
The rise of magma and gas from a source region to the surface of a planet can be simulated by solving the appropriate hydrodynamic equations. Previous treatments\(^1,2\) have essentially assumed a constant mass ratio of exsolved gas to liquid magma at all depths. We treat the general case where gas is exsolved as a function of pressure (and hence depth) from a rising magma to form bubbles. The gas is assumed to be H\(_2\)O on Earth and CO on the Moon.\(^3\) Consideration of the solubility of these gases in silicate liquids\(^4\) and of the thermodynamics of CO production in lunar basalts\(^5\) shows that gas production is appreciable only within a few hundred meters of the surface in most terrestrial eruptions and only within a few km of the surface on the Moon. In each case these depths are only a small fraction of the total rise distance in most eruptions and so the mass eruption rate of magma is dominated by fissure or conduit width, magma viscosity and magma and crustal density. The rise velocity \(u_s\) of a gas-free liquid of density \(\rho_L\) and viscosity \(\eta\) on a planet with gravity \(g\) is given by:

\[
u_s = \left(\frac{2A\eta}{\rho_L^2}ight)\left\{\sqrt{1 + \frac{gr^3(\rho_L - \rho_p)}{A^2 \eta^2}} - 1\right\}
\]  

(1)

where \(A = 8\) for a circular conduit of radius \(r\) and \(A = 3\) for a fissure of half width \(r\). It is stressed that, in practice, the conduit radius, \(r\), is not likely to be constant with depth. The value required in eq. (1) is a suitably weighted average: even a short vertical length with very small \(r\) will significantly reduce \(u_s\). Also, the upward velocity of magma near the surface may be much less than \(u_s\) if the conduit system widens out into a lava lake, as a result of the need to satisfy the continuity equation. \(K\) is a constant dictated by wall friction and close to 0.01. In the simple case of a liquid less dense than the surrounding crust, \(\rho_L\) is the crustal density \(\rho_C\). If the magma is denser than the crust it cannot rise closer to the surface\(^6\) than a depth \(D_L\) given by:

\[
D_L = D_p \frac{(\rho_C - \rho_p) - D_M(\rho_M - \rho_p)}{\rho_C - \rho_M}
\]  

(2)

where \(\rho_C\) and \(\rho_M\) are the crustal and mantle densities and \(D_C\) and \(D_M\) are the crustal thicknesses and the distance of the magma source below the crust/mantle boundary. If the liquid has in fact risen to a height \(D_x\) above this boundary, then the eruption is described by equation (1) with \(\rho_p\) replaced by \(\rho_{p_x} = (\rho_C D_C + \rho_M D_M) / (D_x + D_M)\). Thus the rate of extrusion decreases with time.\(^7\)

Realistic values\(^6\) of lunar basalt density and depth of origin indicate that, in the simple case of a circular conduit, \(u_s\) is likely to be in the range 3 to 70 m/s through conduits with radii between 1 and 100 m leading to mass eruption rates between roughly \(10^6\) and \(10^{10}\) kg/s (say 0.3 km\(^3\)/year to 6 km\(^3\)/hr.). Of course, much higher mass eruption rates are possible for a given rise velocity if the eruption is through a long fissure rather than a circular conduit. A lower limit of about 3 m/s for \(u_s\) is set by the requirement that the magma must not cool to near its solidus before it reaches the surface.\(^8,9\) Table I shows some sets of values of \(r\), \(u_s\) and \(\dot{m}\) for \((\rho_H - \rho_L) = 200\) kg/m\(^3\), \(\eta = 100\) poise and \(\rho_L = 3000\) kg/m\(^3\). In the gas-free case, a liquid fountain would form over the vent reaching a maximum height given by \(u_s^2/2g\). Since there is no direct evidence for lunar mass eruption rates much greater than \(10^6\) kg/s, \(u_s\) is unlikely to be much greater than 40 m/s implying fountain...
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height up to 500 m. Even under the most favorable conditions of detachment, droplets are unlikely to be thrown more than one kilometer from such fountains. Thus unless mass eruption rates very much greater than any yet proposed are invoked, the existence of large pyroclastic deposits or blankets implies the presence of gas in the lunar magmas. Equivalent calculations show that in gas-free flood basalt eruptions on Earth, exit velocities up to 250 m/s are possible at very high mass eruption rates, implying liquid fountains up to 3 km high. More commonly, speeds up to 100 m/s and heights up to 500 m are expected.

Gas exsolution causes an increase in magma rise velocity near the surface, but the mass flow rate, \( \dot{m} \), is still given adequately by \( \dot{m} = \rho_u u_0 X \) where \( X = \alpha \pi \) for a circular conduit and \( X = 2L \) for a fissure of length \( L \). On Earth, exsolution of 5 weight % water can lead to eruption velocities up to 550 m/s. For mass eruption rates between \( 10^6 \) and \( 10^8 \) kg/s (i.e., \( 10^6 \) to \( 10^8 \) m³/hr.) numerical solutions show that the exit velocity in the vent, \( u_e \), is much more strongly dependent on water (or other volatile) content than on any other variable and is given to within about 15% accuracy by:

\[
\begin{align*}
\dot{u}_e &= 125 n_+^{0.75} \\
\end{align*}
\]

where \( u_e \) is in m/s and \( n_+ \) is the total water content of the magma in the chamber of reservoir in weight percent. Chemical considerations imply that lunar basalts may have produced by to 750 ppm CO during their eruption. Numerical results show that the eruption velocity \( u_e \) of gas and small pyroclasts can be approximated to within 10% by:

\[
\begin{align*}
\dot{u}_e^2 &= \dot{u}_s^2 + 2.3n_p \\
\end{align*}
\]

where \( u_s \) is the magma rise velocity at great depth where no gas has been produced, given by eq.(1), \( n_p \) is the weight fraction of CO produced in ppm and both \( u_e \) and \( u_s \) are in m/s. Clearly, if \( u_s \) is unlikely to be more than 70 m/s as discussed earlier, then if \( n_p \) is restricted to 750 ppm, \( u_e \) is not likely to be more than 80 m/s and the maximum range of pyroclasts would be \( R = 4 \) km. A value of \( n_p = 3000 \) ppm is needed to produce \( R = 7 \) km while \( n_p = 1\% (10^4 \) ppm) would produce \( R = 17 \) km.

The simplest method of producing widely dispersed pyroclastic deposits from magmas releasing small amounts of gas is to assume that the magma disrupts into pyroclasts with a range of sizes. Large clasts decouple quickly from the erupting jet above the vent and so the effective gas content of the remaining mixture of gas and small droplets is increased. The segregation must occur early in the gas expansion process to be effective. This process, together with the effects of bubble coalescence, is discussed in a separate paper.11

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TABLE I. For definitions see text.

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<td>9.7</td>
<td>20.3</td>
<td>35.8</td>
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<tr>
<td>( \dot{m} ) (kg/s)</td>
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<td>( 8.3 \times 10^5 )</td>
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<td>( 3.0 \times 10^8 )</td>
<td>( 6.2 \times 10^9 )</td>
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