DYNAMO AMPLIFICATION OF AMBIENT MAGNETIC FIELDS

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Large-scale magnetic fields of planetary objects are thought to be generated by the hydromagnetic dynamo process[1-4]. In this process motions of an electrically conducting fluid interact with a magnetic field through the Lorentz force, $c^{-1}\underline{v} \times \underline{B}$, to produce the electrical currents which generate the magnetic field. The general efficacy of this process and its applicability to magnetic field generation in planetary bodies are well established. A fully quantitative and detailed model of dynamo generation in any particular object has not been produced; but many of the overall properties of the generation mechanism are well known.

Consider the regenerative behavior of a specified dynamo fluid velocity field as the amplitude of the velocity increases from zero[5-7]. At zero velocity, of course, no regeneration occurs and any initial magnetic field decays exponentially to zero through ohmic dissipation of the associated electrical currents. If the amplitude of the fluid velocity is finite but small (where small will be defined presently), the field will still decay to zero exponentially with time, although some components of the field may have their decay rate slowed. Imagine that the amplitude of the fluid velocity continues to grow. Above some finite value of the fluid velocity amplitude a critical transition occurs and the magnetic field ceases to decay and instead grows exponentially with time (The growth of the field continues until the magnetic stresses reach a value at which they can slow the fluid motion and stop further growth of the magnetic field.)

The exact amplitude at which the transition from non-regenerative fluid flow to regenerative dynamo action occurs depends on the geometrical structure of the fluid flow. As a general rule of thumb the transition occurs when

$$R_m \gtrsim 10, \qquad (1)$$

where the magnetic Reynolds number is $R_m \equiv v l \eta^{-1}$; v is the scale of the fluid velocity, l is the length scale of the magnetic field and η is the magnetic diffusivity ($\eta = c^2/4\pi\sigma$, c is the speed of light and σ is the fluid's electrical conductivity). The existence of this well-defined transition to the regenerative state places a constraint on the internal fluid properties of any object known to maintain its own magnetic field; so that it is generally held that any internal dynamo magnetic field implies a relation of the form given in equation (1). Thus it has been suggested[8] that detection of internal magnetic fields in Jovian planet satellites would imply equation (1) for their interiors.

This paper considers the fact that some objects, such as Jovian planet satellites, are embedded in substantial ambient magnetic fields. For example, consider the large satellites of Jupiter: Io, Europa, Ganymede, and Callisto. These reside at distances from Jupiter which are 5.9, 9.4, 15, 26.4 Jovian radii respectively. Taking Jupiter's dipole surface field to be about 10 Gauss, we find the ambient magnetic fields at the locations of the satellites to be respectively 4900γ, 1200γ, 300γ, and 50γ. The listed field values are crude estimates, but are close enough to measured values to indicate that these ambient magnetic fields are comparable to or larger than, say, the apparently internally generated magnetic field of Mercury. These fields are uniform on the scale size of the satellite and are aligned with the satellite's axes.

We will show in this paper that the existence of such an ambient magnetic

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field relaxes the condition in equation (1) and allows an object to produce an internally generated magnetic field which is indistinguishable from the usual internally generated dynamo fields. The reason that the criterion in equation (1) is relaxed is that in the presence of an appreciable ambient magnetic field, the dynamo process need not be fully self regenerating. The ambient field serves as a source which the dynamo fluid motions can locally amplify.

In these few brief pages we will simply indicate the form of the solutions through a dimensional analysis and reserve the formal calculation for the more complete presentation to follow later. For simplicity we use the regenerative flow consisting of differential rotation and cyclonic convection. (The principle involved here is general and will apply in similar form to other dynamo motions.) The relevant dynamo equations are familiar and are given in most of the cited references. In schematic form:

$$\frac{\partial A_\phi}{\partial t} - \eta\nabla^2 A_\phi = \Gamma B_\phi, \tag{2}$$

$$\frac{\partial B_\phi}{\partial t} - \eta\nabla^2 B_\phi = \gamma(B_p + B_o), \tag{3}$$

$$\underline{B_p} = \underline{\nabla} \times \underline{A_\phi}. \tag{4}$$

In the usual way B_ϕ represents the toroidal part of the dynamo field, B_p represents the poloidal part and A_ϕ is a vector potential; Γ and γ are derived from the cyclonic convection and the differential rotation. B_o represents the given external magnetic field which is present independently of the dynamo process. Writing these equations in dimensional form for the steady state and setting the determinant of the resulting algebraic equations to zero, we find the steady state condition

$$N = \frac{\gamma\Gamma R^3}{\eta^2} = \frac{B_p}{B_p + B_o}. \tag{5}$$

N is the commonly defined dynamo number and is related to the magnetic Reynolds numbers associated with the differential rotation and cyclonic convection.

Note that, in the case when the external magnetic field, B_o, vanishes, equation (5) reduces to N=1, which is the usual criterion for a stationary, dynamo generated magnetic field. However in the presence of an external magnetic field, that is when B_o is not zero, the criterion for generation of a stationary magnetic field, with its dipole moment aligned parallel to the ambient magnetic field, always has N, the dynamo number, less than unity. The actual values of N that yield stationary field states depend on the value of the ratio B_p/B_o, that is on the relative strengths of the generated and external fields. The dynamo number is equivalent to the magnetic Reynolds number in equation (1). Thus in the presence of an ambient magnetic field the criterion in equation (1) is relaxed.

The complete, formal analysis is carried out by solving the relations (2) - (5), for field generation by differential rotation and cyclonic convection, or an analogous set of dynamo equations appropriate to another regenerative fluid flow. However, from equation (5) one already can see the shape of the result. If, for example, we observe an internally generated magnetic field associated with a Jovian satellite that has, say, 10% of the intensity of the ambient field, then the dynamo number of the internal fluid motion will be of the order of 10% of the value needed for a truly self-regenerating dynamo.

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