CONVECTION WITH DECAYING HEAT SOURCES: A SIMPLE THERMAL EVOLUTION MODEL.
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It can be argued that the single most important question related to the thermal evolution of a planetary interior is the degree to which a planet remembers its past history. If the thermal evolution results from a relatively inefficient transport mechanism such as conduction, then the present thermal structure will depend to a large extent on the details of the earlier history. If on the other hand an extremely efficient convective transport mechanism is assumed, the present thermal structure will depend almost exclusively on the present rate of heat generation. Assumptions of this last type have become increasingly common with the widespread acceptance of convection in planetary interiors. Two important consequences of this assumed very efficient convection are: (1) The total terrestrial heat flow is very nearly equal to the present day radiogenic heat production and therefore can be used to estimate the concentration of U, Th, and K in the earth. (2) The earth's thermal structure is in effective equilibrium with the rate of heat generation at any stage of its evolution and therefore it is sufficient for modelling evolution to use a series of steady-state models which differ from each other only in the greater heat production at earlier times. Clearly an important issue is to evaluate how realistic is the assumption that convection in the earth is sufficiently efficient to result in an equilibrium between heat production and heat flow.

We have studied the ratio of heat flow to heat production in a series of idealized numerical convection experiments in which the rate of heating is a decaying function of time. The specific model is a layer of Boussinesq fluid heated half from within and half from below, and using either a constant or a temperature-dependent viscosity. Time in the calculation is measured in units of $D^2/K$ where $D$ is the layer depth and $K$ is the thermometric diffusivity. Using values appropriate to the upper mantle ($D\approx700$Km, $K=10^{-2}$cm$^2$/sec), a calculation time of 0.25 is equal to the age of the earth. Each numerical experiment was initialized by establishing a steady state solution at the particular Rayleigh number being studied. At what we choose to call $t=0$, the heating rate begins to decay exponentially with a time constant of 0.1 ($\sim2$ Byrs) and the ratio of heat flow to total heating ($\Gamma(t)$) is monitored as a function of time. An example of the ratio $\Gamma(t)$ for three different initial Rayleigh numbers and constant viscosity is shown in the figure below. Even for the largest initial Rayleigh number considered (and therefore most vigorous convection), the model's "present" heat flow exceeds heat production by a factor greater than two. We have run similar cases with viscosity variations as great as a factor of 250 and the results are not significantly different from those shown below. These calculations are, as far as we know, the first explicit attempt to evaluate the degree to which vigorous convection can maintain a balance between instantaneous heat flow and heat production. We find in all cases a significantly higher heat flow than contemporaneous heat production but must also emphasize that the actual ratios we obtain are model dependent particularly in terms of initial conditions used.
The reason for the long thermal adjustment times implied by our calculations is that as the heat production decreases, the temperature of the entire layer must also decrease and for the central cores of the convection cells this cooling requires that heat be conducted away across flow lines. Therefore even at high Rayleigh numbers a significant part of the thermal evolution still depends on the relatively inefficient conduction mechanism. This conclusion seems sufficiently general to apply also to the more complicated convection process in the mantle. It seems then most reasonable to use heat flow data as an upper bound on the bulk concentration of U, Th, and K in the earth but not as an accurate estimator. It also follows that quasi-steady models of planetary thermal evolution will tend to underestimate internal temperatures.