RELATIVE VELOCITIES OF PLANETESIMALS. W. M. Kaula, University of California, Los Angeles, California 90024.

After initial formation of planetesimals by gravitational instability, a problem of solar system origin is the manner in which planetesimals came together to form planets. The mass growth of planetesimals is necessarily coupled to growth in their relative velocities due to mutual perturbations, which in turn acts to inhibit their rate-of-growth. The obvious solution to this problem is Monte Carlo models for the coupled growth, such as that by Greenberg, et al. (1). However, computer limitations have restricted these models to particular circumstances.

One feature which the Monte Carlo models have produced is that the number density of planetesimal masses can be approximated by an exponential law,

\[ n(m) \, dm = m^{-q} \, dm \]  

(1)

Because velocities are influenced by much more frequent events (close approaches) than are masses (collisions), it is plausible that velocities could approach an equilibrium state for some prescribed mass distributions such as those characterized by equation (1). An equilibrium velocity state is one in which the rate of velocity excitation due to close approaches and elastic collisions, \( 1/T_E \), equals the rate at which velocities are damped by inelastic collisions, \( 1/T_D \).

The only equilibrium velocity theory, by Safronov (2), applies to different circumstances than those generated by the Monte Carlo models: in particular, for \( q<2 \), rather than \( q>2 \). Hence the theory was extended to include cases where the minimum mass \( m_n > 0 \), as is necessary for \( q>2 \). In addition, advantage was taken of computer capability to remove several approximations in the Safronov theory, such as neglect of dissipation factor dependence on impact velocity.

The results of one comparison to the Safronov theory are shown in Figure 1, which pertains to a planetesimal population in which most of the mass is in the largest body. The velocities \( v \) are related to the gravitational energy of the largest body through the parameter \( \theta \):

\[ v^2(m) = \frac{Gm_1}{r_1 \theta(m)} \]  

(2)

where \( m \) and \( r \), are the mass and radius of the largest body. In agreement with Safronov (2), the parameter \( \theta \) is of order unity.

The results of a comparison to the Greenberg et al. (1) Monte Carlo model are shown in Figure 2, which pertains to a planetesimal population in which most of the mass is in the smallest bodies. The same theory obtains, in agreement with (1), that the parameter \( \theta \) is orders-of-magnitude larger for this very different case.

These results encourage the development of models in which the mass growth and velocity growth of the planetesimal population are carried forward in alternate steps, thus allowing significant computer economies. The most fundamental problem outstanding appears to be the velocity and mass ratio dependence of the energy dissipation upon collision.

This work has been submitted for publication (3).
RELATIVE VELOCITIES

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\begin{align*}
q &= 5/3 \\
m_1 &= 1.26 \times 10^{22} \text{ g} \\
m_n &= 0.0
\end{align*}
\]

Fig. 1 Points S are from Safronov (2). Curve MA is a theory intended to approximate Safronov's. Curve EP is a more precise theory.

\[
\begin{align*}
q &= 2.5 \\
m_1 &= 1.7 \times 10^{24} \text{ g} \\
m_n &= 1.6 \times 10^{15} \text{ g}
\end{align*}
\]

Fig. 2 Points on dashed line G are from Greenberg et al. (1). Curves MA and EP are generated by same theories as in Fig. 1.