern polar region. One possibility is a “hot zone” in a rock core [e.g., 8]; another is along major near-surface fractures (the “tiger stripes”) [9]. We argue here for an intermediate depth: the bulk ice shell itself [10], which combines the virtues of maximal tidal flexing (if an ocean exists) and maximal volume. It is not certain if a core and ocean exist, but both are plausible from a thermal history standpoint [10,11] and are required if sufficient tidal heating is to occur in the overlying ice [e.g., 9].
A purely conductive ice shell, with an ocean maintained by tidal heating at the base of the shell, can supply a surface heat flow

\[ q = \frac{621}{D} \ln \left( \frac{T_k}{T_s} \right) \left( 1 - \frac{D}{R_E} \right) \]  

(4)

where \( T_k \) and \( T_s \) are the ocean and surface temperatures, respectively, \( R_E \) the radius of Enceladus, and \( D \) the ice shell thickness [10]. For \( T_k = 270 \) K and \( T_s = 70 \) K, \( q \) varies from 5 to 30 mW m\(^{-2}\) as \( D \) decreases from 100 to 25 km, respectively. These heat flows can be compared to \( \approx 80 \) mW m\(^{-2}\) obtained by averaging the observed thermal emission [2] over the entire South Polar Terrain (SPT) (poleward of 55° S [3]). Thus, to account for the magnitude of the thermal anomaly by viscous tidal dissipation alone, a large vertical fraction of Enceladus’ ice shell (at the SPT) must be “hot” and dissipative. This large fraction is consistent with solid state convection in the shell, in that if convection can initiate, it will advect heat into the shell so that the shell reaches a temperature where it can deform tidally, become dissipative, and heat further [10].

If convecting, the magnitude of the southern polar thermal anomaly implies rather high Nusselt numbers, \( Nu \approx 5–15 \) for shell thicknesses between 100 and 50 km (convection in much thinner shells is less plausible [10]). The stagnant lid (“lithosphere”) thickness should be \( \approx 10 \) km for \( q \approx 80 \) mW m\(^{-2}\). Such a relatively thin lid (with respect to the full convective depth) and low brittle strength (due to Enceladus’ low surface gravity of 0.11 m s\(^{-2}\)) calls the concept of stagnant lid convection into question. Rather, it is more likely that the cold surface boundary layer deforms and participates in the convective cycle (as the Earth’s oceanic lithosphere does [12]). The entire SPT is in fact intensely deformed, with good evidence for compression along its structural boundary [2] and we propose that the central tiger stripes may actually be analogous to terrestrial spreading centers. We are not proposing a direct analogue to terrestrial midocean ridge spreading and plate tectonics, whose major mechanical driver is the negative buoyancy of downgoing slabs; rather, we envision a form of active spreading whereby upwelling viscous ice is directly coupled to the actively deforming surface.

If this concept has merit, CIRS measurements of the heat flux from Enceladus’ surface as a function of distance from the tiger stripes [2] provide us with a unique opportunity to constrain the spreading rate, convective strain rate, and interior geodynamics of Enceladus. The classic solution to the heat flow from terrestrial spreading centers [e.g., 13] can be integrated over distance from the spreading center, \( x \), to yield the average heat flow between \( x = 0 \) and \( x = l \):

\[ \langle q \rangle = \frac{1}{2l} \int_0^l q(x) dx = 2k\Delta T \sqrt{\frac{u}{2\pi kl}} \]  

(5)

where \( u \) is the half-spreading velocity, \( \Delta T \) is the temperature difference from the deep interior to the surface, and \( k \) and \( \kappa \) are the thermal conductivity and diffusivity, respectively, of subsurface ice. While the boundary conditions are different in the Enceladus case (radiation vs. constant \( T_s \)), at long times (large \( l \) \( T_s \)), asymptotes to the background radiative equilibrium temperature (\( T_{eq} \)) and the temperature profile at depth is the same as in the terrestrial solution. Hence the total heat extracted (and \( \langle q \rangle \)) must also be the same in the limit of large \( l \). Using the integral relation (Eq. 5) also frees us from the details of near-surface conductivity and the precise value of \( T_{eq} \). The SPT is equivalent in area to a 270 km x 270 km square; if all four tiger stripes contribute equally to the spreading, then \( l \approx 35 \) km, and \( u \approx 3.5 \) cm yr\(^{-1}\) for \( \Delta T = 200 \) K. This further implies an average time (age) to recycle the SPT of \( \approx 1 \) Myr, and a strain rate \( \tau^{-1} \approx 3 \times 10^{-14} \) s\(^{-1}\), values that appear quite plausible.

**Conclusion:** The tiger stripes on Enceladus may represent the surface manifestation of deep mantle processes – in this case, locations where thermal buoyancy stresses due to tidal-heat-driven solid-state convection in Enceladus’ ice shell have been able to rip its rheologically weak lithosphere and allow hot ice to rise to the surface. This hypothesis is consistent with the CIRS observation that the majority of the thermal emission from the SPT comes from near the tiger stripes themselves. The observed emission and a simple active spreading model are used to derive an age for the SPT of \( \approx 1 \) Myr. This estimate should be refined with a more detailed model of surface temperature evolution near the stripes.

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